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ВОЈНОТЕХНИЧКИ ГЛАСНИК

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САДРЖАЈ

С О Д Е Р Ж А Н И Е

CONTENTS

70 YEARS OF THE MILITARY TECHNICAL COURIER

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DOI: 10.5937/vojtehg70-40505; https://doi.org/10.5937/vojtehg70-40505

ARTICLE TYPE: editorial

Abstract:

This Editorial marks the jubilee which the scientific journal Military Technical Courier celebrates in 2022 - 70 years of regular and continuous publication. On this occasion, the Editorial Office expresses gratitude to institutions and individuals who have contributed most to the progress and success of the Journal.

Key words: Military Technical Courier, scientific journal, jubilee, seventieth anniversary, acknowledgments.

In 2022, the *Military Technical Courier*, a scientific journal of the Ministry of Defence and Serbian Armed Forces, marks the 70th anniversary of its regular and continuous publishing.

The Journal was founded by a decree of the Chief of the General Staff of the Yugoslav Army in August 1952 in order to continue the tradition of five reviews of military branches and services (Artillery Courier, Tank Courier, Military Engineering Courier, Courier of Communications in the Yugoslav Army and Logistics and Support of the Yugoslav Army) which had been covering tactics and technique issues from 1947 to 1952. Chief of the General Staff's decree of 16th December 1952 founding the first editorial board that “has a directive to be responsible for the review editing” was followed by the first issue of the *Military Technical Courier* in January 1953.

The *Military Technical Courier* follows in the footsteps of the outstanding military technical publications published in our previous countries (in the Kingdom of Serbia and Kingdom of Yugoslavia): Artillery

Gaćeša, N. 70 years of the Military Technical Courier, pp.789-796

and Engineering Courier (1905-1906), Artillery Courier (1926-1932), Infantry and Artillery Courier (1933-1941), Engineering Courier (1929-1940), Aviation Courier (1927-1941) and Nautical Courier (1933-1940). (Gaćeša, 2012), (Gaćeša & Jovanović, 2012)

Today, the Journal has an international character. It covers scientific and professional fields within the educational-scientific field of Natural-Mathematical Sciences, as well as within the educational-scientific field of Technical-Technological Sciences, and especially the field of defense sciences and technologies. It publishes theoretical and practical achievements leading to professional development of all members of Serbian, regional and international academic communities as well as members of the military and ministries of defence in particular.

It is indexed in important world databases: DOAJ - Directory of Open Access Journals, Redalyc - Scientific Information System, RINC - Russian Index of Scientific Citation, ISC - Islamic Citation Index, etc. It is a member of OASPA (Open Access Scholarly Publishers Association) and COPE (Committee on Publication Ethics). (Gaćeša, 2017)

The Ministry of Education, Science and Technological Development of the Republic of Serbia categorizes "*Military Technical Courier*" with the categories M51 (reputed national journal) and M52 (quality national journal). The Center for Evaluation in Education and Science (CEON/CEES) categorizes the Journal with category A3 (journal of global potential and top publishing quality).

On the occasion of this year's jubilee, the Editorial Office would like to express their special thanks for the trust and help given to us in the past period by our colleagues from the Redalyc Scientific Information System. Redalyc, supported by the Autonomous University of the State of Mexico (UAEM), has indexed the *Military Technical Courier* in its database guided by its principles of supporting open-access and non-profit journals. Indexing in Redalyc enables a new, exceptional visualization of our journal, given that the indexed articles are visible in multiple formats: Intelligent viewer, Mobile viewer, HTML, iPDF, PDF, and ePUB.

The Editorial Office would like to thank their long-term collaborators for their dedicated and hard work: Dobrila Miletić (reader for the Serbian language), Jasna Višnjić (translator and proofreader for the English language), Karina Avagjan (translator and proofreader for the Russian language), as well as Radovan Mišković (graphic editor of the Journal from the Military Printing House).

The Office is also grateful to all the authors who have put their trust in the *Military Technical Courier*, as well as to previous editors, involved reviewers and members of the editorial boards whose longtime commitment obligates the current and future members of the Editorial Office to work persistently on the further improvement of the Journal.

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СЕМИДЕСЯТИЛЕТИЕ ЖУРНАЛА «ВОЕННО-ТЕХНИЧЕСКИЙ ВЕСТНИК»

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ВИД СТАТЬИ: редакционная статья

Резюме:

Данная редакционная статья представляет обзор, посвященный юбилею научного журнала «Военно-технический вестник», который в 2022 году отмечает 70-летие непрерывной редакционно-издательской и публицистической деятельности. В связи с этой знаменательной датой редакция журнала выражает глубокую благодарность организациям и лицам, которые способствовали прогрессу и успеху журнала на протяжении его существования.

Ключевые слова: Военно-технический вестник, научный журнал, юбилей, семидесятилетие, благодарность.

«*Военно-технический вестник*» – журнал Министерства обороны и Вооруженных сил Республики Сербия в 2022 году отмечает 70-летие своей непрерывной редакционно-издательской и публицистической деятельности.

Журнал «*Военно-технический вестник*» учрежден согласно Приказу начальника Генштаба ЮНА (Югославская Народная Армия), от 27-ого августа 1952 года. И он успешно продолжил работу периодических изданий: «Артиллерийский вестник», «Танковый вестник», «Военно-инженерный вестник», «Вестник связей Югославской армии» и «Тыл и снабжение Югославской армии», выпускаемых в период с 1947 по 1952гг. В соответствии с Приказом Генштаба ЮНА от 16-ого декабря 1952 года создан и первый Редакционный совет – орган руководящий журналом. Первый номер журнала «*Военно-технический вестник*» вышел в январе 1953 года.

«*Военно-технический вестник*» продолжает традицию выдающихся публикаций военно-технического характера, которые выпускались еще во времена существования Королевства Сербия и Королевства Югославия, в виде журналов: «Инженерно-артиллерийский вестник» (1905-1906), «Артиллерийский вестник» (1926–1932), «Сухопутно-артиллерийский вестник» (1933–1941), «Инженерный вестник» (1929–1940), «Вестник военно-воздушных сил» (1927–1941), «Вестник военно-морских сил» (1933–1940). (Gaćeša, 2012), (Gaćeša & Jovanović, 2012)

Сегодня журнал имеет международный характер. Он охватывает научные и профессиональные сферы в рамках учебно-научной области естественно-математических наук, а также в рамках учебно-научной области технико-технологических наук, особенно в области оборонных наук и технологии. В журнале публикуются теоретические и практические достижения, которые способствуют повышению квалификации представителей сербского, регионального и международного академического сообщества, особенно служащих Министерств Обороны и Вооружённых сил.

Журнал индексируется в важных мировых базах данных: ДОАЈ - Директория журналов открытого доступа, Redalyc − Научно-информационная система, РИНЦ − Российский индекс научного цитирования, ISC − Islamic Citation Index и др. Журнал является членом OASPA (Ассоциация издателей научных изданий открытого доступа) и COPE (Комитет по этике публикаций). (Gaćeša, 2017)

Министерство образования, науки и технического развития Республики Сербии присваивает «*Военно-техническому вестнику*» категории М51 (ведущий журнал государственного значения) и М52 (высококачественный национальный журнал). Центр поддержки развития образования и науки (ЦПРОН/CEON) присваивает журналу категорию А3 (журнал с глобальным потенциалом и высочайшим издательским качеством).

По случаю юбилея этого года редакция выражает особую благодарность за доверие и помощь, оказанную в прошедший период нашими коллегами из Научно-информационной системы Redalyc. Redalyc, поддерживаемый Автономным университетом штата Мехико, проиндексировал «*Военно-технический вестник*» в своей базе данных, руководствуясь принципами поддержки общедоступных и некоммерческих журналов. Индексирование в Redalyc обеспечивает новую исключительную визуализацию нашего журнала, учитывая, что проиндексированные статьи видны в нескольких форматах: интеллектуальная программа просмотра, мобильная программа просмотра, HTML, iPDF, PDF, ePUB.

Редакция благодарит своих многолетних сотрудников за самоотверженную и усердную работу: Добрилу Милетич (корректор сербского языка), Ясну Вишнич (переводчик и корректор английского языка), Карину Авагян (переводчик и корректор русского языка), а также Радована Мишковича (графический редактор журнала из «Военной типографии»).

Особую благодарность редакция выражает всем авторам, оказавшим доверие журналу „*Военно-технический вестник*“, публикуя в нем свои работы, а также предыдущим редакторам, рецензентам и членам редколлегии самоотверженный долголетний труд которых обязывает всех членов редколлегии, как в настоящее время, так и в будущем, усердно трудиться в направлении совершенствования журнала.

70 ГОДИНА „ВОЈНОТЕХНИЧКОГ ГЛАСНИКА"

Небојша Н. Гаћеша

Универзитет одбране у Београду, Војна академија,
Београд, Република Србија

КАТЕГОРИЈА (ТИП) ЧЛАНКА: уводник

Сажетак:

Уводник представља осврт на јубилеј који научни часопис „Војнотехнички гласник" обележава у 2022. години – седамдесет година редовног и непрекидног излажења. Тим поводом Редакција се захваљује установама и лицима која су допринела напретку часописа у протеклом периоду.

Кључне речи: Војнотехнички гласник, научни часопис, јубилеј, седамдесета годишњица, захвалност.

Војнотехнички гласник, научни часопис Министарства одбране и Војске Србије, у 2022. години обележава седамдесету годишњицу свог редовног и непрекидног излажења.

Часопис је основан Наредбом начелника Генералштаба ЈНА, од 27. августа 1952. године, као настављач традиције пет дотадашњих часописа родова и служби: *Артиљеријског гласника, Тенковског гласника, Инжињеријског гласника, Гласника веза ЈА и Позадине и снабдевања ЈА*, који су излазили у периоду од 1947. до 1952. године, и објављивали садржаје из тактике и технике. Наредбом начелника Генералштаба ЈНА, од 16. децембра 1952. године, формиран је и први уређивачки одбор „који у директивном смислу руководи уређивањем часописа", а први број *Војнотехничког гласника* изашао је из штампе у јануару 1953. године.

Војнотехнички гласник наставља и традицију публикација војнотехничког карактера које су излазиле у нашим ранијим државама (Краљевини Србији и Краљевини Југославији): Артилериско-инжињерски гласник (1905–1906), Артиљериски гласник (1926–1932), Пешадиско-артилериски гласник (1933–1941), Инжињеријски гласник (1929–1940), Ваздухопловни гласник (1927–1941) и Морнарички гласник (1933–1940). (Gaćeša, 2012), (Gaćeša & Jovanović, 2012)

Часопис данас има међународни карактер. Обухвата научне, односно стручне области у оквиру образовно-научног поља природно-математичких наука, као и образовно-научног поља техничко-технолошких наука, а нарочито области одбрамбених наука и технологија. Објављује теоријска и практична достигнућа која доприносе усавршавању свих чланова српске, регионалне и међународне академске заједнице, а посебно припадника војски и министарстава одбране.

Индексиран је у значајним светским базама: DOAJ – Directory of Open Access Journals, Redalyc – Scientific Information System, РИНЦ – Руски индекс научног цитирања, ISC – Исламски цитатни индекс и др. Члан је OASPA (Open Access Scholarly Publishers Association) и COPE (Committee on Publication Ethics). (Gaćeša, 2017)

Министарство просвете, науке и технолошког развоја Републике Србије категорише *Војнотехнички гласник* категоријама М51 („врхунски часопис од националног значаја") и М52 („истакнути национални часопис"), а Центар за евалуацију у образовању и науци (ЦЕОН) доделио је часопису категорију А3 („часопис међународног потенцијала и врхунског издавачког квалитета").

Поводом овогодишњег јубилеја Редакција се посебно захваљује на указаном поверењу и помоћи коју су у протеклом периоду пружиле колеге из Научно-информационог система Redalyc. Redalyc, којег подржава Autonomous University of the State of Mexico (UAEM), индексирао је *Војнотехнички гласник* у своју базу, вођен принципима подршке часописима који имају отворени приступ и непрофитну оријентацију. Индексацијом у Redalyc-у нашем часопису је омогућена нова изузетна визуелизација, с обзиром на то да су индексирани чланци видљиви у више формата: Intelligent viewer, Mobile viewer, HTML, iPDF, PDF, ePUB.

Редакција се захваљује на преданом и вредном раду и својим дугогодишњим сарадницима: Добрили Милетић (лектору за српски језик), Јасни Вишњић (преводиоцу и лектору за енглески језик), Карини Авагјан (преводиоцу и лектору за руски језик), као и Радовану Мишковићу (графичком уреднику часописа из „Војне штампарије").

Такође, захвалност дугујемо свим ауторима који су указали поверење *Војнотехничком гласнику*, као и претходним уредницима, те ангажованим рецензентима и члановима уређивачких одбора, чији дугогодишњи пожртвовани рад обавезује садашње и будуће чланове Редакције да истрајно раде на даљем унапређењу часописа.

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ОРИГИНАЛНИ НАУЧНИ РАДОВИ

ОРИГИНАЛЬНЫЕ НАУЧНЫЕ СТАТЬИ

ORIGINAL SCIENTIFIC PAPERS

CERTAIN INTEGRALS INVOLVING GENERALIZED MITTAG-LEFFLER TYPE FUNCTIONS

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FIELD: Mathematics
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Abstract:

Introduction/purpose: Certain integrals involving the generalized Mittag-Leffler function with different types of polynomials are established.

Methods: The properties of the generalized Mittag-Leffler function are used in conjunction with different kinds of polynomials such as Jacobi, Legendre, and Hermite in order to evaluate their integrals.

Results: Some integral formulae involving the Legendre function, the Bessel Maitland function and the generalized hypergeometric functions are derived.

Conclusions: The results obtained here are general in nature and could be useful to establish further integral formulae involving other kinds of polynomials.

Key words: Mittag-Leffler function, Generalized hypergeometric function, Bessel Maitland function, Jacobi polynomials, Hermite polynomials.

Introduction

This paper follows the lines of the companion paper (Haq et al, 2019) involving the generalized Galuè-type Struve function in which the same topics are dealt here with the generalized Mittag-Leffler functions. As it is well known, a special function:

$$\mathcal{E}_{\upsilon}(z) = \sum_{k=0}^{\infty} \frac{(z)^k}{\Gamma(\upsilon k + 1)}, \quad z, \upsilon \in \mathbb{C}, \ \Re(\upsilon) > 0, \tag{1}$$

and its general form

$$\mathcal{E}_{\upsilon,\omega}(z) = \sum_{k=0}^{\infty} \frac{(z)^k}{\Gamma(\upsilon k + \omega)}, \quad z, \upsilon \in \mathbb{C}, \ \Re(\upsilon) > 0, \Re(\omega) > 0, \tag{2}$$

are called Mittag-Leffler functions (Erdelyi et al, 1953a), $\mathbb{C}$ being the set of complex numbers. The former was established by Mittag-Leffler (Mittag-Leffler, 1903) in connection with his method of summation of some divergent series. Certain properties of this function were studied and investigated. The function defined by (2) appeared for the first time in the work of Wiman (Wiman, 1905). The functions given by equations (1) and (2) are entire functions of order $\mu = \frac{1}{\upsilon}$ and of type $\sigma = 1$ (see for example (Erdelyi et al, 1953b)). By means of the series representations, a generalization of the functions defined by equations (1) and (2) is introduced by Prabhakar (Prabhakar, 1971) as:

$$\mathcal{E}^{\rho}_{\upsilon,\omega}(z) = \sum_{k=0}^{\infty} \frac{(\rho)_k z^k}{\Gamma(\upsilon k + \omega) k!}, \quad \upsilon, \omega, \rho \in \mathbb{C}, \ \Re(\upsilon) > 0, \Re(\omega) > 0, \tag{3}$$

where

$$(\rho)_k = \rho(\rho + 1) \ldots (\rho + k - 1) = \frac{\Gamma(\rho + k)}{\Gamma(\rho)},$$

whenever $\Gamma(\rho)$ is defined, $(\rho)_0 = 1, \rho = 0$. It is an entire function of order $\mu = (1/\upsilon)[\Re(\upsilon)^{\Re(\upsilon)}]^{-1/\upsilon}$. For various properties of this function with applications, see Prabhakar (Prabhakar, 1971). Further generalization of the

Mittag-Leffler function $\mathcal{E}^{\rho}_{\upsilon,\omega}(z)$ was considered earlier by Shukla and Prajapati (Shukla & Prajapati, 2007) which is given as:

$$\mathcal{E}^{\rho,q}_{\upsilon,\omega}(z) = \sum_{k=0}^{\infty} \frac{(\rho)_{kq} z^k}{\Gamma(\upsilon k + \omega) k!},$$
$$\text{with } z, \omega, \rho \in \mathbb{C},\ \Re(\upsilon) > \max(0, \Re(q) - 1), \Re(q) > 0, \tag{4}$$

which is the special case when $q \in (0,1)$ and $\min\{\Re(\omega), \Re(\rho)\} > 0$.

In continuation of this study, Salim and Faraj (Salim & Faraj, 2012; Nadir et al, 2014) introduced a new generalization of the Mittag-Leffler function which was given as:

$$\mathcal{E}^{\rho,\delta,q}_{\upsilon,\omega,p}(z) = \sum_{k=0}^{\infty} \frac{(\rho)_{kq} z^k}{\Gamma(\upsilon k + \omega)(\delta)_{pk}}, \tag{5}$$
$$(\min\{\Re(\upsilon), \Re(\omega), \Re(\rho), \Re(\delta)\} > 0; \quad p, q > 0; \quad z, \upsilon, \omega, \rho, \delta, \in \mathbb{C}).$$

Numerous generalizations and cases of the Mittag-Leffler function have been studied and investigated, see for details (Singh & Rawat, 2013; Wright, 1935b; Faraj et al, 2013; Dorrego & Cerutti, 2012; Srivastava & Tomovski, 2009; Saxena et al, 2011; Khan & Ahmed, 2012).

Integral formulae involving the Mittag-Leffler functions have been developed by many authors, see for example, (Prajapati & Shukla, 2012; Prajapati et al, 2013; Gehlot, 2021; Purohit et al, 2011). In this sequel, here, we aim to establish certain new generalized integral formulae involving the new generalization of the Mittag-Leffler function. The main result presented here is general enough to be specialized to give many interesting integral formulae which are derived as special cases.

Integrals with the Jacobi polynomials

The Jacobi polynomials $P_n^{\varrho,\sigma}(y)$ (Rainville, 1960; Srivastava & Manocha, 1984) may be defined by

$$P_n^{(\varrho,\sigma)}(y) = \frac{(1+\varrho)_n}{n!} {}_2F_1 \left[\begin{matrix} -n, 1+\varrho+\sigma+n; & \frac{1-y}{2} \\ \varrho+1; & \end{matrix} \right]. \tag{6}$$

When $\varrho = \sigma = 0$, the polynomial in (6) becomes the Legendre polynomial (Rainville, 1960). From (6), it follows that $P_n^{(\varrho,\sigma)}(y)$ is a polynomial of the degree n and that

$$P_n^{(\varrho,\sigma)}(y) = \frac{(1+\varrho)_n}{n!}. \tag{7}$$

Here, we obtain the following integrals.

THEOREM 1. *If* $p, q > 0$ $z, \upsilon, \omega, \rho, \delta, \in \mathbb{C}$, $\Re(\upsilon) > 0, \Re(\omega) > 0, \Re(\rho) > 0, \Re(\delta) > 0$ *and* $\Re(\xi) > -1, \varrho > -1, \sigma > -1$ *then the following integral formula holds true*

$$\begin{aligned}&\int_{-1}^{1} y^{\xi}(1-y)^{\varrho}(1+y)^{\eta+hk}P_n^{(\varrho,\sigma)}(y)\mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(1+y)^h]dy\\ &= \frac{(-1)^n 2^{\varrho+\eta+1}\Gamma(\eta+hk+1)\Gamma(n+\varrho+1)\Gamma(\eta+hk+\sigma+1)}{n!\Gamma(\eta+hk+\sigma+n+1)\Gamma(\eta+hk+\varrho+n+2)}\\ &\times \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}(2^h z) \times {}_3F_2\left[\begin{matrix} -\xi, \eta+hk+\sigma+1, \eta+hk+1; \\ \eta+hk+\sigma+n+1, \eta+hk+\varrho+n+2; \end{matrix} 1\right].\end{aligned} \tag{8}$$

Proof. Naming the left-hand side (LHS) of (8) as I_1 and using the definition (5), we have

$$I_1 = \int_{-1}^{1} y^{\xi}(1-y)^{\varrho}(1+y)^{\eta}P_n^{(\varrho,\sigma)}(y)\mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(1+y)^h]dy$$

$$I_1 = \int_{-1}^{1} y^{\xi}(1-y)^{\varrho}(1+y)^{\eta}P_n^{(\varrho,\sigma)}(y)\sum_{k=0}^{\infty}\frac{(\rho)_{kq}[z(1+y)^h]^k}{\Gamma(\upsilon k+\omega)(\delta)_{pk}}dy,$$

interchanging the order of integration and summation which is permissible under the conditions of the theorem, the above expression becomes

$$\sum_{k=0}^{\infty}\frac{(\rho)_{kq}(z)^k}{\Gamma(\upsilon k+\omega)(\delta)_{pk}}\int_{-1}^{1} y^{\xi}(1-y)^{\varrho}(1+y)^{\eta+hk}P_n^{(\varrho,\sigma)}(y)dy. \tag{9}$$

Apply the following formula (Saxena, 2008) on (9)

$$\int_{-1}^{1} y^{\xi}(1-y)^{\varrho}(1+y)^{\eta}P_n^{(\varrho,\sigma)}(y)dy = \frac{(-1)^n 2^{\varrho+\eta+1}\Gamma(\eta+1)\Gamma(n+\varrho+1)\Gamma(\eta+\sigma+1)}{n!\Gamma(\eta+\sigma+n+1)\Gamma(\eta+\varrho+n+2)}$$

$$\times {}_3F_2\left[\begin{array}{c} -\xi, \eta+\sigma+1, \eta+1; \\ \eta+\sigma+n+1, \eta+\varrho+n+2; \end{array} 1\right], \tag{10}$$

provided that $\varrho > -1$ and $\sigma > -1$, and we get the desired result.

THEOREM 2. *If* $p, q > 0$ $z, \upsilon, \omega, \rho, \delta, \in \mathbb{C}$, $\Re(\upsilon) > 0, \Re(\omega) > 0, \Re(\rho) > 0, \Re(\delta) > 0$ *and* $\Re(\xi) > -1, \varrho > -1, \sigma > -1$ *then the following integral formula holds true*

$$\int_{-1}^{1}(1-y)^{\eta}(1+y)^{\sigma}P_n^{(\varrho,\sigma)}(y)P_n^{(\mu,\theta)}(y)\mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(1-y)^h]dy$$
$$= \frac{2^{\eta+\sigma+1}\Gamma(1+\mu+m)\Gamma(1+\varrho+n)}{m!n!}$$
$$\times \sum_{k=0}^{\infty}\frac{(-m)_k(1+\mu+\theta+m)_k}{\Gamma(1+\mu+k)(k!)}\sum_{l=0}^{\infty}\frac{(-m)_l(1+\mu+\theta+m)_l}{\Gamma(1+\mu+k)(l!)}$$
$$\times \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}(2^h z)\mathbb{B}(1+\eta+hk+k+l, 1+\sigma). \tag{11}$$

Proof. Denoting the LHS of (11) by I_2 and using definition (5), we get

$$I_2 = \int_{-1}^{1}(1-y)^{\eta}(1+y)^{\sigma}P_n^{(\varrho,\sigma)}(y)P_n^{(\mu,\theta)}(y)\mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(1-y)^h]dy$$
$$I_2 = \sum_{k=0}^{\infty}\frac{(\rho)_{kq}(z)^k}{\Gamma(\upsilon k+\omega)(\delta)_{pk}}\int_{-1}^{1}(1-y)^{\eta+hk}(1+y)^{\sigma}P_n^{(\varrho,\sigma)}(y)P_n^{(\mu,\theta)}(y)dy. \tag{12}$$

Now, using (6) in (12), we get

$$I_2 = \sum_{k=0}^{\infty}\frac{(\rho)_{kq}(z)^k}{\Gamma(\upsilon k+\omega)(\delta)_{pk}}\frac{(1+\mu)_m}{m!}\sum_{k=0}^{\infty}\frac{(-m)_k(1+\mu+\theta+m)_k}{(1+\mu)_k 2^k k!}$$
$$\times \int_{-1}^{1}(1-y)^{\eta+hk+k}(1+y)^{\sigma}P_n^{(\varrho,\sigma)}(y)dy. \tag{13}$$

Again using (6) in (13), we attain

$$I_2 = \sum_{k=0}^{\infty}\frac{(\rho)_{kq}(z)^k}{\Gamma(\upsilon k+\omega)(\delta)_{pk}}\frac{\Gamma(1+\mu+m)\Gamma(1+\varrho+n)}{m!n!}$$
$$\sum_{k=0}^{\infty}\frac{(-m)_k(1+\mu+\theta+m)_k}{\Gamma(1+\mu+k)2^k(k!)}\sum_{l=0}^{\infty}\frac{(-m)_l(1+\mu+\theta+m)_l}{\Gamma(1+\mu+l)2^l(l!)}$$

$$\times \int_{-1}^{1} (1-y)^{\eta+hk+k+l}(1+y)^{\sigma} P_n^{(\varrho,\sigma)}(y)dy, \tag{14}$$

but by the formula found in (Rainville, 1960; Srivastava & Manocha, 1984)

$$\int_{-1}^{1} (1-y)^{\eta+n}(1+y)^{\sigma+n} dy = 2^{2n+\sigma+\eta+1}\mathbb{B}(1+\eta+n, 1+\sigma+n), \tag{15}$$

using it in (14), we get the required result.

THEOREM 3. *If* $p, q > 0$ $z, \upsilon, \omega, \rho, \delta, \in \mathbb{C}$, $\Re(\upsilon) > 0, \Re(\omega) > 0$, $\Re(\rho) > 0, \Re(\delta) > 0$ *and* $\varrho > -1, \sigma > -1$, *then*

$$\int_{-1}^{1} (1-y)^{\mu}(1+y)^{\theta} P_n^{(\varrho,\sigma)}(y)\mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(1-y)^h(1+y)^t]dy$$
$$= \frac{2^{\mu+\theta+1}}{n!}\frac{(1+\varrho)_n}{n!}\sum_{k=0}^{\infty}\frac{(-n)_k(1+\varrho+\sigma+n)_k}{(1+\varrho)_k(k!)}$$
$$\times \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}(2^{h+t}z)\mathbb{B}(1+\mu+hk+k, 1+\theta+tk). \tag{16}$$

Proof. Denoting the LHS of (16) by I_3,

$$I_3 = \int_{-1}^{1} (1-y)^{\mu}(1+y)^{\theta} P_n^{(\varrho,\sigma)}(y)\mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(1-y)^h(1+y)^t]dy$$
$$= \sum_{k=0}^{\infty}\frac{(\rho)_{kq}(z)^k}{\Gamma(\upsilon k+\omega)(\delta)_{pk}}\int_{-1}^{1} (1-y)^{\mu+hk}(1+y)^{\theta+tk} P_n^{(\varrho,\sigma)}(y)dy. \tag{17}$$

Now, using (6) in (17) we have

$$I_3 = \sum_{k=0}^{\infty}\frac{(\rho)_{kq}(z)^k}{\Gamma(\upsilon k+\omega)(\delta)_{pk}}\frac{(1+\varrho)_n}{n!}\sum_{k=0}^{\infty}\frac{(-n)_k(1+\varrho+\sigma+n)_k}{(1+\varrho)_k 2^k k!}$$
$$\times \int_{-1}^{1} (1-y)^{n+\mu+hk+k-n}(1+y)^{n+\theta+tk-n}dy, \tag{18}$$

further, using (15) in (18) we attain the desired result.

THEOREM 4. *If* $p, q > 0$ $z, \upsilon, \omega, \rho, \delta, \in \mathbb{C}$, $\Re(\upsilon) > 0, \Re(\omega) > 0, \Re(\rho) > 0, \Re(\delta) > 0$ *and* $\varrho > -1, \sigma > -1$, *then*

$$\int_{-1}^{1} (1-y)^{\mu}(1+y)^{\theta} P_n^{(\varrho,\sigma)}(y)\mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(1+y)^{-h}]dy$$

$$= \frac{2^{\mu+\theta+1}}{n!}\frac{(1+\varrho)_n}{n!}\sum_{k=0}^{\infty}\frac{(-n)_k(1+\varrho+\sigma+n)_k}{(1+\varrho)_k(k!)} \times\mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}(2^{-h}z)\mathbb{B}(1+\mu+k,1+\theta-hk). \quad (19)$$

Proof. Denoting the LHS of (19) by I_4,

$$I_4 = \int_{-1}^{1}(1-y)^{\mu}(1+y)^{\theta}P_n^{(\varrho,\sigma)}(y)\mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(1+y)^{-h}]dy$$
$$= \sum_{k=0}^{\infty}\frac{(\rho)_{kq}(z)^k}{\Gamma(\upsilon k+\omega)(\delta)_{pk}}\int_{-1}^{1}(1-y)^{\mu}(1+y)^{\theta-hk}P_n^{(\varrho,\sigma)}(y)dy. \quad (20)$$

Now, using (6) in (20) we attain

$$I_4 = \sum_{k=0}^{\infty}\frac{(\rho)_{kq}(z)^k}{\Gamma(\upsilon k+\omega)(\delta)_{pk}}\frac{(1+\varrho)_n}{n!}\sum_{k=0}^{\infty}\frac{(-n)_k(1+\varrho+\sigma+n)_k}{(1+\varrho)_k2^kk!} \times\int_{-1}^{1}(1-y)^{n+\mu+k-n}(1+y)^{n+\theta-hk-n}dy \quad (21)$$

further, using (15) in (21) we attain the required result.

THEOREM 5. *If* $p,q>0$ $z,\upsilon,\omega,\rho,\delta,\in\mathbb{C}$*,* $\Re(\upsilon)>0,\Re(\omega)>0,\Re(\rho)>0,\Re(\delta)>0$ *and* $\varrho>-1,\sigma>-1$*, then*

$$\int_{-1}^{1}(1-y)^{\mu}(1+y)^{\theta}P_n^{(\varrho,\sigma)}(y)\mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(1-y)^{h}(1+y)^{-t}]dy$$
$$= \frac{2^{\mu+\theta+1}}{n!}\frac{(1+\varrho)_n}{n!}\sum_{k=0}^{\infty}\frac{(-n)_k(1+\varrho+\sigma+n)_k}{(1+\varrho)_k(k!)} \times\mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}(2^{h-t}z)\mathbb{B}(1+\mu+hk+k,1+\theta-tk). \quad (22)$$

Proof. Denoting the LHS of (22) by I_5,

$$I_5 = \int_{-1}^{1}(1-y)^{\mu}(1+y)^{\theta}P_n^{(\varrho,\sigma)}(y)\mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(1-y)^{h}(1+y)^{-t}]dy$$
$$= \sum_{k=0}^{\infty}\frac{(\rho)_{kq}(z)^k}{\Gamma(\upsilon k+\omega)(\delta)_{pk}}\int_{-1}^{1}(1-y)^{\mu+hk}(1+y)^{\theta-tk}P_n^{(\varrho,\sigma)}(y)dy, \quad (23)$$

now using (6) in (23) we attain

$$I_5 = \sum_{k=0}^{\infty} \frac{(\rho)_{kq}(z)^k}{\Gamma(\upsilon k+\omega)(\delta)_{pk}} \frac{(1+\varrho)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k(1+\varrho+\sigma+n)_k}{(1+\varrho)_k 2^k k!}$$
$$\times \int_{-1}^{1} (1-y)^{n+\mu+hk+k-n}(1+y)^{n+\theta-tk-n} dy \tag{24}$$

further, using (15) in (24) we attain the required result.

Some special cases

If we replace η by $\xi-1$ and put $\varrho=\sigma=\mu=\theta=0$ then the integral I_2 transforms into the following integral involving the Legendre polynomial (Rainville, 1960)

$$I_6 = \int_{-1}^{1} (1-y)^{\xi-1} P_n(y) \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(1-y)^h]dy$$
$$I_6 = \sum_{k=0}^{\infty} \frac{2^{\xi}(-m)_k(1+m)_k}{(k!)^2} \times \sum_{l=0}^{\infty} \frac{(-n)_l(1+n)_l}{l!^2}$$
$$\times \quad \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}(2^h z)\mathbb{B}(1+\xi+hk+k+l,1). \tag{25}$$

If $\sigma=\varrho=0$, μ is replaced by $\mu-1$ and θ by $\theta-1$, then the integral I_3 transforms into the following integral involving the Legendre polynomial (Rainville, 1960)

$$I_7 = \int_{-1}^{1} (1-y)^{\mu-1}(1+y)^{\theta-1} P_n(y) \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(1-y)^h(1+y)^t]dy$$
$$I_7 = \sum_{k=0}^{\infty} \frac{2^{\mu+\theta-1}(-n)_k(1+n)_k}{(k!)^2}$$
$$\times \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}(2^{h+t}z)\mathbb{B}(1+\mu+hk+k,\theta+tk). \tag{26}$$

If $\varrho=\sigma=0$, μ is replaced by $\mu-1$ and θ by $\theta-1$ then the integral I_3 transforms into the following integral involving the Legendre polynomial (Rainville, 1960)

$$I_8 = \int_{-1}^{1} (1-y)^{\mu-1}(1+y)^{\theta-1} P_n(y) \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(1-y)^h(1+y)^{-t}]dy$$

VOJNOTEHNIČKI GLASNIK / MILITARY TECHNICAL COURIER, 2022, Vol. 70, Issue 4

$$I_8 = \sum_{k=0}^{\infty} \frac{2^{\mu+\theta-1}(-n)_k(1+n)_k}{(k!)^2} \times \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}(2^{h-t}z)\mathbb{B}(1+\mu+hk+k,\theta-tk). \tag{27}$$

Integral with the Bessel Maitland function

The special case of the Wright function (Erdelyi et al, 1953b), see also (Wright, 1935a,b) written in the form

$$\phi(-;A,a;z) = {}_0\psi_1\left[\begin{matrix} -; \\ (A,a); \end{matrix} \; z\right] = \sum_{k=0}^{\infty} \frac{1}{\Gamma(Ak+a)} \frac{z^k}{k!}, \tag{28}$$

with complex $z, a \in \mathbb{C}$ and real $A \in \mathbb{R}$. When $A = \eta, a = \nu + 1$ and z is replaced by $-z$, then the function $\phi(\eta, \nu+1; -z)$ is defined by $J_\nu^\eta(z)$

$$\phi(\eta,\nu+1;-z) = J_\nu^\eta(z) = \sum_{k=0}^{\infty} \frac{1}{\Gamma(\eta k+\nu+1)} \frac{(-z)^k}{k!}, \tag{29}$$

and such a function is known as the Bessel Maitland function, or the generalized Bessel function, or the Wright generalized Bessel function, see (Mcbride, 1995).

THEOREM 6. *If* $p, q > 0$ $z, \upsilon, \omega, \rho, \delta, \in \mathbb{C}$ $, \Re(\upsilon) > 0, \Re(\omega) > 0, \Re(\rho) > 0, \Re(\delta) > 0$, $\varrho - \varrho\tau > -1$, $\varrho > 0$, $0 < \tau < 1$ *and* $\Re(\mu+1) > 0$, *then the following integral formula holds true.*

$$\int_{-1}^{1} (y)^\mu J_\nu^\tau(y) \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(y)^\varrho]dy = \frac{\Gamma(\mu+\varrho k+1)}{\Gamma(1+\nu-\tau-\tau(\mu+\varrho k))} \times \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}(z). \tag{30}$$

Proof. Naming the LHS of (30) as I_9, we obtain

$$I_9 = \int_{-1}^{1} (y)^\mu J_\nu^\tau(y) \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(y)^\varrho]dy$$

$$I_9 = \sum_{k=0}^{\infty} \frac{(\rho)_{qk} z^k}{\Gamma(\upsilon k+\omega)(\delta)_{pk}} \times \int_0^\infty (y^{\mu+\varrho k}) J_\nu^\tau(y)dy. \tag{31}$$

Now we know the formula, see (Saxena, 2008)

$$\int_0^\infty (y^\mu) J_\nu^\tau(y)dy = \frac{\Gamma(\mu+1)}{\Gamma(1+\nu-\tau-\tau\mu)}, \tag{32}$$

provided $\Re(\mu) > -1, 0 < \tau < 1.$

Using (32) in (31), we attain

$$I_9 = \sum_{k=0}^{\infty} \frac{(\rho)_{qk} z^k}{\Gamma(\upsilon k + \omega)(\delta)_{pk}} \times \frac{\Gamma(\mu + \varrho k + 1)}{\Gamma(1 + \nu - \tau - \tau(\mu + \varrho k))},$$

hence proved.

Integrals with the Legendre functions

The Legendre functions are solution of Legendre's differential equation, see (Erdelyi et al, 1953a)

$$(1 - z^2)\frac{d^2 w}{dz^2} - 2z\frac{dw}{dz} + [(\nu(\nu + 1)) - \omega^2(1 - z^2)^{-1}]w = 0, \tag{33}$$

where z, ν, ω are unrestricted.

Under the subsitution $w = (z^2 - 1)^{\omega/2}\nu$ in (5.1) becomes

$$(1 - z^2)\frac{d^2\nu}{dz^2} - 2(\omega + 1)z\frac{d\nu}{dz} + [(\nu + \omega)(\nu + \omega + 1)]\nu = 0, \tag{34}$$

and with $\lambda = 1/2 - z/2$ as the independent variable, this differential equation becomes

$$\lambda(1 - \lambda)\frac{d^2\nu}{d\lambda^2} + (\omega + 1)(1 - 2\lambda)\frac{d\nu}{d\lambda} + [(\nu - \omega)(\nu + \omega + 1)]\nu = 0. \tag{35}$$

This is the Gauss hypergeometric type equation with $a = \omega - \nu, b = \nu + \omega + 1, c = \omega + 1.$
Hence it follows that the function

$$W = P_\nu^{(\omega}(z) = \frac{1}{\Gamma(1 - \omega)}\left(\frac{z + 1}{z - 1}\right)^{\omega/2} {}_2F_1\left[\begin{matrix} -\nu, \nu + 1; \\ 1 - \omega; \end{matrix}\ 1/2 - z/2\right], \tag{36}$$

for $\mid 1 - z \mid < 2$
is a solution of (33).

The function $P_\nu^\omega(z)$ is known as the Legendre function of the first kind (Erdelyi et al, 1953a). It is one valued and regular on the $z-$plane, supposed cut along the real axis from 1 to $-\infty$.

THEOREM 7. *If* $p, q > 0$ $z, \upsilon, \omega, \rho, \delta, \in \mathbb{C}$, $\Re(\upsilon) > 0, \Re(\omega) > 0, \Re(\rho) > 0, \Re(\delta) > 0$ *and* $\theta > 0$ *and* η *is a positive integer then*

$$\int_0^1 (y)^{\theta-1}(1-y^2)^{\eta/2} P_\nu^\eta(y) \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(y)^\varrho]dy$$

$$= \frac{(-1)^\eta \sqrt{\pi} 2^{-\theta-\eta} \Gamma(\nu+\eta+1)}{\Gamma(1-\eta+\nu)} \times$$

$$\sum_{k=0}^{\infty} \frac{\Gamma(\theta+\varrho k)}{\Gamma(1/2+\frac{\theta+\varrho k}{2}+\eta/2-\nu/2)\Gamma(1+\frac{\theta+\varrho k}{2}+\eta/2+\nu/2)} \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}(z/2^\theta). \quad (37)$$

Proof. Denoting the LHS of (37) by I_{10},

$$I_{10} = \int_0^1 (y)^{\theta-1}(1-y^2)^{\eta/2} P_\nu^\eta(y) \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(y)^\varrho]dy$$

$$I_{10} = \sum_{k=0}^{\infty} \frac{(\rho)_{qk} z^k}{\Gamma(\upsilon k+\omega)(\delta)_{pk}} \times \int_0^1 y^{\theta-1+\varrho k}(1-y^2)^{\eta/2} P_\nu^\eta(y)dy. \quad (38)$$

Now the integral in (38) can be solved by using the formula (Erdelyi et al, 1953a)

$$\int_0^\infty y^{\theta-1}(1-y^2)^{\eta/2} P_\nu^\eta(y)dy$$

$$= \frac{(-1)^\eta \sqrt{\pi} 2^{-\theta-\eta} \Gamma(\theta)\Gamma(\nu+\eta+1)}{(\Gamma(1/2+\theta/2+\eta/2-\nu/2)(\Gamma(1+\theta/2+\eta/2+\nu/2)(1-\eta+\nu)}, \quad (39)$$

provided $\Re(\theta) > 0, \eta = 1, 2, 3, \ldots$.

Now (38) becomes

$$I_{10} = \sum_{k=0}^{\infty} \frac{(\rho)_{qk} z^k}{\Gamma(\upsilon k+\omega)(\delta)_{pk}} \times$$

$$\frac{(-1)^\eta \sqrt{\pi} 2^{-(\theta+\varrho k)-\eta} \Gamma(\theta+\varrho k)\Gamma(\eta+\nu+1)}{\Gamma(1/2+\frac{\theta+\varrho k}{2}+\eta/2-\nu/2)\Gamma(1+\frac{\theta+\varrho k}{2}+\eta/2+\nu/2)\Gamma(1-\eta+\nu)},$$

which is the desired result.

THEOREM 8. *If* $p, q > 0$ $z, \upsilon, \omega, \rho, \delta, \in \mathbb{C}$, $\Re(\upsilon) > 0, \Re(\omega) > 0, \Re(\rho) > 0, \Re(\delta) > 0$ *and* $\theta > 0$ *and* η *is a positive integer then*

$$\int_0^1 (y)^{\theta-1}(1-y^2)^{-\eta/2} P_\nu^\eta(y) \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(y)^\varrho]dy$$

$$= \sqrt{\pi}2^{-\theta+\eta}\sum_{k=0}^{\infty}\frac{\Gamma(\theta+\varrho k)}{\Gamma(1/2+\frac{\theta+\varrho k}{2}-\eta/2-\nu/2)\Gamma(1+\frac{\theta+\varrho k}{2}-\eta/2-\nu/2)}$$
$$\times\mathcal{E}^{\rho,\delta,q}_{\upsilon,\omega,p}(z/2^{\theta}). \quad (40)$$

Proof. Denoting the LHS of (40) by I_{11},

$$I_{11} = \int_0^1 (y)^{\theta-1}(1-y^2)^{-\eta/2}P^{\eta}_{\nu}(y)\mathcal{E}^{\rho,\delta,q}_{\upsilon,\omega,p}[z(y)^{\varrho}]dy$$

$$I_{11} = \sum_{k=0}^{\infty}\frac{(\rho)_{qk}z^k}{\Gamma(\upsilon k+\omega)(\delta)_{pk}}\times\int_0^1 y^{\theta-1+\varrho k}(1-y^2)^{-\eta/2}P^{\eta}_{\nu}(y)dy, \quad (41)$$

now the integral in (41) can be solved by using the formula (Erdelyi et al, 1953a)

$$\int_0^{\infty} y^{\theta-1}(1-y^2)^{-\eta/2}P^{\eta}_{\nu}(y)dy$$
$$= \frac{\sqrt{\pi}2^{-\theta+\eta}\Gamma(\theta)}{\Gamma(1/2+\theta/2-\eta/2-\nu/2)\Gamma(1+\theta/2-\eta/2-\nu/2)}, \quad (42)$$

provided $\Re(\theta) > 0, \eta = 1, 2, 3, ...$

Again (41) becomes

$$I_{11} = \sum_{k=0}^{\infty}\frac{(\rho)_{qk}z^k}{\Gamma(\upsilon k+\omega)(\delta)_{pk}}\times$$
$$\frac{\sqrt{\pi}2^{-(\theta+\varrho k)+\eta}\Gamma(\theta+\varrho k)}{\Gamma(1/2+\frac{\theta+\varrho k}{2}-\eta/2-\nu/2)\Gamma(1+\frac{\theta+\varrho k}{2}-\eta/2-\nu/2)}.$$

Integrals with the Hermite polynomials

The Hermite polynomials $H_n(y)$, see (Rainville, 1960; Srivastava & Manocha, 1984) may be defined by means of the relation

$$\exp(2yt-t^2) = \sum_{k=0}^{\infty}\frac{H_n(y)t^n}{n!}, \quad (43)$$

valid for all finite y and t. Since

$$\exp(2yt-t^2) = \exp(2yt)\exp(-t^2)$$

$$= \sum_{n=0}^{\infty} \frac{(2y)^n t^n}{n!} \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k}}{k!}$$

$$= \sum_{n=0}^{\infty} \sum_{k=0}^{[n/2]} \frac{(-1)^k (2y)^{n-2k} t^n}{(n-2k)!k!}.$$

It follows from (43) that

$$H_n(y) = \sum_{k=0}^{[n/2]} \frac{(-1)^k (2y)^{n-2k} t^n}{(n-2k)!k!}. \tag{44}$$

The examination of equation (44) shows that $H_n(y)$ is a polynomial of degree precisely n in y and that

$$H_n(y) = 2^n y^n + \pi_{n-2}(y) \tag{45}$$

in which $\pi_{n-2}(y)$ is a polynomial of the degree $(n-2)$ in y.

THEOREM 9. *If* $p, q > 0$ $z, \upsilon, \omega, \rho, \delta, \in \mathbb{C}$, $\Re(\upsilon) > 0, \Re(\omega) > 0, \Re(\rho) > 0, \Re(\delta) > 0$ *and* $h > 0$ $\Re(\mu) = 0, 1, 2 \ldots$ *then*

$$\int_{-\infty}^{\infty} (y)^{2\mu} \exp(-y^2) H_{2\nu}(y) \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(y)^{-2h}]dy$$

$$= \sqrt{\pi} 2^{2(\nu-\mu)} \sum_{k=0}^{\infty} \frac{\Gamma(2\mu - hk + 1)}{\Gamma(\mu - hk - \nu + 1)} \times \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}(2^{2h}z). \tag{46}$$

Proof. Denoting the LHS of (9) by I_{12}, we have

$$I_{12} = \int_{-\infty}^{\infty} (y)^{2\mu} \exp(-y^2) H_{2\nu}(y) \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(y)^{-2h}]dy$$

$$I_{12} = \sum_{k=0}^{\infty} \frac{(\rho)_{qk} z^k}{\Gamma(\upsilon k + \omega)(\delta)_{pk}} \times \int_{-\infty}^{\infty} y^{2\mu - 2hk} (\exp)^{-y^2} H_{2\nu}(y)dy, \tag{47}$$

now the integral in (47) can be solved by using the formula (Saxena, 2008)

$$\int_{-\infty}^{\infty} y^{2\mu} (\exp)^{-y^2} H_{2\nu}(y)dy = \frac{\sqrt{\pi} 2^{2(\nu-\mu)} \Gamma(2\mu+1)}{\Gamma(\mu - \nu + 1)}. \tag{48}$$

Again (47) becomes

$$I_{12} = \sum_{k=0}^{\infty} \frac{(\rho)_{qk} z^k}{\Gamma(\upsilon k + \omega)(\delta)_{pk}} \times \frac{\sqrt{\pi} 2^{2(\nu-(2\mu+2hk))} \Gamma(2\mu - 2hk + 1)}{\Gamma(\mu - hk - \nu + 1)}.$$

THEOREM 10. *If* $p, q > 0$ $z, \upsilon, \omega, \rho, \delta, \in \mathbb{C}$, $\Re(\upsilon) > 0, \Re(\omega) > 0, \Re(\rho) > 0, \Re(\delta) > 0$ *and* $h > 0$ $\Re(\mu) = 0, 1, 2...$ *then*

$$\int_{-\infty}^{\infty} (y)^{2\mu} \exp(-y^2) H_{2\nu}(y) \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(y)^{2h}]dy$$

$$= \sqrt{\pi} 2^{2(\nu-\mu)} \sum_{k=0}^{\infty} \frac{\Gamma(2\mu + 2hk + 1)}{\Gamma(\mu + hk - \nu + 1)} \times \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}(2^{-2h}z). \quad (49)$$

Proof. Denoting the LHS of (10) by I_{13}, we have

$$I_{13} = \int_{-\infty}^{\infty} (y)^{2\mu} \exp(-y^2) H_{2\nu}(y) \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[z(y)^{2h}]dy$$

$$I_{13} = \sum_{k=0}^{\infty} \frac{(\rho)_{qk} z^k}{\Gamma(\upsilon k + \omega)(\delta)_{pk}} \times \int_{-\infty}^{\infty} y^{2\mu+2hk} (\exp)^{-y^2} H_{2\nu}(y) dy, \quad (50)$$

using the formula mentioned in (48), then the above expression (50), we get the desired result.

Integrals with the generalized hypergeometric functions

A generalized hypergeometric function (Rainville, 1960) may be defined by

$${}_pF_q \left[\begin{matrix} (\varrho)_1, (\varrho)_2, ...(\varrho)_p; \\ (\sigma)_1, (\sigma)_2, ...(\sigma)_q, ; \end{matrix} \; z \right] = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p} (\varrho_i)_n}{\prod_{j=1}^{q} (\sigma_j)_n} \frac{z^n}{n!}, \quad (51)$$

in which no denominator parameter σ_j is allowed to be zero or a negative integer. If any numerator parameter ϱ_i in (51) is zero or a negative integer, the series terminates.

THEOREM 11. *The following integral formula holds true,*

$$\int_0^t (y)^{\mu-1} (t-y)^{\theta-1} {}_pF_q[(l_p); (m_q) : ay^{\varrho}(t-y)^{\sigma}] \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}[zy^u(t-y)^v]dy$$

$$= \mathcal{E}_{\upsilon,\omega,p}^{\rho,\delta,q}(zt^{u+v}) t^{\mu+\theta-1} \sum_{k=0}^{\infty} f(k) t^{(\varrho+\sigma)k} \times \mathbb{B}(\mu + uk + \varrho k, \theta + vk + \sigma k), \quad (52)$$

where

$$f(k) = \frac{(l_1)_k, ...(l_p)_k}{(m_1)_k, ...(m_q)_k} \tag{53}$$

provided
(1) $\Re(\upsilon) > 0, \Re(\omega) > 0, \Re(\rho) > 0, \Re(\delta) > 0$ *and* $p, q > 0$*,*
(2)$\Re(\varrho) \geq 0, \Re(v) \geq 0$ *(both are not zero simultaneously),*
(3) ϱ *and* σ *are positive integers such that* $\varrho + \sigma \geq 1$*.*

Proof. Representing the LHS of (11) by I_{15}, we have

$$I_{15} = \int_0^t (y)^{\mu-1}(t-y)^{\theta-1}{}_pF_q[(l_p);(m_q) : ay^{\varrho}(t-y)^{\sigma}]\mathcal{E}^{\rho,\delta,q}_{\upsilon,\omega,p}[zy^u(t-y)^v]dy$$

$$I_{15} = \sum_{k=0}^{\infty} \frac{(\rho)_{qk} z^k t^{\theta+vk-1}}{\Gamma(\upsilon k+\omega)(\delta)_{pk}} \times$$

$$\int_0^t y^{\mu+uk-1}(1-x/t)^{\theta+vk-1}{}_pF_q[(l_p);(m_q) : ay^{\varrho}(t-y)^{\sigma}]dy,$$

putting $x = st$ and $dx = tds$, then we get

$$I_{15} = \sum_{k=0}^{\infty} \frac{(\rho)_{qk}(zt^{u+v})^k t^{\mu+\theta-1}}{\Gamma(\upsilon k+\omega)(\delta)_{pk}} \times$$

$$\int_0^1 (s)^{\mu+uk-1}(1-s)^{\theta+vk-1}{}_pF_q[(l_p);(m_q) : at^{\varrho+\sigma}s^{\varrho}(1-s)^{\sigma}]ds.$$

The remaining theorems could be proved in a completely analogous fashion.

THEOREM 12. *The following integral formula holds true,*

$$I_{16} = \int_0^t (y)^{\mu-1}(t-y)^{\theta-1}{}_pF_q[(l_p);(m_q) : ay^{\varrho}(t-y)^{\sigma}]\mathcal{E}^{\rho,\delta,q}_{\upsilon,\omega,p}[zy^{-u}(t-y)^{-v}]dy$$

$$I_{16} = \mathcal{E}^{\rho,\delta,q}_{\upsilon,\omega,p}(zt^{-u-v})t^{\mu+\theta-1}\sum_{k=0}^{\infty} f(k)t^{(\varrho+\sigma)k} \times \mathbb{B}(\mu-uk+\varrho k, \theta-vk+\sigma k), \tag{54}$$

where $f(k)$ *is defined in (53)*
provided

(1) $\Re(\upsilon) > 0, \Re(\omega) > 0, \Re(\rho) > 0, \Re(\delta) > 0$ *and* $p, q > 0$,
(2)$\Re(\varrho) \geq 0, \Re(v) \geq 0$ *(both are not zero simultaneously),*
(3) ϱ *and* σ *are positive integers such that* $\varrho + \sigma \geq 1$.

THEOREM 13. *The following integral formula holds true,*

$$I_{17} = \int_0^t (y)^{\mu-1}(t-y)^{\theta-1}{}_pF_q[(l_p);(m_q) : ay^{\varrho}(t-y)^{\sigma}]\mathcal{E}^{\rho,\delta,q}_{\upsilon,\omega,p}[zy^u(t-y)^{-v}]dy$$

$$I_{17} = \mathcal{E}^{\rho,\delta,q}_{\upsilon,\omega,p}(zt^{u-v})t^{\mu+\theta-1}\sum_{k=0}^{\infty} f(k)t^{(\varrho+\sigma)k} \times \mathbb{B}(\mu+uk+\varrho k, \theta-vk+\sigma k), \tag{55}$$

where $f(k)$ *is defined in* (53)
provided
(1) $\Re(\upsilon) > 0, \Re(\omega) > 0, \Re(\rho) > 0, \Re(\delta) > 0$ *and* $p, q > 0$,
(2)$\Re(\varrho) \geq 0, \Re(v) \geq 0$ *(both are not zero simultaneously),*
(3) ϱ *and* σ *are positive integers such that* $\varrho + \sigma \geq 1$.

THEOREM 14. *The following integral formula holds true,*

$$I_{18} = \int_0^t (y)^{\mu-1}(t-y)^{\theta-1}{}_pF_q[(l_p);(m_q) : ay^{\varrho}(t-y)^{\sigma}]\mathcal{E}^{\rho,\delta,q}_{\upsilon,\omega,p}[zy^{-u}(t-y)^{v}]dy$$

$$I_{18} = \mathcal{E}^{\rho,\delta,q}_{\upsilon,\omega,p}(zt^{-u+v})t^{\mu+\theta-1}\sum_{k=0}^{\infty} f(k)t^{(\varrho+\sigma)k} \times \mathbb{B}(\mu-uk+\varrho k, \theta+vk+\sigma k), \tag{56}$$

provided
(1) $\Re(\upsilon) > 0, \Re(\omega) > 0, \Re(\rho) > 0, \Re(\delta) > 0$ *and* $p, q > 0$,
(2)$\Re(\varrho) \geq 0, \Re(v) \geq 0$ *(both are not zero simultaneously),*
(3) ϱ *and* σ *are positive integers such that* $\varrho + \sigma \geq 1$.

Conclusions

Certain new generalized integral formulae involving the Generalized Mittag-Leffler Type functions with many types of polynomials were established in this study. The results obtained here are general in nature and yield to many interesting formulae which are derived as particular cases.

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НЕКОТОРЫЕ ИНТЕГРАЛЫ С ОБОБЩЕННЫМИ ФУНКЦИЯМИ ТИПА МИТТАГ-ЛЕФФЛЕРА

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РУБРИКА ГРНТИ: 27.23.17 Дифференциальное и интегральное исчисление
27.23.21 Интегральные преобразования. Операционное исчисление
27.23.25 Специальные функции
ВИД СТАТЬИ: оригинальная научная статья

Резюме:

Введение/цель: В данной статье установлены определенные интегралы, включающие обобщенную функцию Миттага-Леффлера с различными типами многочленов.

Методы: Свойства обобщенной функции Миттаг-Леффлера используются в сочетании с различными видами многочленов, такими как Якоби, Лежандр и Эрмит для оценки их интегралов.

Результаты: Получены некоторые интегральные формулы, включающие функцию Лежандра, функцию Бесселя Мейтланда и обобщенные гипергеометрические функции.

Выводы: Полученные результаты носят общий характер и могут быть полезны для установления дальнейших ин-

тегральных формул, включающих другие виды многочленов.

Ключевые слова: функция Миттаг-Леффлера, обоб- щенная гипергеометрическая функция, функция Бесселя–Мейтленда , многочлены Якоби, многочлены Эрмита.

НЕКИ ИНТЕГРАЛИ КОЈИ УКЉУЧУЈУ ГЕНЕРАЛИЗОВАНЕ МИТАГ-ЛЕФЛЕРОВЕ ФУНКЦИЈЕ

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ОБЛАСТ: математика
КАТЕГОРИЈА (ТИП) ЧЛАНКА: оригинални научни рад

Сажетак:

Увод/циљ: Дефинисани су неки интеграли који укључују генерализовану Митаг-Лефлерову функцију са различитим врстама полинома.

Методе: Својства генерализоване Митаг-Лефлерове функције користе се на различитим врстама полинома, као што су Јакобијеви, Лежандрови, Ермитови, како би одредили њихове интеграле.

Резултати: Изведене су неке интегралне формуле које укључују Лежандрову функцију, Бесел-Мејтландову функцију и генерализоване хипергеометријске функције.

Закључак: Добијени резултати су опште природе и могли би бити корисни за утврђивање других интегралних формула које укључују друге врсте полинома.

Кључне речи: Митаг-Лефлерова функција, генерализована хипергеометријска функција, Бесел-Мејтландова функција, Јакобијеви полиноми, Ермитови полиноми.

EDITORIAL NOTE: The fourth author of this article, Nicola Fabiano, is a current member of the Editorial Board of the *Military Technical Courier*. Therefore, the Editorial Team has ensured that the double blind reviewing process was even more transparent and more rigorous. The Team made additional effort to maintain the integrity of the review and to minimize any bias by having another associate editor handle the review procedure independently of the editor – author in a completely transparent process. The Editorial Team has taken special care that the referee did not recognize the author's identity, thus avoiding the conflict of interest.

КОММЕНТАРИЙ РЕДКОЛЛЕГИИ: Ччетвертый автор данной статьи Никола Фабиано является действующим членом редколлегии журнала *«Военно-технический вестник»*. Поэтому редколлегия провела более открытое и более строгое двойное слепое рецензирование. Редколлегия приложила дополнительные усилия для того чтобы сохранить целостность рецензирования и свести к минимуму предвзятость, вследствие чего второй редактор-сотрудник управлял процессом рецензирования независимо от редактора-автора, таким образом процесс рецензирования был абсолютно прозрачным. Редколлегия во избежание конфликта интересов позаботилась о том, чтобы рецензент не узнал кто является автором статьи.

РЕДАКЦИЈСКИ КОМЕНТАР: Четврти аутор овог чланка Никола Фабиано је актуелни члан Уређивачког одбора *Војнотехничког гласника*. Због тога је уредништво спровело транспарентнији и ригорознији двоструко слепи процес рецензије. Уложило је додатни напор да одржи интегритет рецензије и необјективност сведе на најмању могућу меру тако што је други уредник сарадник водио процедуру рецензије независно од уредника аутора, при чему је тај процес био апсолутно транспарентан. Уредништво је посебно водило рачуна да рецензент не препозна ко је написао рад и да не дође до конфликта интереса.

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INTERPOLATIVE GENERALISED MEIR-KEELER CONTRACTION

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Abstract:

Introduction/purpose: The aim of this paper is to introduce the notion of an interpolative generalised Meir-Keeler contractive condition for a pair of self maps in a fuzzy metric space, which enlarges, unifies and generalizes the Meir-Keeler contraction which is for only one self map. Using this, we establish a unique common fixed point theorem for two self maps through weak compatibility. The article includes an example, which shows the validity of our results.

Methods: Functional analysis methods with a Meir-Keeler contraction.

Results: A unique fixed point for self maps in a fuzzy metric space is obtained.

Conclusions: A fixed point of the self maps is obtained.

Key words: Fuzzy metric space, common fixed points, weak compatibility, Interpolative generalised Meir-Keelar contraction.

Introduction

In 1965 L. Zadeh (Zadeh, 1965) introduced the theory of fuzzy sets. Later on, in 1978, the concept of a fuzzy metric space was introduced by Kramosil and Michalek in (Kramosil & Michalek, 1975), which was modified by George and Veeramani (George & Veeramani, 1994) in order to obtain a Hausdorff topology for this class of fuzzy metric spaces. Then in year 1988, Grabiec (Grabiec, 1988) gave a fuzzy version of the Banach (Banach, 1922) contraction principle in the setting of a fuzzy metric space. Over the past years, various authors have tried to generalize the fixed point theorem by modifying and varying the contractive condition, see, e.g., (Gregori & Sapena, 2002), (Jain & Jain, 2021), (Mihet, 2008), (Saha et al, 2016), (Tirado, 2012) and (Wardowski, 2013) in the sense of George and Veeramani. In 2019, Zheng and Wang (Zheng & Wang, 2019) introduced a Meir-Keeler contraction in the setting of a fuzzy metric (Schweizer & Sklar, 1983) space and proved some fixed point results for a self map.

Inspired with the interpolative theory, Karapinar and Agrawal (Karapinar & Agarwal, 2019) introduced the notion of an interpolative Rus-Reich-$\acute{C}$iri$\acute{c}$ type contraction via the simulation function in a metric space. Motivated by this paper, we introduce an interpolative generalised Meir-Keeler contraction (Gregori & Minana, 2014) for two self maps (Rhoades, 2001) in the setting of a fuzzy metric space, which enlarges, unifies and generalizes the existing Meir-Keelar contraction in a fuzzy metric (Mihet, 2010) space through weak compatibility (Banach, 1922).

The structure of the paper is as follows:

After the preliminaries, we introduce a interpolative generalised Meir-Keeler contraction in the setting of a fuzzy metric space. Then we study the Meir-Keeler contractive mapping due to Zheng and Wang (Zheng & Wang, 2019). In section 4, the existence of a unique common fixed point of an interpolative generalised Meir-Keeler contractive mapping has been established through weak compatibility followed by an example.

Preliminaries

Definition 1. *(George & Veeramani, 1994) A mapping* $* : [0,1] \times [0,1] \to [0,1]$ *is called a continuous triangular norm (t-norm for short) if* $*$ *is continuous and satisfies the following conditions:*

(i) $*$ *is commutative and associative, i.e.* $a * b = b * a$ *and* $a * (b * c) = (a * b) * c$*, for all* $a, b, c \in [0,1]$;

(ii) $1 * a = a$*, for all* $a \in [0,1]$;

(iii) $a * c \leq b * d$*, for* $a \leq b, c \leq d$ *for* $a, b, c, d \in [0,1]$.

The well-known examples of the t-norm are the minimum t-norm $*_m$, $a *_m b = min\{a, b\}$ written as $*_m$ and the product t-norm $*$, $a * b = ab$.

Definition 2. *(George & Veeramani, 1994) A fuzzy metric space is an ordered triple $(X, M, *)$ such that X is a (nonempty) set, $*$ is a continuous t-norm and M is a fuzzy set on $X \times X \times (0, +\infty)$ satisfying the following conditions, for all $x, y, z \in X$ and $t, s > 0$;*

(GV1) $M(x, y, t) > 0$;

(GV2) $M(x, y, t) = 1$ *if and only if* $x = y$;

(GV3) $M(x, y, t) = M(y, x, t)$;

(GV4) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$;

(GV5) $M(x, y, .) : (0, +\infty) \to (0, 1]$ *is continuous.*

Note that in view of the condition (GV2) we have $M(x, x, t) = 1$, for all $x \in X$ and $t > 0$ and $M(x, y, t) < 1$, for all $x \neq y$ and $t > 0$.
The following notion was introduced by George and Veeramani in (George & Veeramani, 1994).

Definition 3. *(George & Veeramani, 1994) A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is said to be M-Cauchy, or simply Cauchy, if for each $\epsilon \in (0, 1)$ and each $t > 0$ there exists an $n_0 \in N$, such that $M(x_n, x_m, t) > 1 - \epsilon$, for all $n, m \geq n_0$. Equivalently, $\{x_n\}$ is Cauchy if $\lim_{n,m\to+\infty} M(x_n, x_m, t) = 1$, for all $t > 0$.*

Lemma 1. *(Grabiec, 1988) Let $(X, M, *)$ be a fuzzy metric space. Then $M(x, y, .)$ is non-decreasing for all $x, y \in X$.*

Theorem 1. *(George & Veeramani, 1994) Let $(X, M, *)$ be a fuzzy metric space. A sequence $\{x_n\}_{n\in N}$ in X converges to $x \in X$ if and only if $\lim_{n\to\infty} M(x_n, x, t) = 1$.*

Definition 4. *(George & Veeramani, 1994) $(X, M, *)$ (or simply X) is called M-complete if every M-Cauchy sequence in X is convergent.*

Lemma 2. *(Saha et al, 2016) If $*$ is a continuous t-norm $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ are sequences such that $\alpha_n \to \alpha, \beta_n \to \beta$ and $\gamma_n \to \gamma$ as $n \to +\infty$ then*

$$\overline{\lim}_{k\to+\infty}(\alpha_k * \beta_k * \gamma_k) = \alpha * \overline{\lim}_{k\to+\infty}\beta_k * \gamma,$$

and

$$\underline{\lim}_{k\to+\infty}(\alpha_k * \beta_k * \gamma_k) = \alpha * \underline{\lim}_{k\to+\infty}\beta_k * \gamma.$$

LEMMA 3. *(Saha et al, 2016) Let $\{f(k,.) : (0,+\infty) \to (0,1]; k = 0,1,2,...,\}$ be a sequence of functions such that $f(k,.)$ is continuous and monotone increasing for each $k \geq 0$. Then $\overline{\lim}_{k\to+\infty}f(k,t)$ is a left continuous function in t and $\underline{\lim}_{k\to+\infty}f(k,t)$ is a right continuous function in it.*

Denote $\triangle = \{\delta|\delta : (0,1] \to (0,1]\}$ where δ is right continuous.

DEFINITION 5. *(Zheng & Wang, 2019) Let $(X, M, *)$ be a fuzzy metric space. A mapping $f : X \to X$ is said to be a fuzzy Meir-Keeler contractive mapping with respect to $\delta \in \triangle$ if the following condition holds:*

$$\text{for all } \epsilon \in (0,1), \epsilon - \delta(\epsilon) < M(x,y,t) \leq \epsilon \ \ implies \ \ M(fx, fy, t) > \epsilon, \quad (1)$$

for all $x, y \in X, t > 0$.

DEFINITION 6. *(Jain et al, 2009) Two self maps f and g in a fuzzy metric space $(X, M, *)$ are said to be weakly compatible if they commute at their coincidence points i.e. for $x \in X, fx = gx = y$ implies $gy = fy$.*

Interpolative generalised Meir-Keeler contraction

DEFINITION 7. *Let $(X, M, *)$ be a fuzzy metric space. A pair (f, g) of self maps in X is said to be an interpolative generalised Meir-Keeler contractive if there exists $\alpha, \beta \in [0,1)$ with $\alpha + \beta < 1$ and for all $x, y \in X, t > 0$*

$$\text{for all } \epsilon \in (0,1), \epsilon - \delta(\epsilon) < M(x,y) \leq \epsilon \ \ implies \ \ M(fx, fy, t) > \epsilon, \quad (2)$$

where

$$M(x,y) = (M(fx,gx,t))^{\alpha}(M(fy,gy,t))^{\beta}(M(gx,gy,t))^{(1-\alpha-\beta)}.$$

REMARK 1. From equation (2) for all $x \neq y \in X, t > 0$ the pair (f, g) is a strict contraction i.e.

$$M(fx, fy, t) > M(x, y).$$

Thus for $x \neq y$.

$$M(fx, fy, t) > (M(fx,gx,t))^{\alpha}(M(fy,gy,t))^{\beta}(M(gx,gy,t))^{(1-\alpha-\beta)} \quad (3)$$

REMARK 2. Taking $g = I$, the identity map in equation (2)we obtain

$$\text{for all } \epsilon \in (0,1), \epsilon - \delta(\epsilon) < M(x,y) \leq \epsilon \ \ implies \ \ M(fx,fy,t) > \epsilon, \qquad (4)$$

where

$$M(x,y) = (M(fx,x,t))^{\alpha}(M(fy,y,t))^{\beta}(M(x,y,t))^{(1-\alpha-\beta)}.$$

which is an interpolative generalised Meir-Keeler contraction, for a self map f.

REMARK 3. Taking $\alpha = 0, \beta = 0$ in equation (4) then $M(x,y) = M(x,y,t)$ and we have

$$\text{for all } \epsilon \in (0,1), \epsilon - \delta(\epsilon) < M(x,y,t) \leq \epsilon \ \ implies \ \ M(fx,fy,t) > \epsilon, \qquad (5)$$

which is precisely the Meir-Keeler contraction, for a self map given by Zheng and Wang (Zheng & Wang, 2019).

LEMMA 4. *(Zheng & Wang, 2019) If $\delta \in \triangle$, then for $t \in (0,1)$, there exists $k = k(t) \in N$ such that*
$t + \frac{\delta(t)}{k} < 1$ and $\delta\left(t + \frac{\delta(t)}{k}\right) - \frac{\delta(t)}{k} > 0.$

Before we prove the main result, we prove the following lemma:

LEMMA 5. *Let (f,g) be a pair of an interpolative Meir-Keeler contractive mapping with respect to $\delta \in \triangle$ and $f(X) \subseteq g(X)$. Construct a sequence $\{y_n\}$, by $fx_n = gx_{n+1} = y_n$, for $n = 0,1,2,\ldots$. Then $\lim_{n\to+\infty} M(y_n, y_{n+1}, t) = 1$.*

Proof. Suppose if possible on the contrary that $\lim_{n\to+\infty} M(y_n, y_{n+1}, t) = p(<1)$. For $\alpha, \beta \in [0,1)$ we have

$$\begin{aligned}
\lim_{n\to+\infty} M(x_n, x_{n+1}) &= \lim_{n\to+\infty} \left\{ \begin{array}{c} ((M(fx_n, gx_n, t))^{\alpha}(M(fx_{n+1}, gx_{n+1}, t))^{\beta} \\ (M(gx_n, gx_{n+1}, t))^{(1-\alpha-\beta)} \end{array} \right\} \\
&= \lim_{n\to+\infty} \left\{ \begin{array}{c} (M(y_n, y_{n-1}, t))^{\alpha}(M(y_{n+1}, y_n, t))^{\beta} \\ (M(y_{n-1}, y_{n+}, t))^{(1-\alpha-\beta)} \end{array} \right\} \\
&= \lim_{n\to+\infty} \left(p^{\alpha} p^{\beta} p^{(1-\alpha-\beta)}\right) \\
&= p.
\end{aligned}$$

By using lemma (4), for $p < 1$ and $\delta \in \triangle$ we can find $k = k(p) \in N$ such that

$p + \frac{\delta(p)}{k} < 1$ and $\delta\left(p + \frac{\delta(p)}{k}\right) - \frac{\delta(p)}{k} > 0.$

Since $\lim_{n\to+\infty} M(x_n, x_{n+1}) = p$, we can find n_0 such that when $n > n_0$,

$$M(x_n, x_{n+1}) > p + \frac{\delta(p)}{k} - \delta\left(p + \frac{\delta(p)}{k}\right). \tag{6}$$

Also, there exists n_1 such that whenever $n > n_1$,

$$M(x_n, x_{n+1}) < p + \frac{\delta(p)}{k}. \tag{7}$$

Let $n > \max\{n_0, n_1\}$. Then both equations (6), and (7) hold for such n. Taking $\epsilon = p + \frac{\delta(p)}{k}$, in equation (2) we get

$$M(fx_n, fx_{n+1}, t) > p + \frac{\delta(p)}{k},$$

i. e. $M(y_n, y_{n+1}, t) > p + \frac{\delta(p)}{k}$, which contradicts the fact that $\lim_{n\to+\infty} M(y_n, y_{n+1}, t) = p$. Therefore, $\lim_{n\to+\infty} M(y_n, y_{n+1}, t) = 1.$ □

Main results

Our first new result is the next one:

THEOREM 2. : *Let f and g be self maps in a fuzzy metric space $(X, M, *)$ satisfying the following conditions:*

(4.11) $f(X) \subseteq g(X)$;

(4.12) The pair (f, g) is an interpolative generalised Meir-Keeler contraction;

(4.13) $f(X)$ is complete;

(4.14) The pair (f, g) is weakly compatible.

Then f and g have a unique common fixed point in X if and only if there exists $x_0 \in X$ such that $\bigwedge_{t>0} M(x_0, f(x_0), t) > 0.$

Proof. : Suppose the pair (f, g) has a unique common fixed point u then $u = fu = gu$. Therefore, $M(u, fu, t) = 1, \forall t > 0$. Hence

Jain, S. et al, Interpolative generalised Meir-Keeler contraction, pp.818–835

$\bigwedge_{t>0} M(u, fu, t) > 0.$

Conversely, suppose that there exists $x_0 \in X$ such that $\bigwedge_{t>0} M(x_0, f(x_0), t) > 0$. Construct a sequence $\{y_n\}$, by defining $fx_n = gx_{n+1} = y_n$, for $n = 0, 1, 2, \ldots$. First we show that if the two maps f and g have a common fixed point then it is unique. Let u and v be two common fixed points of f and g. Then $u = fu = gu$ and $v = fv = gv$. We show that $u = v$.

Suppose, on the contrary that $u \neq v$, then $fu \neq fv$. Now

$$\begin{aligned} M(u,v) &= (M(fu,gu,t))^{\alpha}(M(fv,gv,t))^{\beta}(M(gu,gv,t))^{(1-\alpha-\beta)}, \\ &= (M(u,u,t))^{\alpha}(M(v,v,t))^{\beta}(M(u,v,t))^{(1-\alpha-\beta)}, \\ &= (M(u,v,t))^{(1-\alpha-\beta)}. \end{aligned}$$

Now

$$\begin{aligned} M(u,v,t) &= M(fu,fv,t), \\ &> M(u,v), \text{ using (3)} \\ &= (M(u,v,t))^{(1-\alpha-\beta)} \end{aligned}$$

i. e. $M(u,v,t) > (M(u,v,t))^{(1-\alpha-\beta)}$,
implies

$$(M(u,v,t))^{(\alpha+\beta)} > 1,$$

which is not true as the left hand quantity is less than 1.So $u = v$. Thus, if the pair (f, g) has a common fixed point then it is unique.
Step 1 To see the existence of a common fixed point of the self maps f and g, we consider the following cases.
CASE I Suppose any two terms of the sequence $\{y_n\}$ are equal i. e. for some $n \in N, y_n = y_{n+1}$. As $y_n = fx_n = gx_{n+1} = fx_{n+1} = gx_{n+2} = y_{n+1}$ we have $fx_{n+1} = gx_{n+1}$. Let $fx_{n+1} = gx_{n+1} = z$. So x_{n+1} is a point of coincidence of the pair (f, g). As the pair (f, g) is weakly compatible we have $fz = gz$. Now we show that $fz = z$. Suppose, if possible on the contrary, that $fz \neq z$ so $fz \neq fx_{n+1}$. By using equation (3) we have

$$\begin{aligned} M(z,fz,t) &= M(fx_{n+1},fz,t), \\ &> (M(fx_{n+1},gx_{n+1},t))^{\alpha}(M(fz,gz,t))^{\beta}(M(gx_{n+1},gz,t))^{(1-\alpha-\beta)}, \end{aligned}$$

$$\begin{aligned} &= (M(z,z,t))^{\alpha}(M(fz,fz,t))^{\beta}(M(z,fz,t))^{(1-\alpha-\beta)}, \\ &= (M(z,fz,t))^{(1-\alpha-\beta)}. \end{aligned}$$

i.e.
$M(z,fz,t) > (M(z,fz,t))^{(1-\alpha-\beta)}$, for all $t > 0$ implies $(M(z,fz,t))^{(\alpha+\beta)} > 1$, which is not possible. Hence $fz = z$. Therefore, z is a common fixed point of the pair (f,g) in this case.

So we can assume the consecutive terms of the sequence $\{y_n\}$ are distinct.

Again, to see the existence of a common fixed point in other cases, we first show that all the terms of the sequence $\{y_n\}$ are distinct.

CASE II Suppose $y_n = y_m$, for some $m > (n+1)$, then as all the consecutive terms of the sequence $\{y_n\}$ are distinct, we claim that $y_{n+1} = y_{m+1}$. Suppose if possible on the contrary $y_{n+1} \neq y_{m+1}$ then $y_n \neq y_{n+1} \neq y_{m+1} \neq y_m$ implies $y_n \neq y_m$ which contradicts our assumption. So we have $y_{n+1} = y_{m+1}$. Also and

$$\begin{aligned} M(y_{n+1},y_{n+2},t) &= M(fx_{n+1},fx_{n+2},t) \\ &> \left\{ \begin{array}{c} (M(fx_{n+1},gx_{n+1},t))^{\alpha}(M(fx_{n+2},gx_{n+2},t))^{\beta} \\ (M(gx_{n+1},gx_{n+2},t))^{(1-\alpha-\beta)} \end{array} \right\} \\ &= \left\{ \begin{array}{c} (M(y_{n+1},y_n,t))^{\alpha}(M(y_{n+2},y_{n+1},t))^{\beta} \\ (M(y_n,y_{n+1},t))^{(1-\alpha-\beta)} \end{array} \right\} \\ &= (M(y_{n+1},y_n,t))^{1-\beta}(M(y_{n+2},y_{n+1},t))^{\beta} \end{aligned}$$

i. e.

$$M(y_{n+1},y_{n+2},t) > (M(y_{n+1},y_n,t))^{1-\beta}(M(y_{n+2},y_{n+1}t))^{\beta}.$$

Thus

$$M(y_n,y_{n+1},t) < M(y_{n+1},y_{n+2},t). \tag{8}$$

So

$$M(y_n,y_{n+1},t) < M(y_{n+1},y_{n+2},t) < M(y_{n+2},y_{n+3},t) < ... < M(y_m,y_{m+1},t).$$

i. e. $M(y_n,y_{n+1},t) < M(y_n,y_{n+1},t)$, which is not possible. So this case does not arise.

Thus, we conclude that for distinct $n, m \in N, y_n \neq y_m$. Therefore, the elements of the sequence $\{y_n\}$ are distinct. From equation (8) we have

$$M(y_n, y_{n+1}, t) < M(y_{n+1}, y_{n+2}, t),$$

for all $t > 0$. Thus, $\{M(y_n, y_{n+1}, t)\}$, for each $t > 0$, is a strictly increasing sequence, which is bounded above by 1. Therefore, by lemma 5, for $t > 0$,

$$\lim_{n \to +\infty} M(y_n, y_{n+1}, t) = 1, \tag{9}$$

Now we prove that the sequence $\{y_n\}$ is M-Cauchy. Suppose if possible on the contrary that it is not true; then there exist $\eta \in (0,1), t_0 > 0$ and the sequences $\{p(n)\}, \{q(n)\}$ ($p(n)$ being the smallest ones of the index)

$$n < p(n) < q(n), M(y_{p(n)}, y_{q(n)}, t_0) \leq 1 - \eta, M(y_{p(n)-1}, y_{q(n)}, t_0) > 1 - \eta. \tag{10}$$

STEP 2: In this step, we show that $\lim_{n\to\infty} M(y_{p(n)-1}, y_{q(n)-1}, t_0) = 1 - \eta$. Now for all $n \geq 1, 0 < \lambda < t_0/2$, we obtain,

$$\begin{aligned} 1 - \eta \;&\geq M(y_{p(n)}, y_{q(n)}, t_0), \qquad \text{using (10)} \\ &\geq M(y_{p(n)}, y_{p(n)-1}, \lambda) * M(y_{p(n)-1}, y_{q(n)-1}, t_0, \lambda - 2) * \\ &*M(y_{q(n)-1}, y_{q(n)}, \lambda). \end{aligned} \tag{11}$$

Let

$$h_1(t) = \overline{\lim_{n\to+\infty}} M(y_{p(n)-1}, y_{q(n)-1}, t), t > 0.$$

Taking the limit supremum on both sides of equation (11), and using the properties of M and $*$, and by lemma 3, we obtain

$$\begin{aligned} 1 - \eta \;&\geq 1 * \overline{\lim}_{n\to+\infty} M(y_{p(n)-1}, y_{q(n)-1}, t_0 - 2\lambda) * 1 \qquad \text{using (10)} \\ &= h_1(t_0 - 2\lambda). \end{aligned}$$

Since M is bounded with the range in $(0, 1]$, continuous and non-decreasing in the third variable t, it follows from lemma 3, that h_1 is continuous from the left. Therefore, for $\lambda \to 0$, we obtain

$$h_1(t_0) = \overline{\lim_{n\to+\infty}} M(y_{p(n)-1}, y_{q(n)-1}, t_0) \leq 1 - \eta. \tag{12}$$

Let

$$h_2(t) = \underline{\lim}_{n\to+\infty} M(y_{p(n)-1}, y_{q(n)-1}, t), t > 0.$$

Again, for all $n \geq 1, \lambda > 0$

$$M(y_{p(n)-1}, y_{q(n)-1}, t_0 + \lambda) \;\;\geq\;\; M(y_{p(n)-1}, y_{q(n)}, t_0) * M(y_{q(n)}, y_{q(n)-1}, \lambda)$$

$$> \quad (1-\eta) * M(y_{q(n)}, y_{q(n)-1}, t_0) \quad \text{using (10)}$$

Taking the limit infimum as $n \to +\infty$ in the above inequality, we obtain

$$\begin{aligned} h_2(\lambda + t_0) &= \underline{\lim}_{n\to+\infty} M(y_{q(n)-1}, y_{q(n)-1}, \lambda + t_0), \\ &\geq (1-\eta) * \underline{\lim}_{n\to+\infty} M(y_{p(n)}, y_{q(n)-1}, \lambda), \qquad \text{using (9)} \\ &= (1-\eta) * 1 \\ &= 1-\eta. \end{aligned}$$

Since M is bounded with the range in $(0,1]$, continuous and non-decreasing in the third variable t, it follows from lemma 3, that h_2 is continuous from the right. So letting $\lambda \to 0$, we obtain

$$\underline{\lim}_{n\to+\infty} M(y_{p(n)-1}, y_{q(n)-1}, t_0) \geq (1-\eta). \tag{13}$$

Combining the inequalities (12) and (13), we get

$$\lim_{n\to+\infty} M(y_{p(n)-1}, y_{q(n)-1}, t_0) = (1-\eta). \tag{14}$$

STEP 3 In this step, we show that $\lim_{n\to+\infty} M(y_{p(n)}, y_{q(n)}, t_0) = 1-\eta$. From equation (10) we have

$$\overline{\lim}_{n\to+\infty} M(y_{p(n)}, y_{q(n)}, t_0) \leq 1-\eta. \tag{15}$$

Also for all $n \geq 1$ and $\lambda > 0$ we have

$$\begin{aligned} M(y_{p(n)}, y_{q(n)}, t_0 + 2\lambda) \;&\geq\; M(y_{p(n)}, y_{p(n)-1}, \lambda) * M(y_{p(n)-1}, y_{q(n)-1}, t_0) * \\ &* \; M(y_{q(n)-1}, y_{q(n)}, \lambda) \end{aligned}$$

Taking the limit infimum as $n \to +\infty$ in the above inequality, using (9), (14) and the properties of M and $*$ and by lemma 2, we obtain

$$\begin{aligned} \underline{\lim}_{n\to+\infty} M(y_{p(n)}, y_{q(n)}, t_0 + 2\lambda) &\geq 1 * \underline{\lim}_{n\to+\infty} M(y_{p(n)-1}, y_{q(n)-1}, t_0) * 1 \\ &= 1-\eta, \quad \text{using (14)} \end{aligned} \tag{16}$$

Since M is bounded with the range in $(0,1]$, continuous and non-decreasing in the third variable t, it follows from lemma 3 that $\underline{\lim}_{n\to+\infty} M(y_{p(n)}, y_{q(n)}, t_0)$ is a continuous function of t from the right.

Therefore, for $\lambda \to 0$, we obtain

$$\underline{\lim}_{n\to+\infty} M(y_{p(n)}, y_{q(n)}, t_0) \geq (1-\eta). \tag{17}$$

Combining inequalities (15) and (17), we get

$$\lim_{n\to+\infty} M(y_{p(n)}, y_{q(n)}, t_0) = (1-\eta). \tag{18}$$

STEP 4 In this step, we show that the sequence $\{y_n\}$ is an M-Cauchy sequence.

Using equations (9) and (14) at $t = t_0$, we have

$$\begin{aligned} M(x_{p(n)}, x_{q(n)}) &= \left\{ \begin{array}{c} (M(fx_{p(n)}, gx_{p(n)}, t_0))^{\alpha}(M(fx_{q(n)}, gx_{q(n)}, t_0))^{\beta} \\ (M(gx_{p(n)}, gx_{q(n)}, t_0))^{(1-\alpha-\beta)} \end{array} \right\} \\ &= \left\{ \begin{array}{c} (M(y_{p(n)}, y_{p(n)-1}, t_0))^{\alpha}(M(y_{q(n)}, y_{q(n)-1}, t_0))^{\beta} \\ (M(y_{p(n)-1}, y_{q(n)-1}, t_0))^{(1-\alpha-\beta)} \end{array} \right\} \end{aligned}$$

. (19)

Therefore

$$\lim_{t=t_0, n\to+\infty} M(x_{p(n)}, x_{q(n)}) = (1-\eta)^{(1-\alpha-\beta)}. \tag{20}$$

And

$$\begin{aligned} M(x_{p(n)}, x_{q(n)}) &= \left\{ \begin{array}{c} (M(fx_{p(n)}, gx_{p(n)}, t_0))^{\alpha}(M(fx_{q(n)}, gx_{q(n)}, t_0))^{\beta} \\ (M(gx_{p(n)}, gx_{q(n)}, t_0)^{(1-\alpha-\beta)} \end{array} \right\} \\ &> M(fx_{p(n)}, fx_{q(n)}, t_0) \\ &= M(y_{p(n)}, y_{q(n)}, t_0). \end{aligned}$$

$$M(y_{p(n)}, y_{q(n)}, t_0) > M(x_{p(n)}, x_{q(n)}) \tag{21}$$

For $n \to +\infty$ and using equations (20) and (21), we have

$$(1-\eta) \geq (1-\eta)^{(1-\alpha-\beta)},$$

implies that $(1-\eta)^{(\alpha+\beta)} \geq 1.$, which is not possible as$(1-\eta) < 1$. So,$\{y_n\}$ is an M-Cauchy sequence in $g(X)$ which is M-complete. Therefore, there exists $z \in g(X)$ such that

$$\{y_n\} \to z. \tag{22}$$

i. e.

$$\{fx_n\} \to z \quad \text{and} \quad \{gx_{n+1}\} \to z. \tag{23}$$

As $z \in g(X)$ there exists $v \in X$ such that

$$z = gv. \tag{24}$$

STEP 5 Now we show that $gv = fv$. Suppose, on the contrary, that $fv \neq gv(= w)$. Then exists a positive integer n_0 such that $gv \neq gx_n$, for all $n \geq n_0$.

$$\begin{aligned} M(x_n, v) &= (M(fx_n, gx_n, t))^{\alpha}(M(fv, gv, t))^{\beta}(M(gx_n, gv, t)^{(1-\alpha-\beta)} \\ &= (M(y_n, y_{n-1}, t))^{\alpha}(M(fv, z, t))^{\beta}(M(gx_n, z, t)^{(1-\alpha-\beta)}. \end{aligned}$$

Now

$$\begin{aligned} M(fx_n, fv, t) &> M(x_n, v) \\ &= (M(y_n, y_{n-1}, t))^{\alpha}(M(fv, z, t))^{\beta}(M(gx_n, z, t))^{(1-\alpha-\beta)}. \end{aligned}$$

For $n \to +\infty$ and using equations (9), (23) and (24) we get $M(z, fv, t) \geq [M(fv, z, t)]^{\beta}$ i. e. $M(fv, z, t)]^{1-\beta} > 1$, which is not possible if $fv \neq z$. Hence $fv = u$ and we have

$$fv = gv = z. \tag{25}$$

As the pair of self maps (f, g) is weakly compatible, we have

$$fz = gz. \tag{26}$$

STEP 6 Now we show that $fz = z$. Suppose, on the contrary that $fz \neq z$. Then $gz \neq z$.

$$\begin{aligned} M(z, v) &= (M(fz, gz, t))^{\alpha}(M(fv, gv, t))^{\beta}(M(gz, gv, t)^{(1-\alpha-\beta)} \\ &= (M(fz, z, t))^{(1-\alpha-\beta)} \qquad \text{using (25, 26)} \end{aligned}$$

and

$$M(fz, z, t) = M(fz, fv, t)$$

Jain, S. et al, Interpolative generalised Meir-Keeler contraction, pp.818–835

$$
\begin{aligned}
&> \quad M(z,v), \qquad \text{using (25)}\\
&= \quad [M(fz,z,t]^{(1-\alpha-\beta)}.
\end{aligned}
$$

i. e. $M(fz,z,t)^{(\alpha+\beta)} > 1$ which is not possible as the left hand side is less than 1.Thus, $fu = gu = u$. □

Taking $g = I$ in Theorem 2, then the sequence $\{x_n\} = \{x_0, fx_0, \cdots f^n x_0, , \cdots\}$ becomes a Picard sequence for the self map f and we have

COROLLARY 1. *Let f be an interpolative fuzzy Meir-Keeler contractive map on a M-complete fuzzy metric space $(X, M, *)$.Then the map f has a unique fixed point in X.*

REMARK 4. If we take $\alpha = 0$ and $\beta = 0$ in the above corollary, we obtain Theorem 3.1 of Zheng and Wang (Zheng & Wang, 2019).

EXAMPLE 1. (of Theorem 4.1) Let $X = [0,1]$. Define a self map $f : X \to X$ by $f(x) = \frac{x}{2}$, and $g(x) = x$, the identity map on X. Taking $M(x,y) = \frac{1}{1+d(x,y)}$, then $(X, M, .)$ is a complete stationary fuzzy metric space with the product t-norm. Define δ as follows:

$$
\delta(t) = \begin{cases} \frac{1}{12}; & \text{if } 0 < t < \frac{3}{4}, \\ \frac{1}{n(n+2)}, & \text{if } \frac{n-1}{n} \le t \le \frac{n}{n+1}, \text{for } n \ge 4. \end{cases}
$$

Then $\delta \in \triangle$.

Taking $\alpha = 0 = \beta$. observe that for all values of $x, y \in X, f(x), f(y) \in [0, \frac{1}{3})$. We show that the quadruple (X, M, δ, f) is an interpolative Meir-Keeler contractive. For this we prove the following condition:

$$
forall \ \ \epsilon \in (\frac{3}{4}, 1), \epsilon - \delta(\epsilon) < M(x,y) \le \epsilon \Longrightarrow M(fx, fy) > \epsilon.
$$

If $\epsilon \in (\frac{3}{4}, 1) \Longrightarrow \frac{n-1}{n} \le \epsilon \le \frac{n}{n+1}$, for $n \ge 4$, so $\delta(t) = \frac{1}{n(n+2)}$.

Therefore, the inequality $\epsilon - < M(x,y) \le \epsilon$ gives $\left(\frac{n-1}{n}\right) - \frac{1}{n(n+2)} < \epsilon - \delta(\epsilon) < \frac{1}{1+d(x,y)} \le \epsilon < \frac{n}{n+1}$. Therefore $\frac{1}{n} \le d(x,y) < \frac{2}{n}$

which implies that $x, y \in [0,1]$. Hence

$$M(fx, fy) = \frac{1}{1 + d(fx, fy)} = \frac{1}{1 + \frac{d(x,y)}{3}} > \frac{1}{1 + \frac{1}{n}} = \frac{n}{n+1} > \epsilon.$$

Thus, the quadruple (X, M, δ, f) is an interpolative Meir-Keeler contractive and $x = 0$ is the unique fixed point of the map f.

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ИНТЕРПОЛЯЦИОННОЕ ОБОБЩЕННОЕ СЖАТИЕ МЕИРА-КЕЛЛЕРА

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РУБРИКА ГРНТИ: 27.25.17 Метрическая теория функций,
27.33.00 Интегральные уравнения,
27.39.29 Приближенные методы функционального анализа

ВИД СТАТЬИ: оригинальная научная статья

Резюме:

Введение/цель: Цель данной статьи заключается в введении понятия интерполяционного обобщенного условия сжатия Меира-Келлера для отображений в нечетком метрическом пространстве, которое расширяет, объединяет и обобщает многообразие Меира-Келлера, предназначенное только для одного отображения. При применении устанавливается единая теорема о совместной неподвижной точке для двух отображений через слабую совместимость. В статье приведен пример, доказывающий достоверность результатов исследования.

Методы: Методы функционального анализа с сокращением Меира-Келлера.

Результаты: Получена уникальная неподвижная точка для отображений в нечетком метрическом пространстве.

Выводы: Получена неподвижная точка собственных отображений.

Ключевые слова: нечеткое метрическое пространство, общие фиксированные точки, слабая совместимость, интерполяционное обобщенное сокращение Меира-Келлера.

ИНТЕРПОЛАТИВНА УОПШТЕНА МЕИР-КЕЛЕРОВА КОНТРАКЦИЈА

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КАТЕГОРИЈА (ТИП) ЧЛАНКА: оригинални научни рад

Сажетак:

Увод/циљ: Циљ овог рада је да се уведе појам интерполативног генерализованог Меир-Келеровог контрактивног услова за пресликавања у фузиметричком простору. Он увећава, обједињује и генерализује Меир-Келерову контракцију и служи за само једно пресликавање. Користећи га, успостављамо јединствену заједничку теорему фиксне тачке за два пресликавања кроз слабу компатибилност. Рад садржи пример који показује валидност наших резултата.

Методе: Методе функционалне анализе са Меир-Келеровом контракцијом.

Резултати: Јединствена фиксна тачка за пресликавања у фузипростору је добијена.

Закључак: Фиксна тачка пресликавања самог у себе је добијена.

Кључне речи: фузиметрички простор, заједничке фиксне тачке, слаба компатибилност, интерполативна генерализована Меир-Келерова контракција.

EDITORIAL NOTE: The third author of this article, Stojan N. Radenović, is a current member of the Editorial Board of the *Military Technical Courier*. Therefore, the Editorial Team has ensured that the double blind reviewing process was even more transparent and more rigorous. The Team made additional effort to maintain the integrity of the review and to minimize any bias by having another associate editor handle the review procedure independently of the editor – author in a completely transparent process. The Editorial Team has taken special care that the referee did not recognize the author's identity, thus avoiding the conflict of interest.

КОММЕНТАРИЙ РЕДКОЛЛЕГИИ: Третий автор данной статьи Стоян Н. Раденович является действующим членом редколлегии журнала *«Военно-технический вестник»*. Поэтому редколлегия провела более открытое и более строгое двойное слепое рецензирование. Редколлегия приложила дополнительные усилия для того чтобы сохранить целостность рецензирования и свести к минимуму предвзятость, вследствие чего второй редактор-сотрудник управлял процессом рецензирования независимо от редактора-автора, таким образом процесс рецензирования был абсолютно прозрачным. Редколлегия во избежание конфликта интересов позаботилась о том, чтобы рецензент не узнал кто является автором статьи.

РЕДАКЦИЈСКИ КОМЕНТАР: Трећи аутор овог чланка Стојан Н. Раденовић је актуелни члан Уређивачког одбора *Војнотехничког гласника*. Због тога је уредништво спровело транспарентнији и ригорознији двоструко слепи процес рецензије. Уложило је додатни напор да одржи интегритет рецензије и необјективност сведе на најмању могућу меру тако што је други уредник сарадник водио процедуру рецензије независно од уредника аутора, при чему је тај процес био апсолутно транспарентан. Уредништво је посебно водило рачуна да рецензент не препозна ко је написао рад и да не дође до конфликта интереса.

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SCREEN READING: ELECTROMAGNETIC INFORMATION LEAKAGE FROM THE COMPUTER MONITOR

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FIELD: Computer sciences, Electronics, Telecommunications, Mechanical engineering
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Abstract:

Introduction/purpose: The security of systems can be jeopardized by compromising emanations. This paper provides an overview of computer screen attacks. New technologies can be used to exfiltrate sensitive data from computer screens. Emission security is the prevention of electromagnetic signal attacks that are conducted or radiated.

Methods: This paper examines the impact of a side-channel attack that intercepts compromised information from a computer screen. The leakage of electromagnetic data is also explained. Software-defined radios are described to explain malicious attacks on computer monitors.

Results: The source of the electromagnetic signal determines the nature of the side-channel information they carry. The most well-known issue associated with revealing emissions is the possibility of intercepting visual information displayed on computer monitors.

Conclusion: Visual data displayed on computer monitors could be intercepted by a software-defined radio which can digitize the desired frequency spectrum directly from an antenna, present it to a digital signal processor, and output it to an application for revealing sensitive data. A *variety of countermeasures, such as shielding, zoning, soft TEMPEST, and similar techniques, can be used to prevent data leakage.*

Key words: electromagnetic emission, information leakage, computer monitor.

Introduction

In recent years, new technologies have made it possible to exfiltrate sensitive data from a computer by monitoring the computer screen in a variety of novel ways that do not require network connectivity or physically contacting devices via the invisible channel determined by the computer screen. Because the user does not have a visual perception of what is happening, malware on the compromised computer can obtain sensitive data such as files, images, or passwords. The prevention of attacks using electromagnetic (EM) signals that are either conducted or radiated is referred to as emission security. By formulating that "changing electrical currents induce changing magnetic fields, which induce changing currents and induce a changing magnetic field that propagates as an EM wave through surrounding space," Oersted, Faraday, and Henry discovered the physics of EM emanation (Rowe, 2006). This field can be picked up by nearby electrical conductors and, through EM interference, can impede the operation of other electromagnetic devices. As a result, an antenna with an amplifier can pick up some signal from a computer and reconstruct generated electrical signals (Rowe, 2006). Military and commercial organizations are very concerned about the Transient Electro Magnetic Pulse Emanation Standard (TEMPEST) defence which prevents the stray EM pulses emitted by computers and other electronic devices from being picked up and used to reconstruct the sensitive data (Markagić, 2018, pp.143-153). TEMPEST has recently become a commercial issue for electronic voting machines and smart cards used for digital signatures. Side-channel attacks refer to a variety of attacks that take advantage of optical, thermal and acoustic emanations from the equipment. This happens when information leaks through a channel that is not intended for communication. Electromagnetic eavesdropping attacks can cause a computer to emit a stronger signal than usual and modulate the signal so that it can pass through the firewall.

Electromagnetic compatibility (EMC) and radio frequency interference (RFI) are closely related to EM security measures. All emission security issues are expected to worsen as more devices connect to wireless networks and processor speeds increase into the gigahertz range. There are two types of electromagnetic attacks that are not mutually exclusive:

1) when the signal is transmitted over a circuit such as a power line or phone line, it is known as Highjack and 2) when the signal is transmitted as radio frequency (RF) energy, it is known as TEMPEST. Properly shielded equipment is typically limited in quantity and designed specifically for defence markets, making it extremely expensive. The operating rooms must also be properly filtered.

Screen signals can be found in a variety of locations across computer networks. These signals may contain multiple harmonics, some of which radiate more effectively than others, owing to the designed equipment being certified to not emit any signals beyond a certain distance. Spying on the surface of a screen with a powerful telescope is a very basic approach to spying on the content displayed on it (Lavaud et al, 2021). Khun (2002), Backes et al (2008), and Backes et al (2009), on the other hand, describe several more efficient ways to attack computer monitor content. Computer monitors leak electromagnetic information as a result of three key factors used to reproduce video images: (1) refresh rate, (2) horizontal frequency, and (3) pixel frequency, which is the display principle (Mao et al, 2017). One method for estimating the risk of information leakage is to use multi-resolution spectrum analysis to distinguish and match the spectrum interval from the radiated EM signals.

This paper investigates the impact of how a side-channel attack causes compromised information to be taken from a computer screen. This paper also discusses the leakage of electromagnetic information from computer screens. To explain potential malicious attacks on computer monitors, software-defined radios (SDRs) are described.

Side-channel attacks

The security of a cryptosystem (cryptographic algorithms and protocols, cryptographic keys, and cryptographic devices used for implementation) is dependent on more than just using robust algorithms and parameters, certified protocols, and cryptographic keys that are long enough. Physical attacks on a system can also be used to compromise it. Side-channel attacks are generally physical attacks in which malicious parties extract confidential and protected data by observing how systems physically behave (Barthe et al, 2018). These attacks use the dependency between secret information used in the cryptosystem and physical values measured on/around the cryptosystem (e.g. power consumption, electromagnetic radiation, timing information) to break a system (Mangard et al, 2007). Table 1 depicts the classification of side-channel emanation (Lavaud et al, 2021). Each side-channel attack seeks to exploit an unintentional emission. As a result, the subject of side-channel attacks covers a broad range of techniques (Sayakkara et al, 2018). Side-channel information sources, such as EM emanations from a chip (Agrawal et al, 2003) and timings for various operations performed (Kocher, 1996, pp.104-113) have also been demonstrated to be exploitable (Mangard et al, 2007). Hayashi et al (2014, pp.954-965) conducted a thorough examination of EM emanations from a chip in-depth, including countermeasures. Their primary focus, however, was on recovering sensitive information from inside the computer systems (cryptographic keys, not-the-screen content). Kinugawa et al (2019, pp.62-90) demonstrate how to increase the EM leakage with a (cheap) hardware modification added to potentially any device and spread the attack over a greater distance. The authors show that the additional circuitry (interceptor) increases leakage and forces leakage in devices that are not susceptible to EM leakage.

Table 1 – Side channel emanation
Таблица 1 – Утечка по стороннему каналу
Табела 1 – Еманација успед споредних ефеката

SIDE-CHANNEL EMANATION	
Power line	Keyboard Internal components Cryptosystems
Sound	Speakers Internal components External components
Light	Status LED Internal components Screens
Electromagnetic	Radio radiation Forced broadcast

Goller & Sigl (2015, pp.255-270) proposed to perform side-channel attacks on smartphones using standard radio equipment. The authors also show the ability to distinguish between squaring and multiplications. This discovery may result in the complete recovery of the Rivest, Shamir, and Adelman (RSA) key (Jonsson & Kaliski, 2003). Their setup gathered electromagnetic leaks from an Android phone. Genkin et al (2015, pp.95-112), and Genkin et al (2019, pp.853-869) present the extraction of cryptographic keys such as RSA or ElGamal from laptops using various side channels such as power and EM radiation (Will & Ko, 2015). Furthermore, an adversary may be able to monitor a device's power consumption while it performs secret key operations (Kocher et al, 2011, pp.5-27). Acoustic emanation from various computer system components can be used to exfiltrate data. Genkin et al (2014, pp.444-461) demonstrated that, by listening for acoustic emanation, it is possible to distinguish between CPU operations, resulting in an attack on an RSA algorithm encryption key. Fenkin et al (2019) show how to extract screen content using the acoustic side channel. Microphones can pick up sound from webcams or screens and transmit it during a video conference call or archived recordings. Berger et al (2006, pp.245–254) demonstrated a dictionary attack using keyboard acoustic emanation. Backes et al (2010) investigated acoustic side channels in printers. Asonov and Agrawal (2004) used the sound emitted by different keys to recover information typed on a keyboard. The contribution of Liu et al (2021, pp.1-15) is a side-channel attack analysis that exploits the EM emanations of the display cable from a mobile phone. These signals are more difficult to obtain and may be significantly weaker than those examined in more traditional TEMPEST technique attacks. TEMPEST is a side-channel technique for spying on computer systems via unintentional radio or electrical signals, sounds, and vibrations (Kuhn & Anderson, 1998, pp.124-142). The possibility of intercepting visual information displayed on an electronic device screen is the most well-known issue associated with EM revealing emissions. Van Eck (1985, pp.269-286) is the first to present an unclassified analysis of the feasibility and security risks of computer monitor emanations. He was able to listen in on a real system from hundreds of meters away by measuring electromagnetic emanations with only $15 in equipment and a Cathode-Ray Tube (CRT) television set.

Side-channel attacks have a variety of countermeasures because they are among the most serious threats to embedded crypto devices and frequently target the secret (cryptographic) key in a device that secures sensitive data. The countermeasures' primary goal is to eliminate reliance on sensitive data and the side channel. One method attempts to separate the actual data processed by the device from the data on which the computation is performed (masking) (Prouff & Rivian, 2013, pp.142-159). Another approach attempts to separate the device's computed data from the power consumed by the computations (hiding). One of the countermeasures is also flattening the power consumption of a device. Hardware-based countermeasures propose microarchitecture-based solutions such as providing hardware support for advanced encryption standard (AES) instructions or making caches security-sensitive. Hardware countermeasures are effective, but they can be difficult to implement. In contrast, software countermeasures are simple to implement solutions that can be implemented at the program language level (secure programming guidelines, program transformations). They can also be supported by strict enforcement methods (Bernstein, 2005; Molonar et al, 2005; Barthe et al, 2018).

Electromagnetic information leakage from the computer monitor

EM radiation is the underlying technology for wireless communication, and it is selected based on the distance to be covered, data throughput rate, signal frequency, amount of bandwidth required, modulation technique, power of the transmitted signal, and other factors (Sayakkara et al, 2018). Although wireless communication devices are designed to generate EM radiation at the appropriate frequency and amplitude for the communication technology, as a by-product of their internal operations, these devices also generate EM radiation at unintended frequencies (Genkin et al, 2014, pp.444-461). Unintentional EM emissions from computers can be caused by a variety of factors. The source of each EM signal determines the nature of these EM signals as well as the type of side-channel information they carry. The possibility of intercepting visual information displayed on computer monitors is the most well-known issue associated with the issue of EM revealing emissions. Van Eck (1985, pp.269-286) demonstrated a modified television set that was capable of capturing and visualizing video streams displayed on a nearby television screen. To transmit video data to computer monitors, various protocols are used, necessitating more flexibility than a dedicated hardware-based attack. This article was about CRT monitors. It should be noted that liquid-crystal displays (LCD), which are common output components of computers and currently dominate the market, are not immune to this threat because they are equipped with digital video data (DVD) transmission interfaces. This is not the case, because digital signals, like analogue signals, are susceptible to electromagnetic infiltration and enable non-invasive data acquisition. There is a risk of eavesdropping on the leaked signal because the leakage of the displayed information is quite high. In 2002, Kuhn expands on this eavesdropping concept by conducting an analysis of EM side-channel eavesdropping on modern video display technologies (Kuhn, 2002, pp.3-18). This study employs RF acquisition hardware with fast sampling rates to monitor EM emissions from computer displays. Sekiguchi (2010, pp.127-131) describes receiving EM noise and reconstructing a display image on a touch screen monitor on a personal computer. The experimental results showed that the reconstructed display image can recognize the image of the touched button on the touch screen monitor. Elibol et al (2012, pp.1767-1771) demonstrated a monitor eavesdropping system that remotely reconstructs screen images using RF acquisition hardware. The signal acquisition hardware is a portable platform that can operate at a variety of RF frequencies. In this work, the averaging of adjacent frames is used to improve the readability of the text. In 2016, Lee et al demonstrated the possibility of display information leakage by analysing electromagnetic emissions from desktop and laptop monitors (Lee et al, 2016). By analysing the display mechanism, the characteristics of the leaked signal from the LCD monitor are verified, and electromagnetic emanations are measured over a long distance using an eavesdropping experiment. Using a variety of signal processing techniques, the authors recovered display information.

Software-defined radio: How to spy on?

There have been practical challenges in the more demanding SDR applications, primarily due to analogue to digital conversion (ADC) and digital to analogue conversion (DAC) limitation trade-offs. Many of these compromises are being limited to higher frequencies due to faster ADC/DACs and higher resolution. To avoid being limited to single-frequency ranges and to deal with multiple channels at once, SDR requires a wideband. Wideband performance is required to allow for dynamic spectrum and radio parameter management. The SDR should be able to digitize the desired frequency spectrum directly from an antenna, present it to a DSP processor, and output it to an application, as well as the reverse for a transmitter. The following benefits are provided by SDR: (1) flexibility, (2) interoperability, (3) ease of upgrade, (4) efficiency, and (5) higher-level interfaces. Figure 1 depicts the basic structure of an SDR receiver (Benks, 2016, pp.1-16).

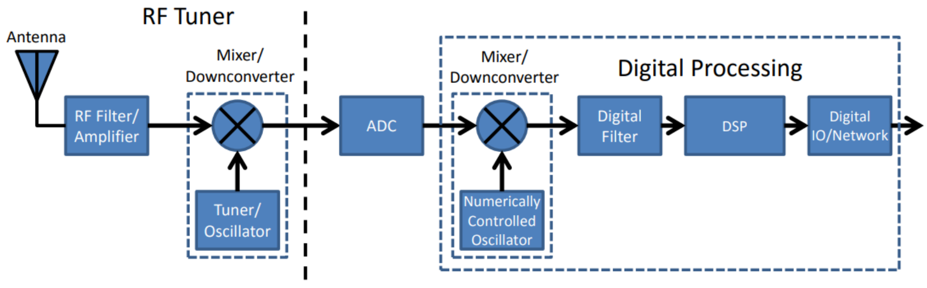

Figure 1 – SDR Receiver (Benks, 2016)
Рис. 1 – SDR приемник (Benks, 2016)
Слика 1 – SDR пријемник

The sampling rate of the RF acquisition hardware is the most important factor in the accuracy of the screen image reconstruction. SDRs provide greater flexibility at a lower cost, but their sampling rate is lower than that of dedicated RF acquisition hardware. Digital processors give radio equipment the flexibility of a programmable system, allowing a communication system to be changed simply by changing its software. Under the SDR paradigm, the task of configuring the radio's behaviour is transferred to software, leaving the hardware only to implement the radiofrequency front end. As a result, the radio is transformed into a dynamic element capable of changing its operational characteristics (bandwidth, modulation, coding rate) based on software configuration. The SDR is defined as "*radio in which some or all physical layer functions are defined by software*" (Garcia Reis et al, 2012). SDR devices use real-time software module execution on microprocessor platforms or digital signal processors, fast programmable gate arrays (FPGA) are commonly used for transmitting or receiving radio signals, the main operational characteristics of SDRs are modified at runtime, and the system can be easily reconfigured to perform different functions (Chamran et al, 2020). SDRs apply to a wide range of radiofrequency technologies, and their standards have made base station software updates more appealing than costly base station replacements. The SDR expands the possibilities by making it easier to implement existing radio applications and enabling new types of applications. The availability of low-cost devices that receive and digitize radiofrequency signals has brought the SDR to both professional and home engineering desks (Stewart et al, 2015, pp.64-71). In their work, Molina-Tenorio et al (2021, pp.1-21) describe the characteristics of the SDR-RTL (Nooelec, 2021), HackRF One (Great scott gadgets, 2021), and LimeSDR Mini (Lime microsystems, 2021) devices. Table 2 lists the main characteristics of these devices.

Table 2 – SDR device characteristics
Таблица 2 – Характеристики устройства SDR
Табела 2 – Карактеристике SDR уређаја

Device	HackRF One	RTL-SDR	LimeSDR Mini
Frequency range	1 MHz-6 GHz	22 MHz-2.2 GHz	10 MHz-3.5 GHz
RF bandwidth	20 MHz	3.2 MHz	30.72 MHz
Sample depth	8 bit	8 bit	12 bit
Sample rate	20 MSPS	3.2 MSPS	30.72 MSPS
Tx channels	1	0	1
Rx channels	1	1	1
Duplex	Half	-	Full
Transmit power	- 10 dBm	-	Max 10 dBm

Rugeles Uribe et al (2021, pp.1-13) compare 19 more commercially available SDR platforms in terms of ADC/DAC, Tx/Rx, Fmin-Fmax and Max RF Bandwidth, all of which were collected in 2019 and 2020: FUNcube Dongle, RSPduo, Airspy-mini, Airspy-R2, Pluto, BladeRF 2.0 Micro, AD-FMCOMMS4-EBZ, USRP-1, PicoSDR, WARP-V3, USRP-N210, TMDSSFFSDR, USRP X310, USRP-2974, AIR-T, Sidekiq X4, USRP N320 and CRIMSON Cyan, all of them being commercially available SDR platforms. The authors also discuss how hardware is evolving to increase computational capacity. Furthermore, the authors present the Bastille Network classification and descriptions of wireless vulnerabilities (Bastille Networks, 2020).

Electromagnetic compatibility standards and regulations in consumer products address emerging threats from eavesdropping attacks using EM side-channels. According to the International Organization for Standardisation (ISO) ITU-T advisory notice K.841, when considering the EMC requirements of consumer devices, information leakage from EM emissions must be considered (ITU, 2014). Many hardware-software tools, however, use SDR platforms for side-channel attacks on computer monitors. TempestSDR, an open-source software library that uses SDR platforms for EM side-channel attacks on computer monitors, is one of the most well-known. It is capable of automatically detecting the dimensions and frame rate of a target when the target monitor details are unknown by identifying repeating patterns in the EM signal that correspond to the individual frames of the video (see Figure 2).

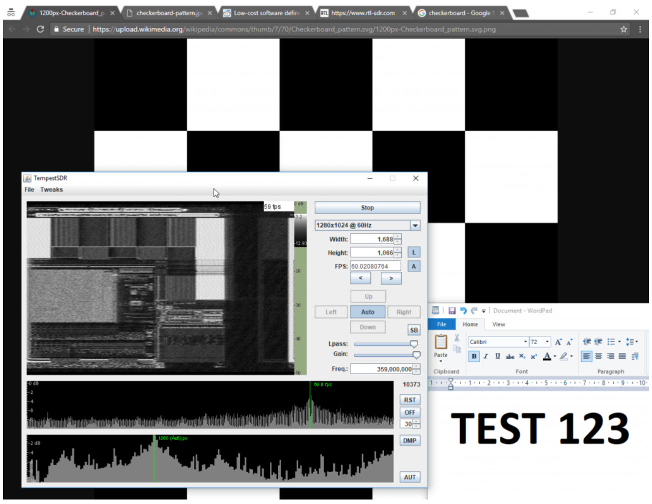

Figure 2 – TempestSDR (RTL-SDR, 2017)
Рис. 2 – TempestSDR (RTL-SDR, 2017)
Слика 2 – Tempest SDR (RTL-SDR, 2017)

TempestSDR allows the user to use any SDR that supports ExtIO (such as those described in Table 2) to receive unintentional signal radiation from the screen and convert that signal back into a live image, allowing them to see what is on screen without a direct connection (RTL-SDR, 2017).

How to defend against computer monitor attacks?

Electronic security protection, like all security measures, must be cost-effective, that is, it must eliminate security issues without interfering with system performance (Doychev, 2016; Rowe, 2006). The sources of unwanted signals must be protected by implementing solutions that effectively prevent the infiltration process from taking place (Levina et al, 2019, pp.393-400; Ometov et al, 2017, pp.2591-2601). Rowe summarizes the suitability of various electronic security methods for mitigating various threats. Table 3 shows the results for the monitor, power, and cables.

Table 3 – Summary of electronic protection methods (Rowe, 2006)
Таблица 2 – Краткое изложение методов электронной защиты (Rowe, 2006)
Табела 2 – Резиме метода електронске заштите (Rowe, 2006)

Threat	Monitor	Cable	Power
Electromagnetic shielding	Yes	Yes	No
Source suppression	Yes	Yes	Yes
Noise generation and encryption	Yes	Yes	Yes
Signal irregularity	Yes	Yes	Yes
Deliberate deception	Yes	Yes	No
Bug detectors	Yes	Yes	Yes

It is essential to emphasize the significance of differences in military and civil security standards. In 2006, Khun predicted that simple eavesdropping tools for compromising emanations would soon be available for free download from the Internet. For today's information security professionals, the question is: What protective countermeasures are available for computers that display extremely sensitive data regularly? These could be jamming devices used to intentionally increase environmental noise, metal shielding, zoning, soft TEMPEST, and other similar techniques. Today, jamming devices are rarely used because the jamming signal must be carefully selected and synchronized with the signal to be covered, and jamming devices may draw the attention of eavesdroppers to the location of equipment. The EM shielding protects devices, cables and rooms against compromising emanations (Kuhn, 2005, pp.265-279; Kuhn, 2006, pp.1-10). To eliminate emanations, the source of the emanations should be placed in metal boxes made of conductive materials (copper, aluminium, steel), also known as Faraday cages. However, perfect protection requires that the conductive enclosure remain intact. Because gaps are required for ventilation, power lines, keyboards, and network connections, these gaps may allow signals to leak out (Warne & Chen, 1992, pp.173-182). Molyneux-Child (1997) recommends at least one-tenth to prevent significant radiation from escaping and one-hundredth to provide a 60 dB reduction. Creating meandering channels through the gaps, as well as using waveguides in the form of conductive pipes through the gaps, can help to reduce the emanations at these gaps. Power lines can be filtered through these gaps, and fibfibretic cables can transmit data without requiring an electromagnetic channel. A conductive film can be applied to monitor screens, but keyboards are more difficult to protect. Due to the difficulty of shielding, both devices and their locations can be classified to indicate how close an eavesdropper can get (Zone 0: eavesdropper can be within 1–20 m; Zone 1: eavesdropper can be within 20–100 m; Zone 2: eavesdropper can be 100 m to 1 km; Zone 3: eavesdropper cannot be closer than a kilometre). These measurements are based on the assumption that only space exists between the eavesdropper and the target. Przybysz et al (2021, pp.1-15) discuss publicly available fragments of the American military requirements NSTISSAM TEMPEST/1-92 (Cryptome, 2008) and NSTISSAM TEMPEST/2-95 (Cryptome, 2000), which define three levels of security for devices that could be used in information processing zones and whose R rays meet the following conditions: R 20 m, R 100 m, and R > 100 m. (Emission levels must be measured from a distance of one meter). The MIL-STD-461G document (EverySpec, 2015) also recommends this measurement distance. The authors concluded promising emission signals emitted by commercial devices can be detected from a few dozen meters away. De Meulemeester et al (2020) confirmed this by demonstrating that visual information could be recovered from a distance of approximately 80 m. Various software countermeasures, such as deliberate softening of font edges or randomization of less significant bits in the frame buffer, can be used to protect against EM leakage from the computer monitor (Duc et al, 2019, pp.1263-1297). Safe fonts are one of the security measures developed using Kubiak's safety criteria (Kubiak, 2020), and they have the following characteristics: (1) The lines that form the characters intersect at right angles, implying that each character is made entirely of vertical and horizontal lines, (2) font characters are devoid of decorative and diagonal elements, and (3) the general contour of safe font characters is rectangular. In the case of a graphic source of valuable emissions, safe fonts protect processed information from EM penetration. These fonts safeguard the Video Graphics Array (VGA) and Digital Video Interface (DVI).

Conclusion

New technologies enable malicious scanning of information emitted by the computer monitor. The TEMPEST defence, which prevents stray EM pulses emitted by computers and other electronic devices from being picked up and used to reconstruct sensitive data, is causing concern among military and commercial organizations. There are two types of EM attacks: highjack and RF energy attacks. Screen signals contain multiple harmonics, some of which radiate more effectively than others. The refresh rate, horizontal frequency, and pixel frequency of computer monitors all leak electromagnetic information. This paper describes how a side-channel attack causes compromised information to be taken from a computer screen. The SDR is used to explain how visual data can be intercepted. To describe the possibility of protecting the data radiated from the computer monitor, a variety of countermeasures such as shielding, zoning, soft TEMPEST, and similar techniques are described.

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ЧТЕНИЕ С ЭКРАНА: ЭЛЕКТРОМАГНИТНАЯ УТЕЧКА ИНФОРМАЦИИ С КОМПЬЮТЕРНОГО МОНИТОРА

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РУБРИКА ГРНТИ: 20.23.25 Информационные системы с базами знаний,
30.03.17 Физические проблемы механики,
30.19.17 Оболочки,
47.01.11 Современное состояние и перспективы развития,
47.43.21 Влияние различных факторов среды на распространение радиоволн,
47.53.35 Электростатические системы записи и воспроизведения сигналов

ВИД СТАТЬИ: оригинальная научная статья

Резюме:

Введение/цель: Безопасность систем может оказаться под угрозой компрометирующих излучений. В данной статье представлен обзор различных атак на информацию, излучаемую компьютерным монитором. Новые технологии отслеживания излучения экрана могут быть использованы в извлечении (эксфильтрации) конфиденциальных данных с экранов компьютеров. Меры по безопасности способствуют предотвращению электромагнитных атак на излучаемую или передаваемую информацию.

Методы: В данной статье исследуется воздействие атак по сторонним каналам, которые перехватывают скомпрометированную информацию с экрана компьютера. Также в статье объясняется утечка данных вследствие электромагнитных излучений. Программно-определяемые радиосистемы описаны с целью объяснения вредоносных атак на компьютерные мониторы.

Результаты: Источник электромагнитного сигнала определяет характер передаваемой им информации по побочному каналу. Наиболее известная проблема, связанная с выявлением излучений, заключается в возможности перехвата визуальной информации, отображаемой на мониторах компьютеров.

Выводы: Визуальные данные, отображаемые на компьютерных мониторах, могут быть перехвачены программно-определяемой радиосистемой, которая может оцифровывать нужный частотный спектр непосредственно с антенны, передавать его в цифровой сигнальный процессор и выводить в приложение для выявления конфиденциальных данных. Для предотвращения утечки данных можно использовать различные контрмеры, такие как: экранирование, зонирование, программный комплекс TEMPEST и прочие аналогичные методы.

Ключевые слова: электромагнитное излучение, утечка информации, компьютерный монитор.

ЧИТАЊЕ СА ЕКРАНА: ЦУРЕЊЕ ЕЛЕКТРОМАГНЕТНИХ ИНФОРМАЦИЈА СА МОНИТОРА РАЧУНАРА

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ОБЛАСТ: рачунарске науке, електроника, телекомуникације, информационе технологије, машинство
КАТЕГОРИЈА (ТИП) ЧЛАНКА: оригинални научни рад

Сажетак:

Увод/циљ: Безбедност система може бити угрожена компромитујућим зрачењем. У раду је приказан преглед напада на информације које зрачи монитор рачунара. Праћењем зрачења са екрана рачунара, нове технологије се могу користити за ексфилтрацију осетљивих података. Емисиона безбедност представља начин за спречавање напада електромагнетних сигнала који настају зрачењем или се преносе.

Методе: У раду се испитује утицај side-channel напада који пресреће компромитоване информације са екрана рачунара. Објашњено је „цурење" података услед електромагнетног зрачења. Софтверски дефинисани радио описан је како би били објашњени злонамерни напади на мониторе.

Резултати: Извор електромагнетног сигнала одређује природу информација које оне носе. Најпознатији проблем повезан са откривањем емисија јесте могућност пресретања визуелних информација приказаних на мониторима рачунара.

Закључак: Визуелни подаци приказани на мониторима могу бити пресретнути софтверски дефинисаним радиом, који може дигитализовати жељени спектар директно са антене, у дигиталном облику га представити процесору и проследити апликацији за откривање осетљивих података. За спречавање цурења података могу се користити разне противмере заштите, као што су зонирање, софт TEMPEST и сличне технике.

Кључне речи: електромагнетна емисија, цурење информација, монитор.

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FREQUENCY HOPPING SPREAD SPECTRUM: HISTORY, PRINCIPLES AND APPLICATIONS

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FIELD: Telecommunications
ARTICLE TYPE: Original scientific paper

Abstract:

Introduction/purpose: The frequency hopping spread spectrum (FH-SS) technique assumes the carrier generated by the syntesizer to hop from frequency to frequency over a wide bandwidth, according to a pseudonoise code sequence defined by the code sequence generator. The article presents the history, principles and applications of the FH-SS technique. Both military and commercial applications are discussed.

Methods: This article presents an overview of data from the technical literature, with appropriate comments.

Results: After presenting the history and principles of the FH-SS technique, the article summarizes its use with examples of military and commercial applications. The importance of using FH-SS in the described applications is highlighted.

Conclusion: The FH-SS technique has been successfully implemented in many military and commercial technologies due to its high protection against interference, making communication difficult for reconnaissance and eavesdropping, and its ability to provide code division multiple access.

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Key words: radio communication, spread spectrum, frequency hopping, bluetooth, global system for mobile communication, unmanned aerial vehicles.

Introduction

Spread spectrum is a modulation widely used in military communications. In spread spectrum modulation, the bandwidth of the transmitted signal is much greater than the bandwidth of the original message, and is determined by the spreading code (Yahya et al, 2006). The spreading code is a digital signal that is independent of the message and is known to both the transmitter and the receiver. There are two the most frequently used methods for performing the spreading of the spectrum: frequency hopping (FH-SS) and direct sequence (DS-SS).

Spread spectrum modulations provide high protection against interference, make unauthorized reception (reconnaissance and eavesdropping) difficult, allow the formation of a code multiplex in which all users operate simultaneously in the same frequency bandwidth, and enable selective addressing of individual users (Todorović, 2021).

In this article, the history, principles, and applications of the spread spectrum frequency hopping technique are presented. A review of the military and commercial applications of frequency hopping is given.

After the Introduction, in the second section, the history of frequency hopping is presented. The third section gives the principles of FH-SS. Sections after that present applications of frequency hopping, such as military radio communications, control of unmanned aerial vehicles, the global system for mobile communication, and Bluetooth technology. In the last section, the most important conclusions are made.

History of FH-SS

More than a century separates Nikola Tesla's frequency-based method of signalling from today's widely used spread spectrum data transmissions. In 1901, Tesla considered a method of signalling that consists of two, or more, transmitters operating at different frequencies. The receiver is designed to respond only when both signals are received (Rothman, 2018).

On December 18, 1920, Otto B. Blackwell, De Loss K. Martin, and Gilbert S. Vernam applied for a patent "Secrecy communication system". Successive portions of a message are transmitted on waves of different frequencies. Secrecy is obtained by the transmission of signals on a plurality of waves of different frequencies. A station tuned to one of the frequencies receives only a part of the message. The frequency shifting is not accomplished in a cyclic order, but rather in a random and variable manner (Rothman, 2018).

In 1922, Emory-Leon Chaffee described a system of radiocommunication that proposes rapid changing of the carrier frequency in an "erratic manner" in order to provide secrecy (Price, 1983).

On October 11, 1929, Broertjes submitted a patent application in Germany for a "Method of maintaining secrecy in the transmission of wireless telegraph messages". He said that the known methods of maintaining secrecy operate, in most cases, with codes or cryptograms and with a periodically modified transmission frequency, which is received by means of a receiving apparatus, the tuning of which is modified in synchronism. According to Broertjes, the idea of changing transmission frequencies is already commonplace, but he argues that this approach does not prevent the interception and decipherment of the message because a broadband receiver could pick up all the frequencies. Broertjes proposed a system in which a number of operating frequencies, known to the transmitter and receiver alone, can be varied in a random or variable manner. In a method of this kind, secrecy is ensured by the fact that an unauthorized receiver, which at first is tuned to only a single frequency, picks up only disconnected portions of the message. The transmitter includes a code wheel that selects a new frequency each time a telegraph key is pressed. One can only assume that the receiver contains a duplicate wheel, but Broertjes prefered manual operation, so it remains unclear how he intended synchronization to be effected. Despite its flaws, Broertjes's invention proves that the frequency hopping concept was available in Germany before World War II (Rothman, 2018).

In September 1940, Ellison Purington, who had done graduate work in physics at Harvard University and had worked on torpedo guidance systems at the Hammond Laboratory during World War I, filled an application for a "System for reducing interference" (Rothman, 2018). In this patent, Purington proposes changing the carrier frequency to reduce the ability of other transmitters to interfere with the signal (Scholtz, 1983).

Actress Hedy Lamarr, born Hedwig Kiesler in Austria, soon after the beginning of World War II escaped and ended up in Hollywood where she met George Antheil. Hedy revealed that she possessed a flair for inventing weapons and shared with him an idea for a secure torpedo guidance system that employed a novel technique known as frequency hopping. Hedy lacked the technical expertise to put her idea into practice (Rothman, 2018). George was likewise no engineer, but in the 1920's he had written a concert piece Ballet Mecanique, which included parts for synchronized player pianos (Price, 1983). They had an idea to place a player-piano roll punched with 88 rows of randomly placed perforations in the transmitter to control the hopping among 88 radio frequencies. They would place the identical roll in the receiver, and then they would synchronise them. George and Hedy applied for a patent in June 1941. Despite the novelty of their approach, the pneumatic player-piano mechanism made their system unwieldy and certainly unworkable in a battle. Antheil made strenuous lobbying efforts to get the invention adopted by the Navy, but it was shelved (Rothman, 2018).

Early in January 1943, U.S. army signal corps officer Henry P. Hutchinson applied for a patent on frequency hopping signalling for maintaining the secrecy of telephone conversations or for privately transmitting information (Price, 1983). His scheme utilized cryptographic machines to produce a pseudorandom hopping sequence on demand (Rothman, 2018).

After World War II, another major effort at developing a secure, jam-proof system was put forth by Sylvania with its Buffalo Laboratories Application of Digitally Exact Spectra (BLADES). Its development began in 1955, as a system for communicating with Polaris submarines. It was tested in 1957, and in 1963 it was installed on the flagship U.S.S. Mount McKinley, for operational development tests where It successfully thwarted intentional jamming efforts. Thus, BLADES was quite likely the earliest FH-SS communication system to reach an operational state (Scholtz, 1982).

A practical application of frequency hopping began in the years to follow. Radio systems with FH-SS for battle conditions were developed by Racal, Marconi Communication Systems, Single Channel Ground and Airborne Radio System (SINCGARS) and other companies.

In 1992, the early 2G cellular networks did use a form of frequency hopping. In 1997, Jaap Haartsen patented the Bluetooth technology that employs frequency hopping.

Principles of FH-SS

During the communication using the FH-SS technique, the receiver and the transmitter change their carrier frequency according to a predefined order and rate, which should remain a secret for everyone except for them. The carrier frequency change takes place under the control of the code sequence generator within the transmission bandwidth. The FH-SS technique can be used to transmit messages both in analogue and digital formats (Todorović, 2021).

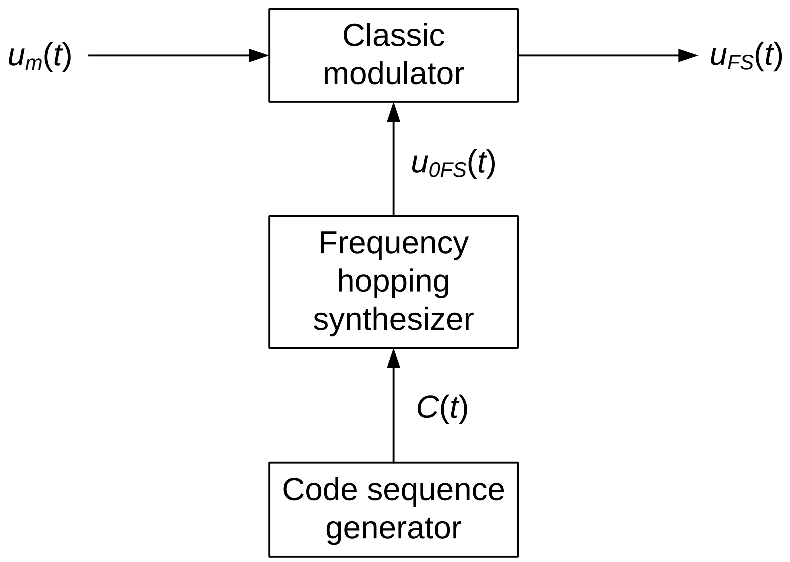

Figure 1 – FH-SS signal modulator
Рис. 1 – Модулятор сигнала со скачкообразной перестройкой частоты
Слика 1 – Модулатор сигнала са фреквенцијским скакањем

A block diagram of the FH-SS signal modulator is given in Figure 1 (Todorović, 2021). The FH-SS signal modulator consists of a code sequence generator, a frequency hopping synthesizer and a modulator, where some of the classic analogue or digital modulations is applied.

The most important block of FH-SS signal modulators is the frequency synthesizer. Under the control of the code sequence generator, a signal $u_{0FS}(t)$ is generated at its output. That signal has a variable carrier frequency. The rate of carrier frequency change varies from a few hops per second to 100000 hops per second. The frequency synthesizer generates the frequency hopping pattern known to both the transmitter and the receiver (Čisar et al, 2020).

When FH-SS is used, the frequency bandwidth for signal transmission is significantly wider than necessary to transmit the message when using some of classic modulations with the carrier at a fixed frequency.

The spreading ratio of FH-SS modulations is defined as follows:

$$\eta = \frac{B_{FS}}{B_m} \tag{1}$$

where B_{FS} denotes the frequency bandwidth of the FH-SS transmission, while the bandwidth of one channel is denoted by B_m.

If there is no frequency gap between adjacent channels, the spreading ratio is equal to the number of channels:

$$\eta = N \tag{2}$$

If another signal is present in any of the channels within the frequency bandwidth B_{FS}, that signal and the FH-SS signal interfere with each other. This interference is of very short duration and occurs in the time interval when the FH-SS signal is present in that channel.

The block diagram of the FH-SS signal demodulator is shown in Figure 2 (Todorović, 2021). The FH-SS signal demodulator consists of a code sequence generator, a frequency hopping synthesizer, a demodulator and a code synchronization block.

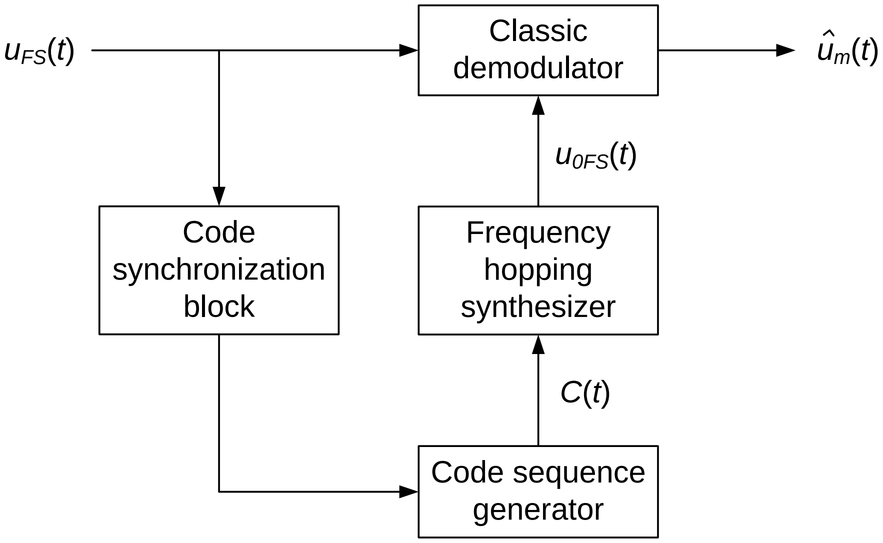

Figure 2 – FH-SS signal demodulator
Рис. 2 – Демодулятор сигнала скачкообразной перестройки частоты
Слика 2 – Демодулатор сигнала са фреквенцијским скакањем

The basic task of the FH-SS receiver is to despread the spectrum of the incoming signal. The process of despreading has to be performed before demodulation. This is accomplished by correlating the incoming signal with a synchronized locally generated code sequence. Code synchronization is a necessary step in order to be able to despread the spectrum of the incoming signal.

The task of the code synchronization block is to ensure that the change of the carrier frequency in the transmitter and the receiver takes place at the same time.

FH-SS systems may be classified as fast or slow. Fast FH-SS occurs if one information symbol is transmitted over several frequency hops. Slow FH-SS occurs if one or more information symbols are transmitted within one frequency hop (Torrieri, 2018).

Applications of FH-SS

FH-SS techniques have been employed intensively for highly secured data transmission in military and commercial wireless communications (Aung et al, 2012). Military applications include HF, VHF and UHF radios, and unmanned aerial vehicles (UAV) control and data signals. Among commercial applications, GSM mobiles and Bluetooth have to be mentioned.

Military radio communications

Spread spectrum signals are highly resistant to unintentional or intentional jamming. Therefore, they are suitable for military applications. FH-SS has a good near-far performance and can be implemented easier than DS-SS (Hasan et al, 2016). It has good resistance to various forms of electronic attacks, and it is a suitable technique for overcoming interception, direction finding, and jamming.

Military radios use the FH-SS technique to transmit messages in the analogue or digital format, with hopping rates from a few hops per second to several hundred hops per second. Military radios generate the FH-SS pattern under the control of a secret transmission security key (TRANSEC) that the transmitter and receiver share in advance. Frequency changes are made in the programmed frequency band that the transceiver selects based on the current conditions of radio wave propagation. The number of programmed bands may vary depending on the manufacturer and the frequency bandwidth in which the radio operates. By analyzing the characteristics of the radio wave propagation at a given time, the control station selects one of these frequency bands. The transceiver uses the best frequencies in the defined frequency band by analyzing the interference. Some radios also allow the entry of forbidden frequency bands, which the transceiver also takes into account when selecting operating frequencies. The best use of these transceiver characteristics depends on a careful selection of frequency bands.

The FH-SS technique is used by many companies, especially in the production of military radio communication devices. Some of such companies are Racal, Marconi Communication Systems, SINCGARS, Aselsan, Thales Group and L3Harris Technologies.

Among the first military radio systems with FH-SS was a series of JAGUAR radios, developed by Racal. Racal was a British electronics company founded in 1950. JAGUAR-H, JAGUAR-V and JAGUAR-U were developed for different frequency bands. JAGUAR-H was developed in 1982. It operates in the 2-30 MHz frequency band. It has a 5 W manpack and 100 W vehicle configurations. FH-SS is performed according to the pseudorandom law within the subband. The width of the subband goes from 400 kHz in the lower frequencies and increases in the central and upper part of the HF band. The hopping rate for the JAGUAR-H is 10-20 hops per second (Todorović, 1987). The JAGUAR-V system was developed in 1981. There are a 4 W manpack and vehicular 50 W radio stations. It is designed to hop over a bandwidth of 6.4 MHz, corresponding to 256 channels at 25 kHz spacing. It provides 9 hop subbands in the combat network radio band of 30-88 MHz. The hopping rate for the JAGUAR-V system is in the region of 50-500 hops per second (Munday & Pinches, 1982). The 15 W vehicle radio station JAGUAR-U, developed in 1982, operates within the 225-400 MHz band. FH-SS is performed on 768 channels in 9 subbands of 19.2 MHz (Todorović, 1987).

Marconi Communication Systems is a British company which developed SCIMITAR-H and SCIMITAR-V radios in 1982. The SCIMITAR-H radio operates in the 1.6-30 frequency band. It has a 20 W manpack and 100 W vehicular configurations. FH-SS is performed within the entire frequency band with a hopping rate of 150 hops per second. The SCIMITAR-V radio operates in the 30-88 MHz frequency band. There are a 5 W manpack and vehicular 50 W radio stations. FH-SS is performed on 2320 channels within the entire frequency band with a hopping rate of 150 hops per second (Todorović, 1987).

The SINCGARS is a combat network radio used by the U.S. and other military forces. Among the first SINCGARS radios was the SINCGARS-V radio developed in 1986. It was developed to work in the frequency band from 30-88 MHz. There are a 5 W manpack and vehicular 50 W radio stations. It supports FH-SS which is performed according to the pseudorandom law within the entire frequency band with a hopping rate of 150 hops per second (Todorović, 1987).

Aselsan is a defence electronics company in Turkey. Among others, it is engaged in the production of communication and information technologies (Aselsan, 2022). The 9661 HF radios and PRC/VRC 9661

V/UHF software-defined radios are representatives of military radio communications systems. The 9661 HF radios are software-defined radios covering the HF 1.6-30 MHz band. It is possible to communicate under intentional or unintentional interference by using the frequency hopping mode of operation. The 9661 HF radio family has three configurations for manpack, vehicular and fixed station usage. The 20 W station can be used for manpack and vehicle configurations and the 150 W station can be used for vehicular and fixed station configurations. PRC/VRC 9661 V/UHF software-defined radios are tactical radios capable of communicating in clear, encrypted and frequency hopping voice and data in VHF and UHF bands from 30-512 MHz. There are 5 W handheld, 10 W manpack and 50 W vehicular/base station configurations of 9661 V/UHF radios.

Thales Group is French company that produces HF and VHF radio communications systems for defence (Thales Group, 2022). Both HF and VHF radios provide frequency hopping encrypted voice and data transmission. A digital HF software-defined radio operates in the 1.5-30 MHz frequency band. There are 20 W (TRC 3700), 125 W (TRC 3730) and 400 W (TRC 3740) radio stations. VHF radios operate in the 30-88 MHz frequency band. There are several types of VFH radios: handheld 2 W radio (TRC 9110), manpack 10 W radio (TRC 9210), vehicular 50 W station (TRC 9310 A/AP) and vehicular 2x50 W dual fit station (TRC 9310 B/C).

L3Harris Technologies (L3Harris) is an American technology company. Among others, it is engaged in the production of tactical communications systems (L3harris, 2022). The L3Harris Falcon III AN/PRC-160(V) presents a military radio which uses frequency hopping for transmission. It covers the HF/VHF 1.5-60 MHz band. It provides 20 W output power for HF and 10 W for the VHF frequency band.

Unmanned aerial vehicles (UAVs)

Unmanned aerial vehicles, or drones, are aircraft without a human pilot aboard. UAVs are a component of an unmanned aircraft system (UAS), which includes a UAV, a ground-based controller, and a system of communications between them (Nived Maanyu et al, 2020). There are two versions of UAVs: UAVs controlled from a remote location and UAVs which fly autonomously based on a pre-programmed flight plan (Todorovic & Orlic, 2009). Due to the absence of a pilot, UAVs always have a certain level of autonomy (Vergouw et al, 2016).

UAVs were most often associated with the military until a few years ago. They were used initially for intelligence gathering, anti-aircraft target practice, and then, more controversially, as weapons platforms. Their high cost and restricted market meant they were extremely difficult to provide. This situation has changed dramatically in recent years, as UAVs are becoming cheaper, making it easier to provide for ordinary people (Mototolea, 2019).

Commercial UAVs typically are built on a small platform, they are easy to operate and flexible to execute different kinds of missions (Fan & Ala Saadeghvaziri, 2019). UAVs can carry multiple sensors, transmitters and imaging equipment. As the use of UAVs continues to proliferate, they will impact industries ranging from entertainment to agriculture, from construction to delivery markets (Rao et al, 2016).

There are several links for communication with UAVs: control, video transmission, and telemetry links. These links use radio-frequency transmission to transmit and receive information to and from the UAV. These transmissions can include location, remaining flight time, distance and location to target, payload information, airspeed, altitude, and many other parameters.

Table 1 – UAVs types, frequencies they operate on and the used modulation
Таблица 1 – Виды дронов, их рабочие частоты и используемая модуляция
Табела 1 – Врсте дронова, фреквенције на којима раде и модулација коју користе

Brand	Frequency	Modulation
DJI Phantom	2.4/ 5.8 GHz	FH/ DSSS
Futaba	2.4 GHz	FH/ DSSS
Spektrum	2.4 GHz	FH/ DSSS
JR	2.4 GHz	FH/ DSSS
Hitec	2.4 GHz	FH/ DSSS
Graupner	2.4 GHz	FH/ DSSS
Yuneec	2.4 GHz	DSSS
Parrot AR2	2.4 GHz	OFDM
Immersion	433 MHz	FH

The 2.4 GHz band is the most often used frequency for remote control (Šević et al, 2020). The 2.4 GHz systems usually use spread spectrum technology which provides much more resistance to interference. The transmitter and the receiver are paired which eliminates the possibility that another transmitter connects to the receiver in use.

Table 1 (Mototolea, 2019) shows that the majority of controllers use the 2.4 GHz band with proprietary FH-SS modulation, since it is resistant to narrowband interference and is difficult to intercept. Some UAVs use the 2.4 GHz band with DS-SS modulation or with orthogonal frequency division multiplexing (OFDM) modulations.

Video links are used to send information from cameras, microphones, or different sensors. The OFDM is the most popular modulation technique for transmitting video (Mototolea, 2019). Simple frequency modulation is also used for transmitting video information.

The telemetry link is used to send information regarding the voltage of the supply, altitude, vertical and horizontal speed, temperature, and other critical parameters. Using such a module, the controller can receive real-time data from the UAV. DS-SS, FH-SS, and OFDM modulations are used to transmit the telemetry data (Mototolea, 2019).

While most popular UAV brands use only one type of modulation in their proprietary technology, some brands combine DS-SS and FH-SS to make the communication more robust.

Global system for mobile communication (GSM)

The GSM is a digital mobile network that is widely used by mobile phone users in Europe and other parts of the world. It is a standard that specifies how second generation (2G) cellular networks operate. When the GSM was first introduced in Europe in 1991, these 2G networks created faster, more secure wireless connections. For the first time, voice communications became encoded into digital signals before being transmitted through the network. For years, the GSM reigned as the world’s most widely used standard for mobile communications. But today, 2G networks are significantly slower than other cellular networks, and in several countries, 2G networks are being switched off.

Two frequency bands are defined for the GSM: the band from 890 MHz to 915 MHz is used for the uplink and the band from 935 MHz to 960 MHz is used for the downlink. Besides these 900 MHz bands, there are two bands in the 1800 MHz, from 1710 MHz to 1785 MHz and from 1805 MHz to 1880 MHz (Wigard et al, 1996).

The GSM system combines time division multiple access (TDMA) and frequency division multiple access (FDMA) for effective use of the frequency band. FH-SS is a feature of the GSM system used to decrease the simultaneous usage of the same frequencies and in this way averages the interference level. The GSM system divides the carrier frequencies into eight timeslots, according to TDMA. Each time slot is used to handle the call of one subscriber (Mishra, 2007).

The slow FH-SS method is implemented in the GSM system (Ivanov et al, 1996). The frequency can be changed after every TDMA frame. By changing the frequency after each 4.615 ms (length of one TDMA frame), in one second the TDMA frame makes 217 hops per second (Mishra, 2007).

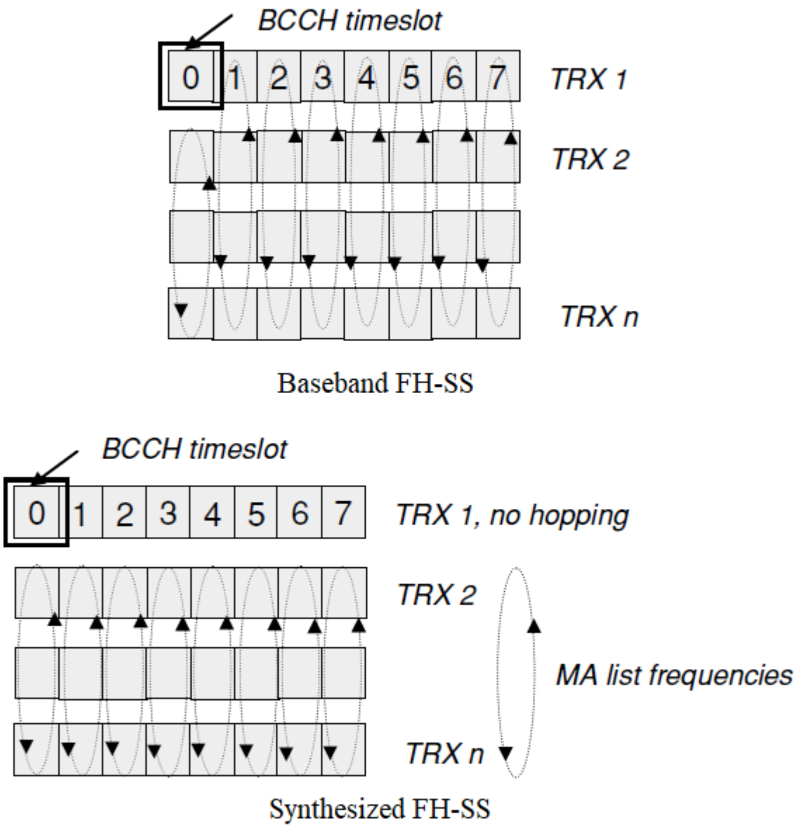

Figure 3 – Baseband FH-SS and synthesized FH-SS
Рис. 3 – Скачкообразная перестройка частоты основной полосы и синтезированная скачкообразная перестройка частоты
Слика 3 – Фреквенцијско скакање основног опсега и синтетизонавно фреквенцијско скакање

A base station contains one or more cells. Each cell may contain one or more transceivers (TRXs). The first time slot in the first transceiver of a cell is used as the broadcast control channel (BCCH). The remaining time slots in that transceiver and the time slots in the other transceivers are used as traffic channels (Bourjolly et al, 2002).

Figure 3 (Mishra, 2007) presents the FH-SS types of the GSM system, baseband FH-SS and synthesised FH-SS. This depends on the type of equipment installed at the transceiver.

In baseband FH-SS, the transceivers actually have fixed frequencies and frequency hopping operates so that bursts are shifted from one transceiver to another according to the hopping sequence. The number of hopping frequencies, therefore, comes from the number of transceivers. The length of the mobile allocation (MA) list is the number of transceivers. The broadcast control channel in the first transceiver does not hop and therefore is excluded from the hopping groups. With baseband FH-SS there are two actual hopping groups, one for the first timeslots of each transceiver excluding the BCCH transceiver and another for all the other timeslots. The first hopping group has TRX-1 frequencies, due to the dedicated BCCH frequency (Mishra, 2007).

In synthesized FH-SS, the frequency of the transceiver changes. Any call goes through one transceiver, but the frequency is changing over time. This allows the number of frequencies in the mobile allocation to be larger than the number of transceivers (Mishra, 2007). A maximum of 64 hopping frequencies is allowed (Olofsson et al, 1995). The synthesized FH-SS pattern can be either random or cyclic. In random frequency hopping, the frequencies are randomly selected from the available set of frequencies. Cyclic hopping has a defined cycle for the frequencies in the mobile allocation list (Mishra, 2007).

Bluetooth

Bluetooth technology is a standard for short range radio links between personal computers, mobile phones, and other portable devices (Đukić et al, 2012). The name Bluetooth comes from the Danish king Haruld Blitund (Bluetooth). King Bluetooth is credited with uniting the Scandinavian people during the 10th century. Similarly, the Bluetooth wireless technology aims to unite personal computing devices. The name was chosen temporarily to describe the yet unannounced development project. However, the search for a new name never came to a successful fruition and the temporary name became permanent (Bisdikian, 2001).

Robustness, low consumption, and low price are the key features of Bluetooth. The Bluetooth specifications were established by the joint effort of over two thousand industry-leading companies, including 3Com, Ericsson, IBM, Intel, Lucent, Microsoft, Motorola, Nokia, Toshiba, etc.

under the umbrella of Bluetooth Special Interest Group (Chlamtac et al, 2003). Bluetooth is standardized within the IEEE 802.15 Working Group for Wireless Personal Area Networks formed in early 1999 (IEEE 802.15, 2022). The IEEE 802.15.1 standard defines the basics of Bluetooth wireless technology.

Bluetooth technology supports both point-to-point and point-to-multipoint connections. Several piconets can be established and linked together ad hoc, where each piconet is identified by a different FH-SS sequence. All users participating on the same piconet are synchronized to this FH-SS sequence. A piconet supports up to 8 devices, where one device acts as the master. The master controls the traftic up to a maximum of 7 units, defined as slaves in a piconet (Nusser & Pelz, 2000). Inside a piconet, Bluetooth stations can establish up to three 64 Kbps synchronous (voice) channels or an asynchronous (data) channel supporting data rates of maximal 723 Kbps asymmetric or 433 Kbps symmetric (Chlamtac et al, 2003).

The Bluetooth radio channel frequency hopping rate is about 1600 hops per second for data/voice links and 3200 hops per second during page and inquiry scanning. A channel is used for a very short period (e.g. 625 microseconds for data/voice links), followed by a hop designated by a pre-determined pseudo-random sequence to another channel. This process is repeated continuously according to the FH-SS sequence (Scarfone & Padgette, 2008).

Bluetooth also provides radio link power control, where devices can negotiate and adjust their radio power according to signal strength measurements. Each device in a Bluetooth network can determine its received signal strength indication and request the other network device to adjust its relative radio power level. This is performed to conserve power and to keep the received signal characteristics within a preferred range (Scarfone & Padgette, 2008). Transmit power goes from 1 mW to 100 mW. The designed operating range is from 1 to 100 m.

Figure 4 (Bluetooth, 2022) presents the types of Bluetooth radios. There are two designs of Bluetooth radios: the Bluetooth Classic radio and the Bluetooth Low Energy (LE) radio.

The Bluetooth Classic radio, also referred to as Bluetooth Basic Rate/Enhanced Data Rate (BR/EDR), is a low-power (less than 100 mW) radio that streams data over 79 channels with 1 MHz spacing in the 2.4 GHz unlicensed industrial, scientific, and medical (ISM) frequency band (2.402 – 2.480 GHz Utilized). The data rate can be from 1 to 3 Mbps. Bluetooth versions 1.1 and 1.2 only support transmission speeds of up to 1 Mbps, which is known as the Basic Rate (BR), and can achieve throughput of approximately 720 Kbps. Introduced in Bluetooth version 2.0, the Enhanced Data Rate (EDR) specifies data rates up to 3 Mbps and throughput of approximately 2.1 Mbps. The BR uses Gaussian Frequency-Shift Keying (GFSK) modulation to achieve a 1 Mbps data rate. The EDR uses π/4 rotated Differential Quaternary Phase Shift Keying (DQPSK) modulation to achieve a 2 Mbps data rate, and 8 phase Differential Phase Shift Keying (8DPSK) to achieve a 3 Mbps data rate (Scarfone & Padgette, 2008). Supporting point-to-point device communication, Bluetooth Classic is mainly used to enable wireless audio streaming and has become the standard radio protocol behind wireless speakers, headphones, and in-car entertainment systems. The Bluetooth Classic radio also enables data transfer applications, including mobile printing.

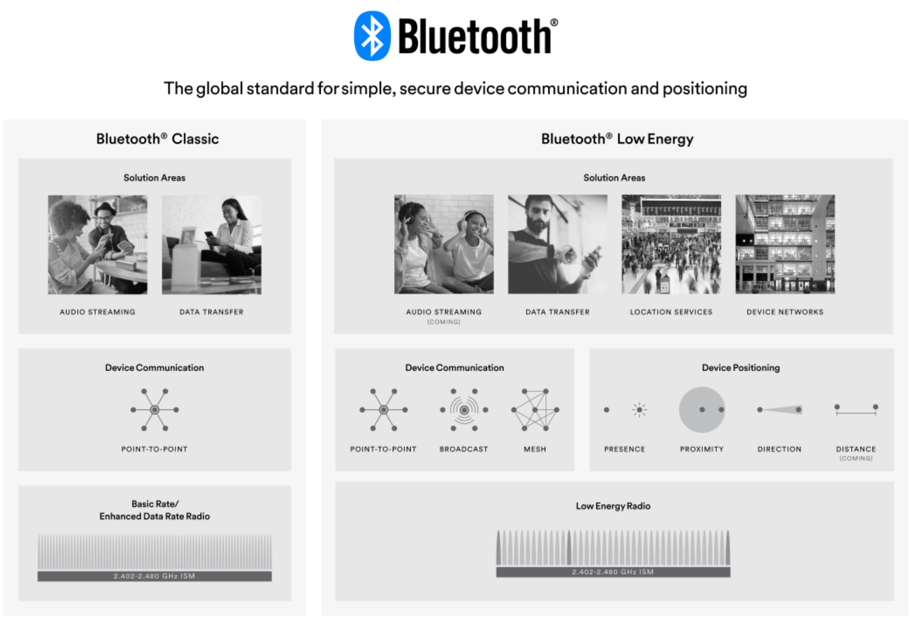

Figure 4 – Bluetooth technology
Рис. 4 – Bluetooth технология
Слика 4 – Блутут технологија

The Bluetooth Low Energy (LE) radio is designed for very low power (less than 100 mW) operation, transmitting data over 40 channels (3 advertising channels and 37 data channels) in the 2.4 GHz unlicensed ISM frequency band. The data rate can be from 0.125 to 2 Mbps via GFSK modulation. Bluetooth LE was introduced in the Bluetooth 4.0 specification. The key technology goals of Bluetooth LE (compared with Bluetooth BR/EDR) include lower power consumption, reduced memory requirements, efficient discovery and connection procedures, short packet lengths, and simple protocols and services (Scarfone & Padgette, 2008). Bluetooth LE supports multiple communication topologies, expanding from point-to-point to broadcast and, most recently, mesh, enabling Bluetooth technology to support the creation of reliable, large-scale device networks. While initially known for its device communications capabilities, Bluetooth LE is now also widely used as a device positioning technology to address the increasing demand for high accuracy indoor location services. Initially supporting simple presence and proximity capabilities, Bluetooth LE now supports Bluetooth Direction Finding and, soon, high-accuracy distance measurement.

Conclusion

The FH-SS technique is based on a change of the carrier frequency in a wide bandwidth according to a sequence defined by the code sequence generator. The FH-SS technique has been successfully implemented in many military and commercial applications, due to its high protection against interference, making communication difficult for reconnaissance and eavesdropping, and its ability to provide code division multiple access. The history, principles and applications of the FH-SS technique are presented in this article.

Military radio communications require a technique that has good resistance to various forms of electronic attacks. Communication links for the control of UAVs are very important, so they need to use technology that is difficult to interfere with. Mobile personal computers, mobile phones, and other portable devices use Bluetooth technology. The FH-SS technique is used for all these needs.

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РАСПРОСТРАНЕНИЕ СПЕКТРА СО СКАЧКООБРАЗНОЙ ПЕРЕСТРОЙКОЙ ЧАСТОТЫ: ИСТОРИЯ, ПРИНЦИПЫ И ПРИМЕНЕНИЕ

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РУБРИКА ГРНТИ: 49.27.00 Система передачи,
49.43.00 Радиосвязь и радиовещание
ВИД СТАТЬИ: оригинальная научная статья

Резюме:

Введение/цель: Скачкообразная перестройка частоты основана на скачкообразном изменении несущей частоты в очень широком диапазоне, которое происходит под управлением кодовой последовательности. В статье представлены история, принципы и применение метода скачкообразной перестройки частоты.

Методы: В данной статье представлен обзор данных из технической литературы с соответствующими комментариями.

Результаты: После представления истории и принципов метода скачкообразной перестройки частоты в статье подводятся итоги по ее использованию на примерах военного и коммерческого применения. Подчеркнута важность использования скачкообразной перестройки частоты в описанных приложениях.

Выводы: Метод скачкообразной перестройки частоты успешно применяется во многих военных и коммерческих технологиях, благодаря наличию высокой защиты от помех, успешно противостоящей разведке и прослушиванию.

Ключевые слова: радиосвязь, расширенный диапозон, скачкообразная перестройка частоты, Bluetooth, глобальная система мобильной связи, беспилотные летательные аппараты.

ПРЕНОС У ПРОШИРЕНОМ СПЕКТРУ – ФРЕКВЕНЦИЈСКО СКАКАЊЕ: ИСТОРИЈА, ПРИНЦИПИ И ПРИМЕНА

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ОБЛАСТ: телекомуникације
КАТЕГОРИЈА (ТИП) ЧЛАНКА: оригинални научни рад

Сажетак:

Увод/циљ: Фреквенцијско скакање се заснива на скоковитој промени фреквенције носиоца у врло широком опсегу, која се одвија под контролом кодне секвенце. У раду су представљени историјат, принципи и примена технике фреквенцијског скакања.

Методе: Приказан је преглед података из техничке литературе, уз одговарајуће коментаре.

Резултати: Након представљања историје и принципа технике фреквенцијског скакања, сумира се њена употребу са примерима војне и комерцијалне примене. Истиче се важност употребе фреквенцијског скакања у описаним апликацијама.

Закључак: Техника фреквенцијског скакања је успешно имплементирана у многим војним и комерцијалним технологијама јер пружа већу заштиту од сметњи, што отежава извиђање и прислушкивање.

Кључне речи: радио-комуникације, пренос у проширеном спектру, фреквенцијско скакање, блутут, глобални систем за мобилну комуникацију, беспилотне летелице.

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COMPONENT SIZING AND ENERGY MANAGEMENT FOR A SERIES HYBRID ELECTRIC TRACKED VEHICLE

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FIELD: Mechanical engineering

ARTICLE TYPE: Original scientific paper

Abstract:

Introduction/purpose: The paper presents a systematic approach to the development of a series hybrid electric tracked vehicle (HETV) including powertrain sizing and adequate energy management strategy (EMS) selection.

Methods: Powertrain elements were sized considering key performance requirements. Three energy management strategies were proposed: Thermostat Control Strategy (TCS), Power Follower Control Strategy (PFCS), and Optimal Power Source Strategy (OPSS). The evaluation of the powertrain configuration and the three proposed EMSs was performed in the Simulink environment using a driving cycle containing significant acceleration, braking and steering.

Results: The results showed that the OPSS proved to be the best due to increased fuel economy and a low battery state of charge (SOC) variation. Compared to the previous research of the same vehicle with a parallel hybrid configuration, significantly better results were achieved. The investigation of the results indicates that the proposed powertrain and control strategy offer 53.79% better fuel economy which indicates that the powertrain sizing was properly performed.

Conclusions: The results of this work are of great importance for understanding the effect of proper powertrain sizing on fuel economy. Compared to the reference vehicle, the proposed configuration achieves significant improvement, most of which is attributed to adequate sizing. The OPSS proved to be the best strategy, thus confirming the theoretical hy-

pothesis. The series hybrid configuration with the OPSS as the EMS proved to be a major candidate for use in HETVs.

Key words: tracked vehicle, hybrid electric vehicle, energy management, control strategy, fuel economy.

Introduction

Hybrid drive is the most practical and realistic alternative to conventional transmission at the moment (Jimenez-Espadafor et al., 2011; Ehsani et al., 2018). In the field of wheeled vehicles, hybrid propulsion systems have been very common for many years (Hannan et al., 2014). On the other hand, due to different technological and economical reasons, the research of hybrid technology for tracked vehicles has not been the focus of many researchers. However, in the last decade, the defense industry started showing interest in the military vehicle hybridization (Rizzo, 2014) and hybrid electric tracked vehicles (HETVs). Hybrid drive for HETVs offers advantages such as better fuel economy, additional onboard electric power, silent watch capability and decreased noise and thermal signature (Khalil, 2009).

In (Galvagno et al., 2012), the authors presented a mathematical model and a dynamic analysis of a single-drive series hybrid tracked tank, while in (Zou et al., 2012a) the authors developed *bi-level* optimization consisting of two nested optimizations, one for optimal powertrain sizing, and the other one for optimal power management of the HETV. In (Liu et al., 2015; Zou et al., 2016), a control-oriented model of an HETV was developed and the EMS based on reinforcement learning was proposed, which achieved results comparable to dynamic programming, while the authors in (Randive et al., 2019, 2021) presented a systematic approach to powertrain sizing which reduced transmission weight by 16% and proposed a novel rule-based strategy which achieved over 30% better fuel economy when compared to the previous original powertrain.

Research on the topic of hybrid propulsion has also appeared in Serbian military circles. Driven by the idea of achieving better performance and less fuel consumption with minimal changes to the original conventional transmission, in (Milićević & Muždeka, 2021) the authors proposed a conceptual hybridization model of the Serbian infantry fighting vehicle BVP M80A. In this paper, the performance of the vehicle was analyzed, and the later research (Milićević et al., 2021) proposed an energy management strat-

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egy (EMS) based on the Power Follower Control Strategy (PFCS) which achieved 12.8% better fuel economy than conventional transmission, and even better result (23.2%) was achieved with introducing additional generator in the powertrain. These two research studies laid the foundation to hybridization of the BVP M80A, the former focusing on conceptualization of hybrid powertrain and the latter focusing on improving efficiency of the designed powertrain. The constraints set in the first research study led to the complex powertrain and very complex multi-mode EMS designed in the second one. The motivation for this work stems from the mentioned fact. The aim of this paper is to design a series hybrid powertrain which would be efficient and simple by considering key performance requirements, proper sizing of all powertrain elements and adequate selection of an EMS.

Proposed powertrain configuration

The most common configuration in HETVs is series (Zou et al., 2012b; Zhang et al., 2020; Randive et al., 2019; Qin et al., 2018; Randive et al., 2021; Zou et al., 2016). The lack of shafts, gears and other mechanical elements is very important from the aspect of reliability and better utilization of space. The increased power rating of battery offers more onboard electric power. These advantages and the possibility of increased fuel economy were the main factors for the proposed series configuration. The major components of the proposed configuration are (Fig. 1):

1. Traction motors,
2. Simple two-stage transmission,
3. Engine-generator unit, and
4. Energy storage.

The proposed powertrain adopts a dual-drive variant of the series configuration. Two traction motors independently drive the two sprockets, a generator is driven by the diesel engine which is a primary source of energy (PS) while the battery pack supplies or absorbs energy when needed and acts as a secondary source (SS). Between the electric motor and the wheels there is a simple two-speed transmission. Compared to the reference vehicle and the parallel configuration designed in (Milićević & Muždeka, 2021), this powertrain is much simpler. It has no planetary gear sets and no complex gearbox. Lack of the engine-sprocket mechanical connection enables the engine to work at the optimal operating point which is the main advantage of this configuration.

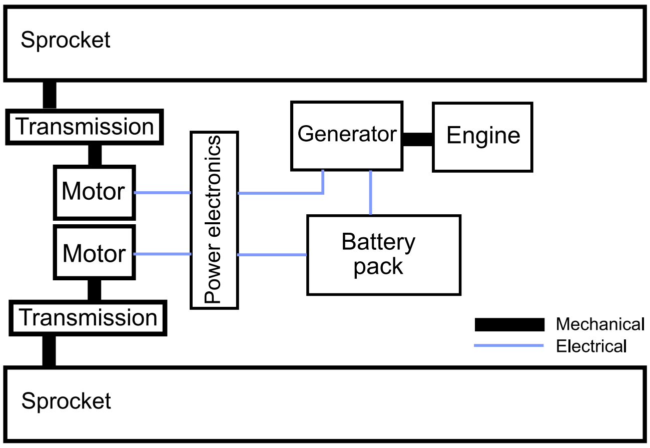

Figure 1 – Proposed powertrain configuration
Рис. 1 − Предлагаемая конфигурация гибридного привода
Слика 1 – Предложена конфигурација хибридног погона

Component sizing

The main rationale of the hybrid drive concept shown in (Milićević & Muždeka, 2021) was as few changes to the transmission as possible. Therefore, the authors of the aforementioned research retained numerous parameters of the initial transmission, and the parameters of the battery and the electric motor were adapted to the initial system parameters. Such approach to sizing certainly results in an unbalanced and disproportionate powertrain. Due to their specific purpose, tracked vehicles have unique requirements, such as high gradeability, off-road mobility and satisfactory skid-steering and acceleration performance. The design of the HETV powertrain should be made based on a critical performance analysis, something that is rarely done in literature (Randive et al., 2019). Based on the analysis, the adequate sizing of the components can be done to achieve satisfactory performance.

Power and torque demand

Movement resistances have a complex nature and depend on the type and slope of the terrain, pressure distribution, turning radius, etc. For this paper, the simplest case of movement on hard terrain will be considered. To propel the wheels of the HETV in a straight-line motion, traction force F_{tr} has to be equal to the sum of various resistances as follows:

$$F_{tr} = F_{rol} + F_{aero} + F_{grade} + F_{in}, \tag{1}$$

where $F_{rol}, F_{aero}, F_{grade}, F_{in}$ are rolling resistance, aerodynamic resistance, road grade resistance and inertial force, respectively. These forces are given as follows:

$$\begin{aligned} F_{rol} &= f m_v g \cos\alpha \\ F_{aero} &= \frac{1}{2} C_d A \rho v^2 \\ F_{grade} &= m_v g \cos\alpha\alpha \\ F_{in} &= \delta m_v a \end{aligned} \tag{2}$$

where f is the coefficient of the rolling resistance, m_v is the vehicle curb weight, α is the road slope angle, C_d is the aerodynamic resistance coefficient, A is the vehicle frontal area, ρ is the air density, V is the vehicle speed, δ is the mass coefficient and a is the vehicle acceleration.

During turning, the vehicle needs to overcome additional turning resistances which are obtained as (Wong, 2022):

$$M_R = \frac{1}{4} \mu m_v g L \tag{3}$$

where μ is the coefficient of the lateral resistance and L is the contact length of the track. The coefficient μ is calculated as:

$$\mu = \mu_{max}(0.925 + 0.15R/B)^{-1} \tag{4}$$

where μ_{max} is the maximum value of μ, B is the vehicle tread and R is the turning radius. The turning radius can be calculated from the equation:

$$R = \frac{B}{2} \frac{\omega_2 + \omega_1}{\omega_2 - \omega_1} \tag{5}$$

where ω_1, ω_2 are the sprocket angular velocities.

Power demand is obtained by multiplying the sum of the resistance forces with the vehicle velocity:

$$P_{req} = F_{tr}V + M_R\omega. \tag{6}$$

The maximum power is required during the maximum acceleration. Assuming acceleration on the level ground, the power required is given as:

$$P_{max} = (fm_vg + \frac{1}{2}C_dA\rho v^2 + \delta m_va)V_f, \tag{7}$$

where V_f is the final vehicle speed.

The maximum torque is determined based on the gradeability requirements. On a slope, at a constant low speed, the vehicle must overcome road grade resistance and rolling resistance as follows:

$$F_{gra,max} = (fm_vg\cos\alpha + m_vg\sin\alpha). \tag{8}$$

The required torque is expressed as:

$$T_{max} = F_{gra,max}\cdot r = (fm_vg\cos\alpha + m_vg\sin\alpha)\cdot r, \tag{9}$$

where r is the sprocket radius.

The performance requirements and the vehicle parameteres are given in Table 1.

Table 1 – Overview of the vehicle parameters
Таблица 1 – Обзор характеристик машины
Табела 1 – Преглед параметара возила

Parameter	Value	Parameter	Value
Vehicle mass m $[kg]$	13850	Sprocket radius r $[m]$	0.2577
Track contact length L $[m]$	3.3	Vehicle tread B $[m]$	2.526
Vehicle frontal area A $[m^2]$	5.4	Drag coeff. C_d $[-]$	1.1
Air density ρ $[kg/m^3]$	1.2258	Rolling resistance coeff. f $[-]$	0.07
Maximum gradeability $[\%]$	60	Maximum speed V_{max} $[km/h]$	65
Maximum acceleration	0-32km/h in $8s$	Silent watch autonomy $[km]$	25

Transmission

Most of electric and series electric hybrid vehicles have a single-stage transmission between the traction motor and the wheel (or the final drive) due to high efficiency and a favorable torque curve of the electric motor (Wu et al., 2013). However, with HETVs, the required power and torque performances are in relative disproportion, so it is difficult to find an electric motor with a sufficiently broad operating range. In addition, it was shown that the dual-stage transmission for HETVs is significantly more efficient than single-stage (Randive et al., 2021). The adopted gear ratios depend on the maximum torque and the maximum required speed of the vehicle, and have a direct impact on the sizing of the traction motor. The minimum gear ratio must meet the condition of gradeability, that is:

$$i_{min} \geq \frac{1}{2}\frac{F_{grade,max} \cdot r}{T_{m,peak}}, \tag{10}$$

where $Tm, peak$ is the peak torque rating of the two traction motors. On the other hand, the maximum gear ratio must meet the condition of simultaneously achieving the maximum speed of the traction motor and the vehicle, that is:

$$i_{max} \leq \frac{\omega_{EM,max}}{V_{max}/r}, \tag{11}$$

where $\omega_{EM,max}$ is the maximum speed of the traction motor.

Traction motor

The maximum power and the torque rating of the two traction motors need to satisfy performance requirements expressed with Eq. (7) and Eq. (8). The combined maximum power of the two traction motors should be equal to or greater than the maximum power required P_{max}. The maximum and rated torque must be sufficient to satisfy gradeability performance and to enable continuous and smooth motion at the maximum speed, respectively. Adopting the data from Table 1, and substituting into Eq. (7) and Eq. (8), the maximum required power of $P_{req} = 227.34kW$ and the maximum required torque of $20113Nm$ are obtained. The torque required to maintain the maximum speed is obtained from Eq. (1) when acceleration and road grade resistances are ignored and amounts to $3062.3Nm$. The maximum sprocket speed is calculated as $670rpm$. In accordance with this data, two electric motors of a maximum speed of $9000rpm$, a maximum torque of

$600Nm$ and a rated power of $120kW$ were considered in this study. After

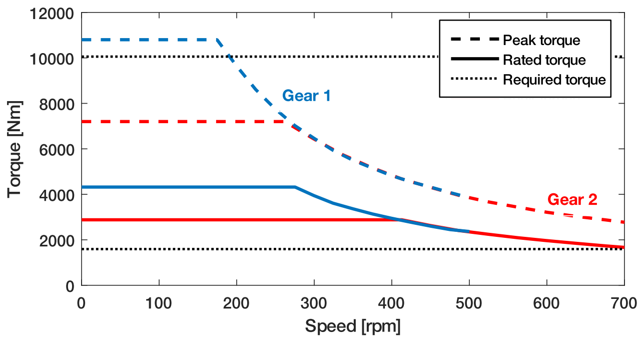

Figure 2 – Combined torque-speed curve of the traction motor and the transmission
Рис. 2 − Общая характеристика электродвигателя и трансмиссии
Слика 2 – Заједничка карактеристика електромотора и трансмисије

adopting gear ratios of $i_1 = 12$ and $i_2 = 18$, it is confirmed that the combined torque-speed characteristic of the motor and transmission satisfies the required performances as shown in Fig. 2.

Engine-generator set

In (Randive et al., 2019) engine sizing is based on the condition of meeting the required power for constant speed operation. However, in this work, an identical engine as in the reference vehicle will be adopted for the reason of achieving the validity of the comparison with the existing parallel hybrid configuration. Therefore, an engine with the power rating of $235kW$ is adopted.

Energy storage sizing

Considering the size of the engine-generator set in this case, the energy storage has the role of meeting required vehicle performance in the electric only mode. The power of the storage is calculated as (Arıkan, 2019):

$$P_{es} = \frac{P_{max}}{\eta}, \tag{12}$$

where P_{max} is the maximum power needed in the silent watch mode and η is the efficiency of transmission and electric motors.

The battery energy required to satisfy the performance of the silent watch mode is calculated as (Borthakur & Subramanian, 2016):

$$E_{es} = P_{ele} \cdot \frac{S}{V_{ele}}, \tag{13}$$

where S is the required silent watch autonomy, V_{ele} is the vehicle speed during silent watch (35 km/h) and P_{ele} is the power needed to drive the vehicle in a pure electric mode defined as

$$P_{ele} = \frac{1}{\eta}(m_v g f + \frac{1}{2} C_d \rho A V^2) \cdot V, \tag{14}$$

Using Eq. (12) and Eq. (13) with the desired performance parameters from Table 1, a battery with the power rating of $105kW$ and the energy rating of $75kWh$ is selected for this work. The summary of component sizes is given in Table 2.

Table 2 – Overview of the powertrain specifications
Таблица 2 – Обзор характеристик гибридного привода
Табела 2 – Преглед параметара хибридног погона возила

Item	Specification
Transmission	Dual-stage with ratios: $i_1 = 12$, $i_2 = 18$
Traction motor	Max.power: $120kW$, Max.speed: $9000rpm$, Max.Torque: $600Nm$
Engine-generator set	Max.power: $235kW$@$2500rpm$
Energy storage	Max.power: 105kW, Energy capacity: 75kWh

Energy management

The most common rule-based strategy in hybrid vehicles is the Thermostat Control Strategy (TCS). Other frequently represented strategies are the Power Follower Control Strategy (PFCS) and the Maximum SOC of Peak Power Supply (Max.SOC-of-PPS) (Ehsani et al., 2018). However, some more advanced rule-based strategies such as the Optimal Primary Source Strategy (OPSS) (Shabbir & Evangelou, 2019) have appeared in recent times. These strategies have a simple implementation, are robust and achieve good results, which makes them adequate candidates for implementation in military tracked vehicles.

1. Thermostat Control Strategy is based on on/off switching of the PS depending on the battery state of charge (SOC) value. The battery SOC can vary in a predefined range $[SOC_L, SOC_U]$. Then, when the SOC reaches the lower limit, the PS turns on and recharges the battery up to the SOC_U value, when it turns off again. The PS is typically set at the most efficient operating point P_{PSopt}. The mathematical implementation is based on the state $S(t)$ which determines if the PS is active:

$$S(t) = \begin{cases} 0, & SOC(t) \geq SOC_U \\ 1, & SOC(t) \leq SOC_L \\ S(t^-) & SOC_L < SOC(t) < SOC_U \end{cases} \tag{15}$$

   where SOC_L and SOC_U are the lower and upper limits of the battery SOC, and the $S(t^-)$ is the state S in the previous time sample. The PS will also supplement power if the power demand exceeds the power rating of the SS without changing the state $S(t)$.
2. Power Follower Control Strategy employs the power-following approach which means that the PS follows the load with some deviation in order to correct the battery SOC. The PS power follows the load when the SOC is between SOC_L and SOC_U but biases the PS operation in favor of charging or discharging the battery when the SOC leaves the predefined range. Mathematical implementation is similar to the TCS:

$$S(t) = \begin{cases} 0, & SOC(t) \geq SOC_U \text{ and } P_L < P_{PSmin} \\ 1, & SOC(t) \leq SOC_L \text{ or } P_L > P_{SSmax} \\ S(t^-) & SOC(t) \geq SOC_L \text{ and } P_L < P_{SSmax} \end{cases} \tag{16}$$

   where P_L is the power demand, P_{PSmin} is the tunable minimum power of the PS and P_{SSmax} is the maximum power of the SS. For $S(t) = 0$ the PS power is always $P_{PS} = 0$, while for $S(t) = 1$ the PS operation is defined as:

$$P_{PS}(t) = \begin{cases} P_{PSmin}, & SOC(t) \geq SOC_U \\ P_m(t), & SOC_L < SOC(t) < SOC_U \\ P_{PSmax} & SOC(t) \leq SOC_L \end{cases} \tag{17}$$

where P_{PSmax} is the maximum power of the PS and $P_m(t)$ is given as:

$$P_m(t) = P_L + P_{ch}\Big(\frac{SOC_U + SOC_L}{2} - SOC(t)\Big), \tag{18}$$

where P_{ch} is the charging factor.

3. Optimal Primary Source strategy employs the load-leveling approach using a threshold changing mechanism instead of state changing as in the TCS or the PFCS. In that way, a charge sustaining mechanism is obtained. The strategy design is strongly based on solutions gained via optimization strategies and by utilizing effectiveness of modern start-stop engine systems in HEVs (Shabbir, 2015). The threshold value for the activation of the PS is defined as

$$P_{PSmin}(SOC) = P_{th} + P_{th}\Big(\frac{SOC - SOC_{initial}}{SOC_{range}}\Big), \tag{19}$$

where P_{th} is the threshold value that needs to be tuned to achieve the best results. Applied to a small passenger vehicle, this strategy managed to achieve fuel consumption only 1% lower than the optimization based Equivalent Consumption Minimization Strategy (ECMS) (Shabbir & Evangelou, 2019).

Results analysis

Based on the mathematical model, a backward-looking model of the series HETV was created in the Simulink environment. For the evaluation of the model and EMS, a drive cycle was artificially constructed using the data available. The drive cycle contains the speed profiles for both tracks moving on the hard ground (Fig. 3).

It includes significant acceleration, braking, and steering. The average vehicle speed is $18.5 km/h$, and the travel distance is $11.12 km$. Since the drive cycle is assumed to be known, gear shifting is also predetermined such that the vehicle meets the required performance. Three proposed

EMSs were evaluated with the same initial data on the same driving cycle. The power profiles are shown in Fig. 4, and the SOC change over time is presented in Fig. 5.

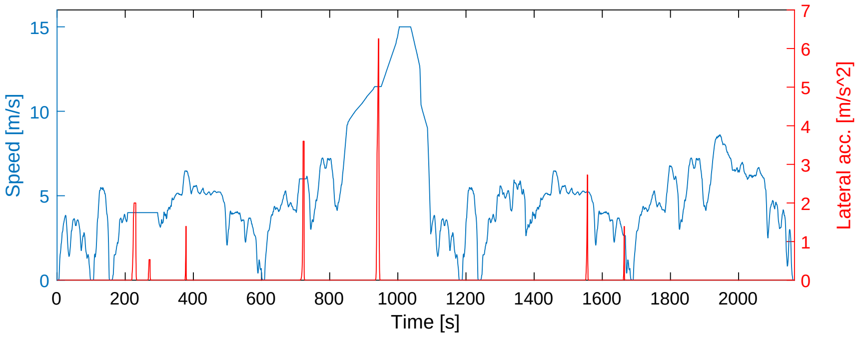

Figure 3 – Created drive cycle for the EMS evaluation
Рис. 3 – Разработанный ездовой цикл для оценки стратегии управления энергопотреблением
Слика 3 – Креирани циклус вожње за евалуацију стратегије управљања енергијом

The power profiles of the PFCS and the TCS are very similar (Fig. 4). Although the PFCS should follow the load, in this case it does not happen due to the specific vehicle exploitation conditions causing the elements of the powertrain to be oversized in order to achieve the required performance. Therefore, it seems that the PFCS is not a good choice for use in HETVs. Instead, the TCS would be a more adequate choice for HETVs due to its simplicity. On the other hand, the power profile of the OPSS is significantly different from those of the PFCS and the TCS. The load-levelling approach is noticeable and the PS always works at the optimal operating point. Also, the SOC varies significantly less than in the PFCS and the TCS, thus extending the battery life. Despite the significantly less SOC variation, with the OPSS, the engine is active about 6% time less compared to the other two strategies (Table 3). This fact directly reflects in fuel consumption, which is lower than in the other two strategies (Table 4).

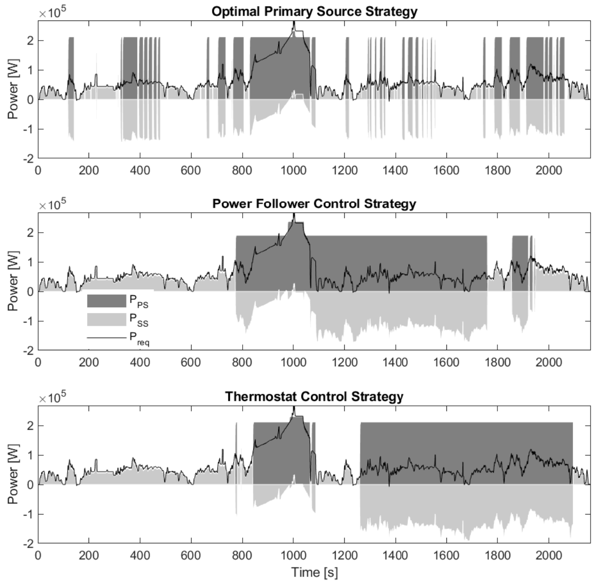

Figure 4 – Power time histories of the tested EMS
Рис. 4 − Профиль изменения мощности протестированных стратегий управления энергопотреблением
Слика 4 – Профил промене снаге код тестираних стратегија управљања енергијом

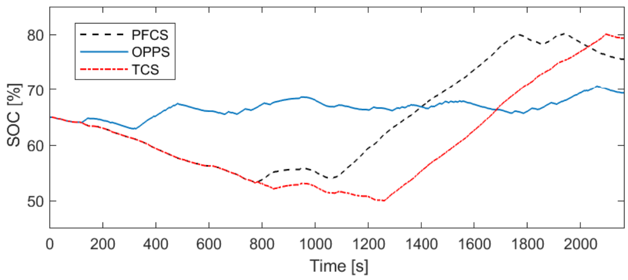

Figure 5 – Charge profiles for the TCS, the PFCS, and the OPSS
Рис. 5 − Диаграмма изменения значений SOC относительно стратегий TCS, PFCS и OPSS
Слика 5 – Дијаграм промене вредности SOC за стратегије TCS, PFCS и OPSS

The OPSS proved to be the best of the tested strategies. In addition to the best fuel economy, the small SOC variation reduces battery voltage fluctuation and increases the battery life. A constantly high battery SOC ensures the availability of electric power for auxiliary loads on or off the vehicle. In comparison with the reference vehicle from (Milićević et al., 2021), the proposed configuration with the OPSS achieves a significantly better economy of as much as 53% (Table 5).

Table 3 – Engine usage time as the percentage of the total drive cycle time
Таблица 3 – Время использования двигателя СУС, выраженное в процентах от общего времени ездового цикла
Табела 3 – Време активираности мотора СУС изражено као проценат укупног времена трајања циклуса вожње

EMS	Engine ON time as % of total drive cycle time
TCS	50.5%
PFCS	50.51%
OPSS	44.09%

Table 4 – Fuel consumption comparison for the tested EMSs
Таблица 4 – Сопоставление расхода топлива протестированных стратегий управления энергопотреблением
Табела 4 – Поређење потрошње горива тестираних стратегија управљања енергијом

EMS	Relative fuel consumption [-]	**Improvement [%]**
TCS	100	**-**
PFCS	97.86	**2.14%**
OPSS	92.02	**7.98%**

This result is a consequence of much better powertrain sizing and a simple and adequate EMS. The initial hypothesis for the reference vehicle to change the transmission as little as possible, and then to optimize the operation of such a transmission, proved to be very unsuccessful, which showed that when hybridizing a vehicle, one must take into account the proper sizing of the powertrain elements.

The reference vehicle ended up with a slightly oversized engine and undersized electric motors and battery, which caused the engine to be active during most of the drive cycle. By switching to the series configuration and with proper sizing, significantly better results were achieved. The main reason for this is a much larger battery and more efficient operation of the engine due to the absence of a mechanical connection between the engine and the wheels. In this work, the engine was left unchanged for the purpose of comparison with the reference vehicle; however, as Fig. 4 shows, it is clear that excess energy is created and that the engine should be downsized, which would also achieve additional fuel savings.

Table 5 – Comparison between the reference and proposed powertrain configuration and the EMS
Таблица 5 – Сопоставление серийного и предлагаемого гибридного привода и стратегии управления энергопотреблением
Табела 5 – Поређење референтног и предложеног хибридног погона и стратегије управљања енергијом

	Engine ON time	Relative fuel consumption [-]	**Improvement [%]**
Reference	92%	100	**-**
Proposed	44.09%	46.21	**53.79%**

Conclusion

This paper presents a systematic approach to the development of a series configuration hybrid electric tracked vehicle. All powertrain elements were systematically sized based on the key performance requirements except the engine, which remained of the same size as in the reference vehicle with the parallel hybrid configuration. Three energy management strategies were proposed, namely the TCS, the PFCS, and the OPSS. The evaluation of the powertrain configuration and the proposed strategies was performed in the Simulink environment using a driving cycle containing significant acceleration, braking and steering. The results showed that the OPSS proved to be the best due to the best fuel economy and a low battery SOC variation. The load-leveling approach enabled this strategy to have an engine usage time of 6.4% less than the other two EMSs relative to the total drive cycle time. In comparison with the same vehicle with the parallel configuration where the sizing was constrained in the sense of harnessing hybrid drive advantages with as little change of the powertrain as possible, the proposed powertrain configuration achieved a significantly better fuel economy of 53.79%. The main reasons for improved fuel economy are proper sizing, a much simpler powertrain and therefore a much more efficient EMS which enabled the engine to be active only 44.09 % of the total drive cycle time compared to 92% of the time of the parallel configuration. The main conclusion of this work is that proper sizing of powertrain elements must be taken into account when hybridizing a vehicle. Potential fuel savings and increased efficiency outweigh the cost of radical powertrain changes. Also, the series hybrid configuration presented itself as a major candidate for use in hybrid electric tracked vehicles.

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МОЩНОСТЬ И УПРАВЛЕНИЕ ЭНЕРГОПОТРЕБЛЕНИЕМ ГИБРИДНОЙ ГУСЕНИЧНОЙ МАШИНЫ ОБЫЧНОЙ КОНФИГУРАЦИИ

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РУБРИКА ГРНТИ: 78.25.09 Военная автомобильная техника,
78.25.10 Бронетанковая техника
ВИД СТАТЬИ: оригинальная научная статья

Резюме:

Введение/цель: В данной статье представлен системный подход к разработке серийного гибридного электрического гусеничного транспортного средства (ГЭТС), включая выбор привода и соответствующей стратегии управления энергопотреблением (ЭМС).

Методы: Размеры элементов силового агрегата были подобраны с учетом ключевых требований к производительности. Были предложены три стратегии управления энергопотреблением: стратегия управления с помощью термостата (TCS), стратегия управления повторителем мощности (PFC) и стратегия оптимального источника питания (OPSS). Оценка конфигурации трансмиссии и трех предложенных стратегий была выполнена в среде Simulink с использованием ездового цикла, включающего значительное ускорение, торможение и рулевое управление.

Результаты: Результаты показали, что OPSS оказалась лучшей стратегией, благодаря экономии топлива и низкому уровню изменчивости заряда батареи (SOC). Надо подчеркнуть, что по сравнению с предыдущими испытаниями того же автомобиля с параллельной гибридной конфигурацией были достигнуты значительно лучшие результаты. Анализ результатов показывает, что предложенная кон-

фигурация привода и стратегия управления обеспечивают снижение расхода топлива на 53,79 %, что свидетельствует о правильном выборе размера гибридного привода.

Выводы: Результаты данного исследования представляют большую значимость для понимания влияния правильного выбора размера привода на экономичность транспортного средства. По сравнению с серийным транспортным средством, предлагаемая конфигурация обеспечивает значительное улучшение, в частности, благодаря соответствующему выбору размера. Согласно выдвинутой гипотезе, наилучшей стратегией оказалась OPSS. Серийная гибридная конфигурация с OPSS в качестве ЭМС оказалась лучшим кандидатом для использования ГЭТС.

Ключевые слова: гусеничная машина, гибридная машина, энергопотребление, стратегия управления энергопотреблением, расход топлива.

ДИМЕНЗИОНИСАЊЕ ПОГОНА И УПРАВЉАЊЕ ЕНЕРГИЈОМ ХИБРИДНОГ ГУСЕНИЧНОГ ВОЗИЛА РЕДНЕ КОНФИГУРАЦИЈЕ

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ОБЛАСТ: машинство
КАТЕГОРИЈА (ТИП) ЧЛАНКА: оригинални научни рад

Сажетак:

Увод/циљ: У раду је представљен систематски приступ развоју редног хибридног електричног гусеничног возила (ХЕТВ) укључујући димензионисање погона и избор одговарајуће стратегије управљања енергијом (ЕМС).

Методе: Елементи погонског склопа су димензионисани, узимајући у обзир кључне захтеве перформанси. Предложене су три стратегије управљања енергијом: термостатска стратегија (ТЦС), стратегија управљања праћењем оптерећења (ПФЦС) и стратегија оптималног извора енергије (ОПСС). Евалуација конфигурације погона и три предложене ЕМС извршене су у окружењу Симулинк ко-

ришћењем циклуса вожње који садржи делове са знатним убрзањима, кочењима и управљањем.

Резултати: Резултати су показали да се ОПСС показала као најбоља стратегија због повећане уштеде горива и ниске варијације стања напуњености батерије (СОЦ). У поређењу са претходним истраживањем истог возила са паралелном хибридном конфигурацијом, постигнути су знатно бољи резултати. Анализа резултата показује да се предложеном конфигурацијом погона и стратегијом управљања потрошња горива смањује за 53,79 %, што указује на то да је димензионисање хибридног погона правилно изведено.

Закључак: Резултати овог рада су од великог значаја за разумевање утицаја правилног димензионисања погона на економичност возила. У поређењу са референтним возилом, предложена конфигурација постиже значајно побољшање, од којег се највећи део приписује адекватном димензионисању. ОПСС се показала као најбоља стратегија, чиме је потврђена теоријска хипотеза. Показало се да је редна хибридна конфигурација са ОПСС као ЕМС најбоља за употребу у ХЕТВ-у.

Кључне речи: гусенично возило, хибридно возило, управљање енергијом, стратегија управљања енергијом, потрошња горива.

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Paper accepted for publishing on / Дата окончательного согласования работы / Датум коначног прихватања чланка за објављивање: 15.10.2022.

NUMERICAL ANALYSIS OF A FRONTAL IMPACT OF A 12.7 mm PROJECTILE ON AN ARMOR PLATE

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FIELD: Mechanical engineering, Materials
ARTICLE TYPE: Original scientific paper

Abstract:

Introduction/purpose: The paper presents a numerical simulation of an impact of a 12.7 mm projectile on an armored metal plate with a velocity of 500 m/s at a distance of 900 m. Numerical simulations offer the possibility of drastically reducing the time required to obtain results in comparison to the time required for planning, organization and execution of experiments. The numerical simulation is done by variations in the thickness of the armor metal plate, specifically an armor metal plate of a thickness of 10 mm, 17 mm, 18 mm, and 23 mm. The mentioned armored plate thicknesses were chosen based on the results in order to determine the limit thickness of the *armored plate for the projectile perforation limit, as well as for complete ballistic protection.*

Methods: Finite element modeling is used for analyzing stresses and deformations of the armored plates. The mentioned method calculates the impact of the projectile on the obstacle, precisely the collision of the projectile and the armor plate.

Results: For the comparative analysis, the parameters used are the values of the stress and the displacement. For each of the above-mentioned thicknesses of the armored metal plate, the values of stress and displacement during projectile impact were determined. The results of this study show how the thickness of the armor plate affects the interaction of the projectile and the armor plate.

Conclusion: If the physical and chemical characteristics of the armored plate remain unchanged, as the thickness of the armored plate increases, the possibility of projectile penetration decreases, and vice versa. This research is of essential importance because it analyzes the stresses and deformation of armor plates whose basic role is the protection of personnel and equipment from the projectile impact. In this regard, the thickness of the armored plate for semi-penetration of the projectile is determined.

Keywords: armor plate, projectile, impact, finite element modeling.

Introduction

Small-caliber bullet protection is a key concern for both military and civilian facilities, especially at distances up to 100 m. The main task is how to protect infantry from the effects of anti-materiel rifles in calibers of 10 mm to 20 mm. Modern war implies that infantry is transported by combat vehicles such as Infantry Fighting Vehicles (IFVs), also known as Mechanized Infantry Combat Vehicles (MICVs), or Mine-Resistant Ambush-Protected (MRAP) wheeled armored vehicles.

Troops transported by such vehicles are a very easy group target, and because of that, it is very important to protect troops inside vehicles from the effect of projectiles. In order to reduce the penetrability of vehicles, armored steel plates are added. Metallic armor plates are often used to protect moving and stationary platforms from a variety of projectiles. However, it is necessary to be careful, because the addition of armor plates affects the overall weight of the vehicle and reduces the mobility and passability of the vehicle.

Large deformation, erosion, high strain rate, dependent nonlinear material behavior, and fragmentation are all problems associated with high-velocity impact and projectile penetration.

The basic task of this paper is to determine the thickness of the plate that will be resistant to the impact of a projectile of 12.7 mm, thus protecting the infantry, and which will not affect the performance of the vehicle. For this study, only a frontal impact of a projectile into a plate of various thicknesses was considered.

The bullet used in this analysis is 12.7 mm and it is shown in Figure 1.

The core of the bullet is made of an alloy of copper and zinc, and the core of this bullet is the projectile used in the simulation. The ballistic characteristics of the core are presented in Table 1.

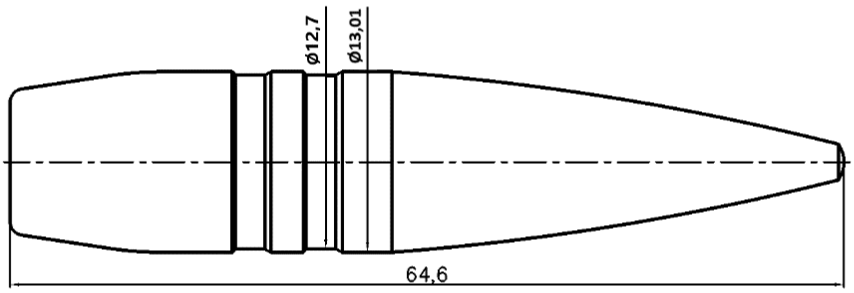

Figure 1 – Bullet 12.7 x 108
Рис. 1 – Пуля 12.7 x 108
Слика 1 – Метак 12,7 x 108

Table 1 – Ballistic characteristics of the bullet core
Таблица 1 – Баллистические характеристики сердечника пули
Табела 1 – Балистичке карактеристике зрна метка

Projectile velocity (at a distance of 25 m)	V_{25}	805 m/s
Projectile velocity (at a distance of 300 m)	V_{300}	720 m/s
Pressure	p_{max}	304 MPa
Precision	R_{s300}	10 cm
Core weight	m	51,3 g

The material characteristics of the core of the bullet for the explicit dynamic analysis are given in Table 2.

Table 2 – Material characteristics of the core of the bullet for the explicit dynamic analysis
Таблица 2 – Характеристики материала сердечника пули и для явного динамического анализа
Табела 2 – Материјалне карактеристике зрна метка и параметри потребни за експлицитну динамичку анализу

	Parameters		Values
Johnson-Cook parameters	Yield stress	*A [MPa]*	112
	Proportionality coefficient	*B [MPa]*	505
	Strain rate Impact parameter	*C*	0.009
	Temperature impact parameter	*m*	1.68
	Reinforcement exponent	*n*	0.42
	Melting temperature	T_m *[K]*	1189
	Room temperature	T_r *[K]*	293
	Constant	$\dot{\varepsilon}_0$	1
Johnson-Cook damage parameters	Damage parameters	D_1	0.54
		D_2	4.89
		D_3	3.03
		D_4	0.014
		D_5	1.12
EOS parameters	Mie-Gruneisen equations of state parameters	*M [m/s]*	3667
		S_1	1.507
		S_2	0.000
		S_3	0.000
		Γ	2.086
		a	0.485
General parameters	Density	ρ *[t/mm³]*	8.52E-9
	Young’s modulus	*E [MPa]*	110
	Shear modulus	G *[GPa]*	40
	Poisson’s ratio	*v*	0.375
	Specific heat	Cp [J/kgK]	385

Finite element modeling

The penetration, damage, and failure mechanisms when the projectile impacts the armor plate were investigated using a computational model based on finite elements (Jena et al, 2019). Theoretical models are used in simulations with real material properties (projectiles and armor plates) to show how the projectile interacts with the armor plate. LS-DYNA (Livermore Software Technology, 2014), a commercially available finite element software, was used for finite element modeling and analysis (Mahfuz et al, 1999).

A two-dimensional finite element model of the armor plate and the bullet core was developed as shown in Figure 2.

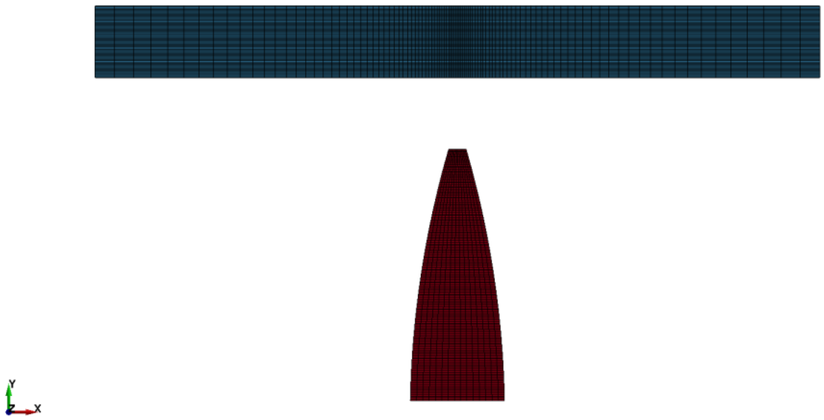

Figure 2 – Finite element model
Рис. 2 – Модель конечных элементов
Слика 2 – Модел коначних елемената

The armor plate was meshed with four-node continuum hexahedral elements. Two-dimensional finite elements provide better computational performance/cost than fully integrated 3D elements. The element size was smallest in the region where the projectile impacted the armor plate and the element size was increased in regions away from the impact point. The overall finite element model had 8400 2D four-noded hexahedral elements. Contact was defined between the projectile and the armor plate with a hard contact definition for normal contact. The developed finite element model was used to investigate the penetration of the projectile through the base armor plate. The numerical calculation was performed at a distance of 900 m when the projectile velocity was 500 m/s.

Theoretical basis

Penetration is the motion process of a penetrator through an obstacle (armor plate for this study). The term, penetrator, means anything that is intended for penetration, and the obstacle is the environment that is exposed to the action of the penetrator. The study of the penetration process is of great importance both in the field of military technology and in the field of civilian application (Feng et al, 2020). Terminal ballistics is one of the basic disciplines that deals with defining the mechanisms of penetration, which significantly contributes to the optimization of the design of the projectile, penetrating, and destructive action, as well as for the design of armor protection (Meng et al, 2021).

Depending on the outcome of the penetration process, there are four different cases:

Perforation - means the penetration of the entire penetrator through the obstacle (armor plate for this study), forming a regular, approximately cylindrical hole in the obstacle.

Limit perforation - represents the limit case of penetration because the hole in the obstacle is of irregular shape and a smaller area than the cross-sectional area of the penetrator, unlike the perforation, i.e. only parts of the broken penetrator pass through the hole.

Semi penetration - characterizes the stopping (jamming) of the penetrator in the obstacle or its breaking during penetration.

Ricochet - is the repulsion of the penetrator due to sliding on the surface of the obstacle if it is tilted.

The penetrating power of a penetrator is the ability to break through an obstacle. Increasing the penetrating power of the penetrator can be achieved by increasing the length and density of the penetrator, as well as by reducing its diameter. In opposition to this, the ability to resist penetration is the resistance of an obstacle. Increasing the resistance of the obstacle is achieved by increasing its thickness and density, as well as by improving the mechanical properties of the material. When considering penetration, the impact velocity, output velocity, and velocity of the ballistic limit are of greatest importance.

Impact velocity V_s (or V_0) is the instantaneous value of the penetrator line velocity at the moment of initial contact with the obstacle. It is assumed that the impact velocity vector is collinear with the penetrator axis, i.e. the flight of the penetrator with zero angle of attack is always assumed. The effects of the angular velocity of the penetrator around its own axis, in the case of gyro-stabilized penetrators, are not taken into account (Rajole et al, 2020).

Output (residual) velocity V_r is the velocity of the penetrator at the moment of passing the bottom of the penetrator through the plane determined by the rear surface of the obstacle.

The velocity of the ballistic limit is one of the basic characteristics of the penetrator-obstacle system and can be defined in several ways. Theoretically, this is the minimum value of the impact velocity at which the penetration occurs, or the maximum value of the impact velocity at which the penetration through the obstacle does not occur.

Johnson-Cook material model

The Johnson-Cook plasticity model was used to calculate the strain rate-dependent plastic deformation of the projectile core and armor plate material. Metal high-strain rate deformation has been successfully defined using the Johnson-Cook plasticity model (Wang & Shi, 2013). The effects of strain, strain rate, and adiabatic heating on flow stress are included in the Johnson-Cook plasticity model. The Johnson-Cook plasticity model is represented by Equation 1.

$$\sigma = \left[A + B\varepsilon^n \right]\left[1 + C\ln\dot{\varepsilon}^*\right]\left[1 - (T^*)^m\right] \quad (1)$$

where A, B, C, n, and m are the material parameters determined from experimental data. The temperature is determined from equation 2.

$$T^* = \frac{\left(T - T_{ref}\right)}{\left(T_{melt} - T_{ref}\right)} \quad (2)$$

where T_{ref} is the temperature below which material shows no temperature dependence on flow stress. The strain rate is given by equation 3.

$$\dot{\varepsilon}^* = \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \quad (3)$$

The initiation of damage is determined by equation 4, which gives the equivalent plastic strain at the onset of damage.

$$\varepsilon^{pl} = \left[d_1 + d_2 e^{(-d_3\eta)} \right]\left[1 + d_4 \ln\left(\frac{\dot{\varepsilon}^{pl}}{\dot{\varepsilon}_0} \right) \right]\left(1 + d_5 T^*\right) \quad (4)$$

where d_1, d_2, d_3, d_4, and d_5 are the material damage parameters and $\dot{\varepsilon}_0$ is the reference strain rate. The damage in the material is defined by using a parameter D with a value between 0 and 1 where 0 means no damage and 1 means fully damaged material. Material failure occurs when D reaches a value of 1.

Material characteristics and initial conditions

AISI 4340 steel material characteristics were used for the armor plate and are shown in Table 3, while the projectile material characteristics were the ones from a copper alloy and are shown in Table 2.

Table 3 – Material characteristics of the armor plate for the explicit dynamic analysis
Таблица 3 – Характеристики материала бронелиста и характеристики для явного динамического анализа
Табела 3 – Материјалне карактеристике балистичке плоче и параметри потребни за експлицитну динамичку анализу

	Parameters		Values
Johnson-Cook parameters	Yield stress	*A [MPa]*	792
	Proportionality coefficient	*B [MPa]*	510
	Strain rate Impact parameter	*C*	0.014
	Temperature impact parameter	*m*	1.03
	Reinforcement exponent	*n*	0.26
	Melting temperature	T_m *[K]*	1793
	Room temperature	T_r *[K]*	293
	Constant	$\dot{\varepsilon}_0$	1
Johnson-Cook damage parameters	Damage parameters	D_1	0.05
		D_2	3.44
		D_3	-2.12
		D_4	0.002
		D_5	0.61
EOS parameters	Mie-Gruneisen equations of state parameters	*M [m/s]*	3850
		S_1	1.354
		S_2	0.000
		S_3	0.000
		Γ	1.707
		a	0.430
General parameters	Density	ρ *[t/mm^3]*	7.85E-9
	Young's modulus	*E [MPa]*	210
	Shear modulus	G *[GPa]*	80
	Poisson's ratio	*ν*	0.29
	Specific heat	Cp [J/kgK]	477

Results and discussion

In accordance with theoretical and practical knowledge, it is very easy to conclude that with increasing the thickness of the obstacle (it is important to mention that the same physical and chemical characteristics are maintained) the probability of achieving the effect of penetration decreases.

Within this paper, a numerical simulation of the penetration of a 12.7 mm projectile was performed for four different cases, i.e. for four different obstacle thicknesses.

Model 1

Figures 3-7 show the field of distribution of the von Misses equivalent stress for model 1. For this case, the impact projectile velocity was 500 m/s, the armor plate thickness 10 mm, and the simulation time 0.2 ms.

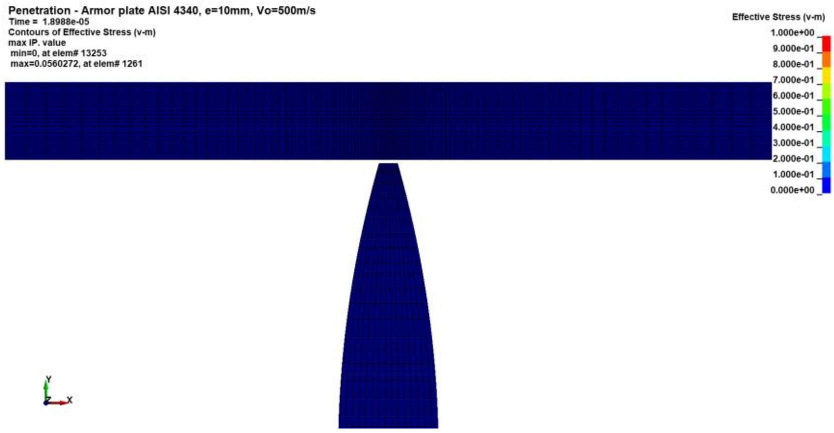

Figure 3 – Von Misses equivalent stress, time of analysis 0.01 ms – Model 1
Рис. 3 – Von Misses эквивалентное напряжение, время анализа 0.01 ms – Модель 1
Слика 3 – Von Misses-ов еквивалентни напон, време анализе 0,01 ms – модел 1

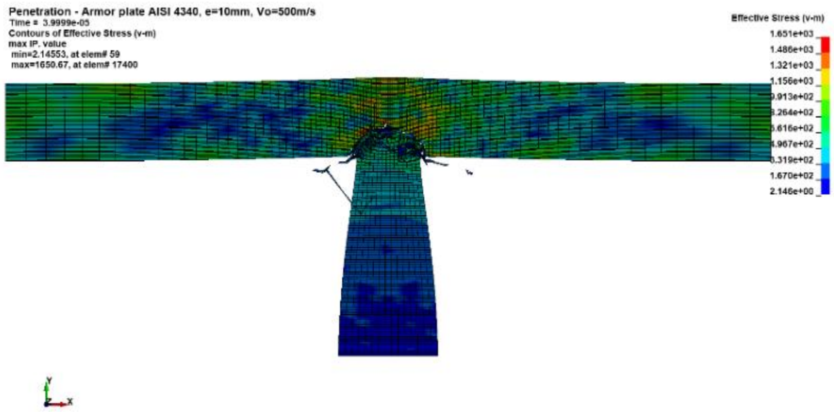

Figure 4 – Von Misses equivalent stress, time of analysis 0.03 ms – Model 1
Рис. 4 – Von Misses эквивалентное напряжение, время анализа 0.03 ms – Модель 1
Слика 4 –Von Misses-ов еквивалентни напон, време анализе 0,03 ms – модел 1

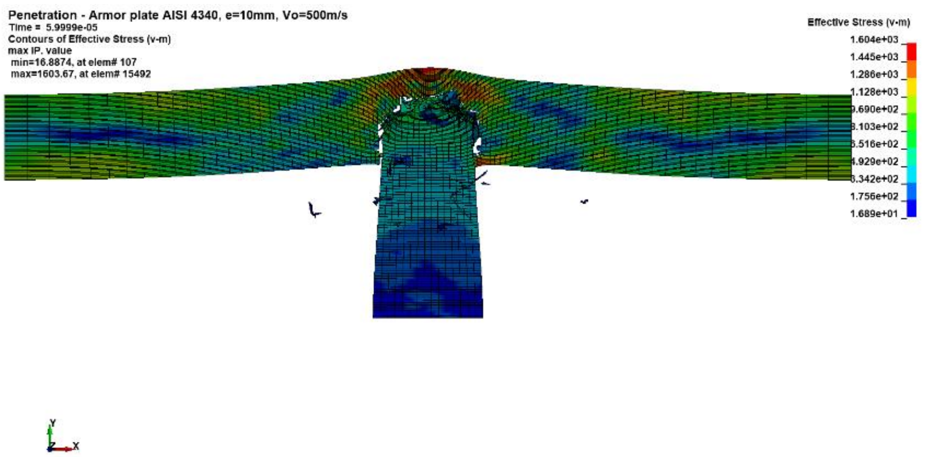

Figure 5 – Von Misses equivalent stress, time of analysis 0.05 ms – Model 1
Рис. 5 – Von Misses эквивалентное напряжение, время анализа 0.05 ms – Модель 1
Слика 5 –Von Misses-ов еквивалентни напон, време анализе 0,05 ms – модел 1

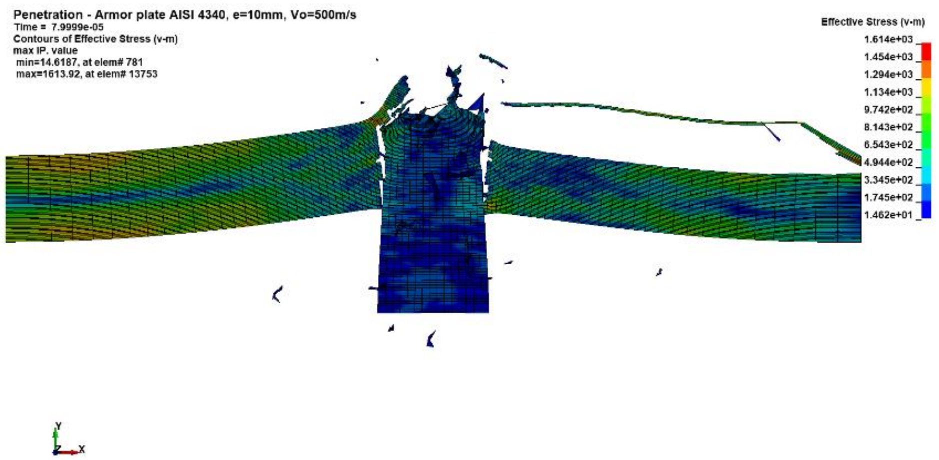

Figure 6 – Von Misses equivalent stress, time of analysis 0.07 ms – Model 1
Рис. 6 – Von Misses эквивалентное напряжение, время анализа 0.07 ms – Модель 1
Слика 6 –Von Misses-ов еквивалентни напон, време анализе 0,07 ms – модел 1

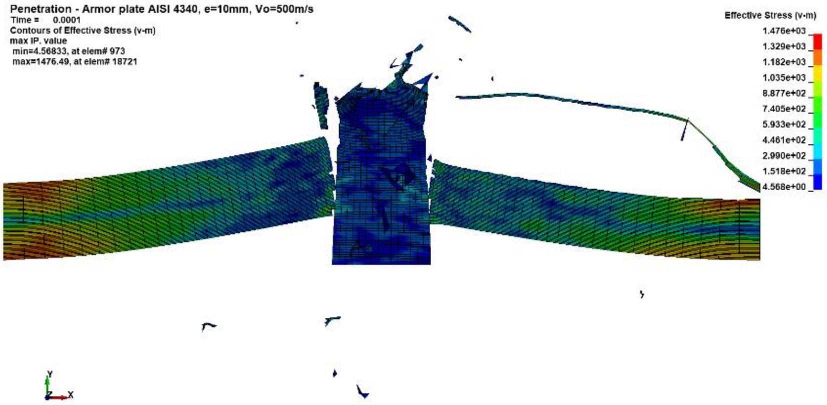

Figure 7 – Von Misses equivalent stress, time of analysis 0.1 ms – Model 1
Рис. 7 – Von Misses эквивалентное напряжение, время анализа 0.1 ms – Модель 1
Слика 7 – Von Misses-ов еквивалентни напон, време анализе 0,1 ms – модел 1

As it can be seen from the previous figures, for the case when the thickness of the armor plate is 10 mm, the penetration of the projectile occurs.

The projectile velocity after the impact and penetration is shown on the diagram in Figure 8.

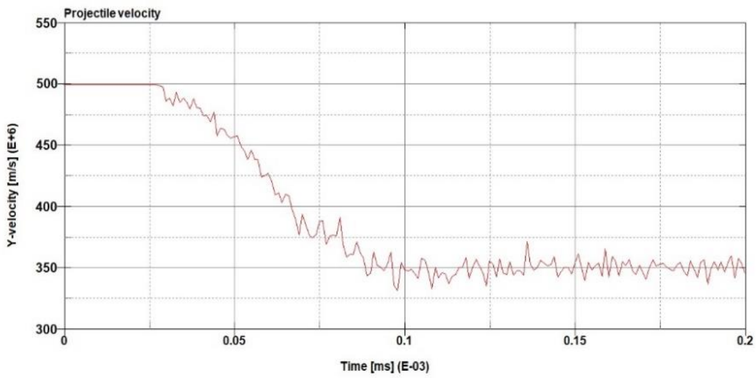

Figure 8 – Velocity of the projectile in relation to time
Рис. 8– Скорость снаряда в зависимости от времени
Слика 8 – Брзина пројектила у зависности од времена

From the diagram in Figure 8, it can be seen that the projectile perforates the armor plate after 0.03 ms. It can be noticed that the projectile velocity decreases between 0.03 ms and 0.1 ms, for the perforation required time. The projectile velocity after 0.1 ms is 350 m/s.

The displacement of the armor plate after the impact and perforation is shown on the diagram in Figure 9. The first displacements occur after 0.03 ms.

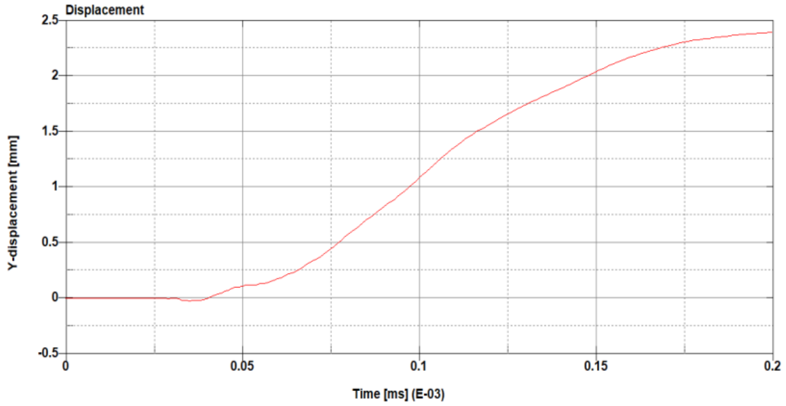

Figure 9 – Displacement of the armor plate in relation to time
Рис. 9 – Смещение в зависимости от времени
Слика 9 – Померање у зависности од времена

Model 2

Figures 10-14 show the field of distribution of the von Misses equivalent stress for model 2. For this case, the impact projectile velocity was 500 m/s, the armor plate thickness 23 mm, and the simulation time 0.2 ms.

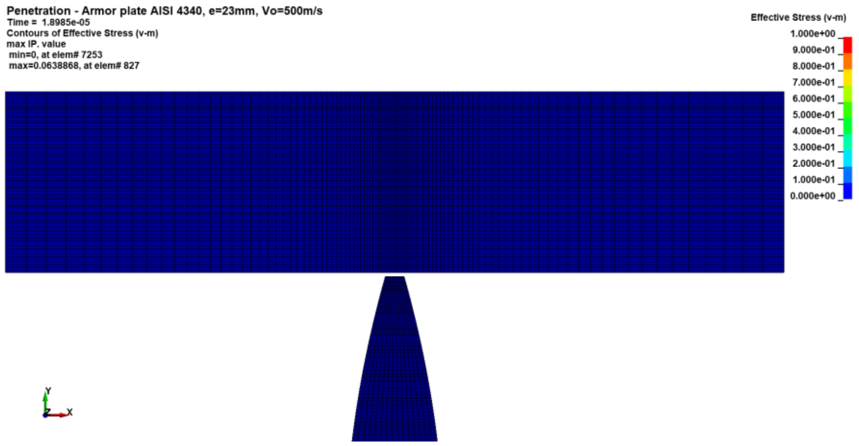

Figure 10 – Von Misses equivalent stress, time of analysis 0.01 ms – Model 2
Рис. 10 – Von Misses эквивалентное напряжение, время анализа 0.01 ms – Модель 2
Слика 10 – Von Misses-ов еквивалентни напон, време анализе 0,01 ms – модел 2

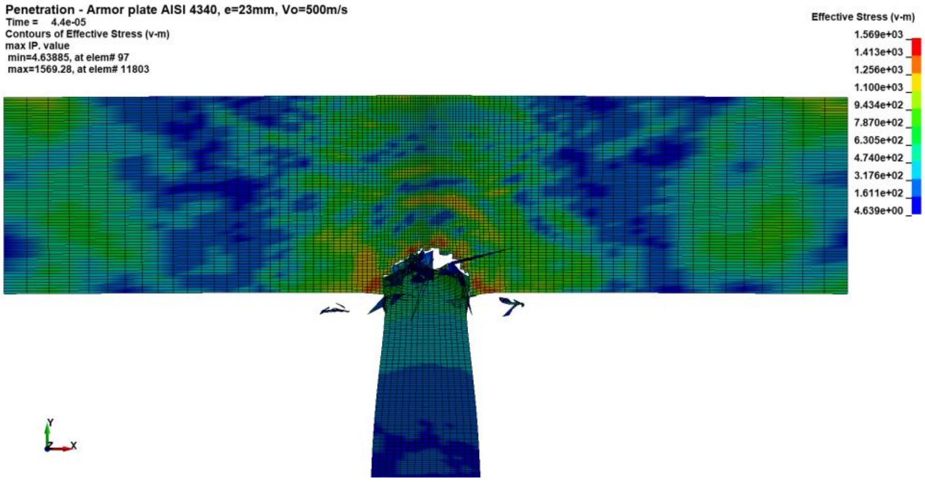

Figure 11 – Von Misses equivalent stress, time of analysis 0.04 ms – Model 2
Рис. 11 – Von Misses эквивалентное напряжение, время анализа 0.04 ms – Модель 2
Слика 11 – Von Misses-ов еквивалентни напон, време анализе 0,04 ms – модел 2

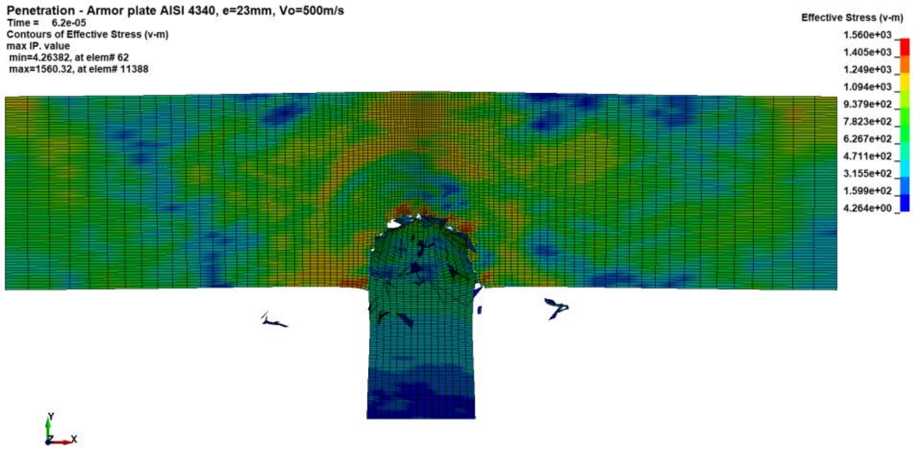

Figure 12 – Von Misses equivalent stress, time of analysis 0.06 ms – Model 2
Рис. 12 – Von Misses эквивалентное напряжение, время анализа 0.06 ms – Модель 2
Слика 12 – Von Misses-ов еквивалентни напон, време анализе 0,06 ms – модел 2

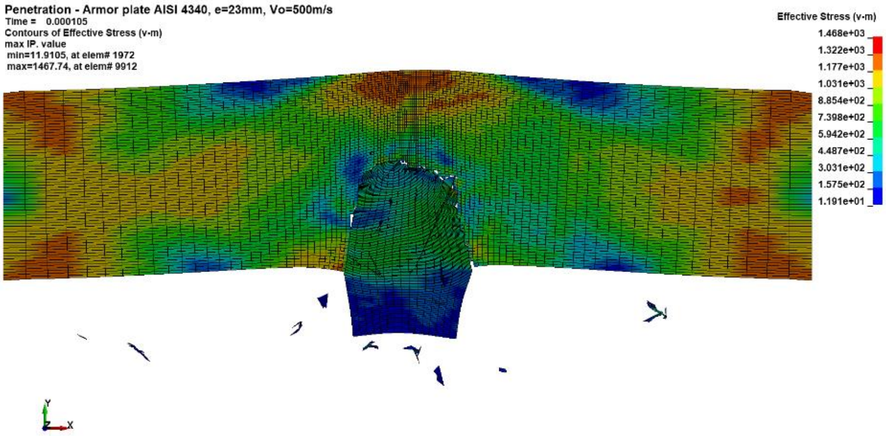

Figure 13 – Von Misses equivalent stress, time of analysis 0.1 ms – Model 2
Рис. 13 – Von Misses эквивалентное напряжение, время анализа 0.1 ms – Модель 2
Слика 13 – Von Misses-ов еквивалентни напон, време анализе 0,1 ms – модел 2

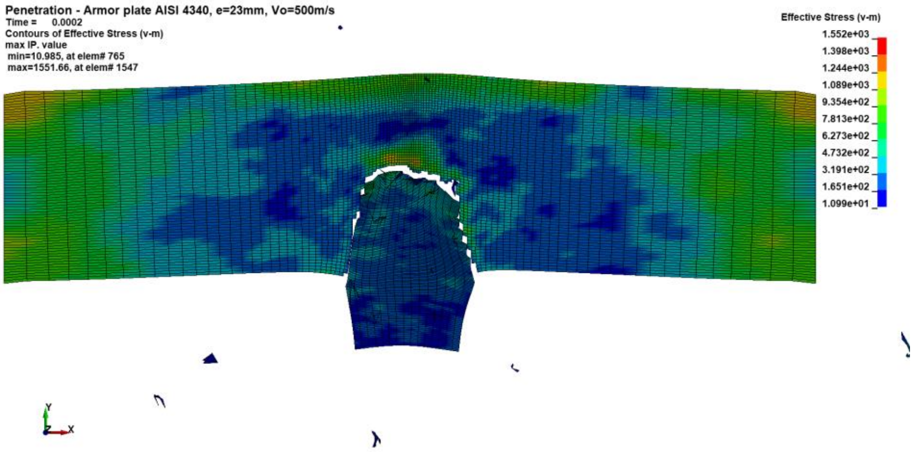

Figure 14 – Von Misses equivalent stress, time of analysis 0.2 ms – Model 2
Рис. 14 – Von Misses эквивалентное напряжение, время анализа 0.2 ms – Модель 2
Слика 14 – Von Misses-ов еквивалентни напон, време анализе 0,2 ms – модел 2

As it can be seen from the previous figures, for the case when the thickness of the armor plate is 23 mm, the perforation of the projectile does not occur.

The projectile velocity after the impact and semi-penetration is shown on the diagram in Figure 15.

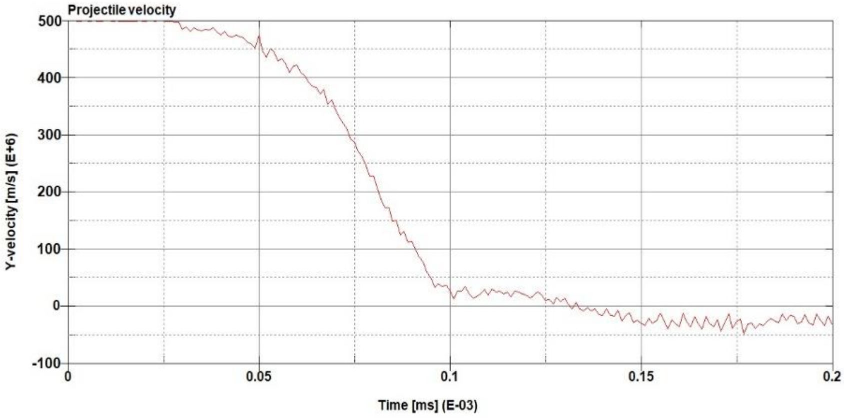

Figure 15 – Velocity of the projectile in relation to time
Рис. 15 – Скорость снаряда в зависимости от времени
Слика 15 – Брзина пројектила у зависности од времена

From the diagram in Figure 15, it can be seen that semi-penetration occurs.

The displacement of the armor plate after the impact and semi-penetration is shown on the diagram in Figure 16. The first displacements occur after 0.03 ms. After 0.13 ms of the analysis, the maximum values of the displacements are achieved.

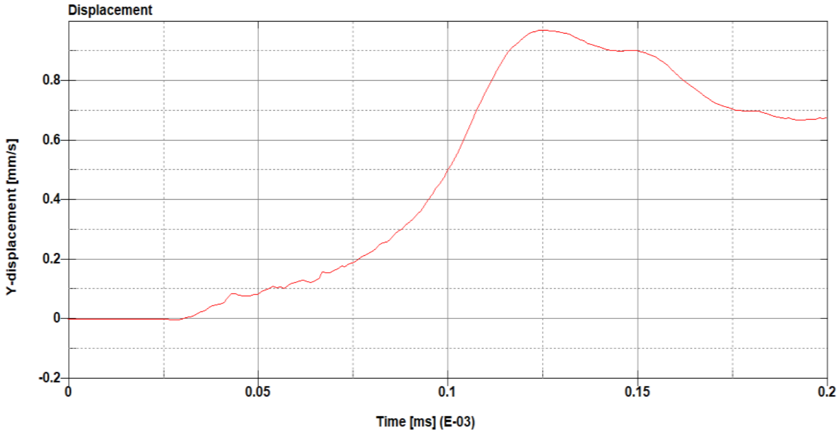

Figure 16 – Displacement of the armor plate in relation to time
Рис. 16 – Смещение в зависимости от времени
Слика 16 – Померање у зависности од времена

Additional numerical simulations

It is of great importance to determine the maximum value of the plate thickness at which the penetration effect occurs, as well as the minimum value of the plate thickness at which the semi-penetration effect occurs, at the same impact velocity.

After presenting the results obtained by the numerical simulation of the penetration process, it is easy to conclude in which cases the projectile has enough energy to break through obstacles of certain thicknesses. In this case, an armor plate made of AISI 4340 alloy was used as an obstacle and it was determined that the 12.7 mm armor projectile at an impact velocity of 500 m/s achieves the effect of penetration on the armor plate with a thickness of 10 mm, while in the case of an armor plate with a thickness of 23 mm it achieves the effect of semi penetration, i.e. no penetration occurs.

In accordance with the previously defined models, using the same initial and boundary conditions, additional numerical simulations were performed and on that occasion, it was determined that the penetration effect is realized on up to 17 mm thick plates, and then the limit penetration effect occurs.

Model 3

Figures 17-21 show the field of distribution of the von Misses equivalent stress for model 3. For this case, the impact projectile velocity was 500 m/s, the armor plate thickness 17 mm, and the simulation time 0.2 ms.

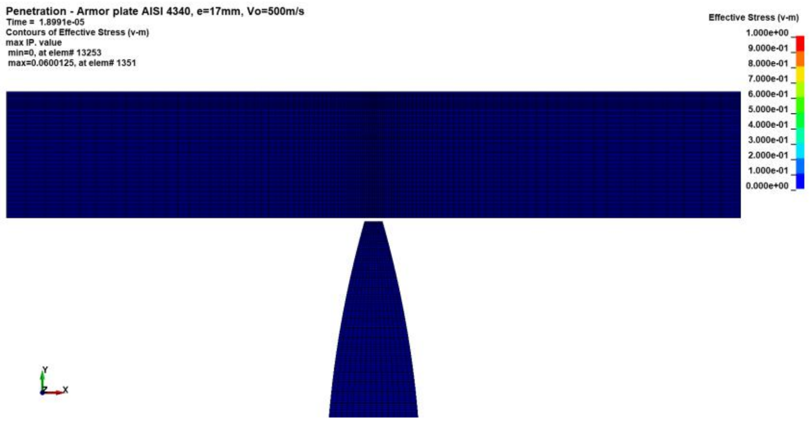

Figure 17 – Von Misses equivalent stress, time of analysis 0.01 ms – Model 3
Рис. 17 – Von Misses эквивалентное напряжение, время анализа 0.01 ms – Модель 3
Слика 17 – Von Misses-ов еквивалентни напон, време анализе 0,01 ms – модел 3

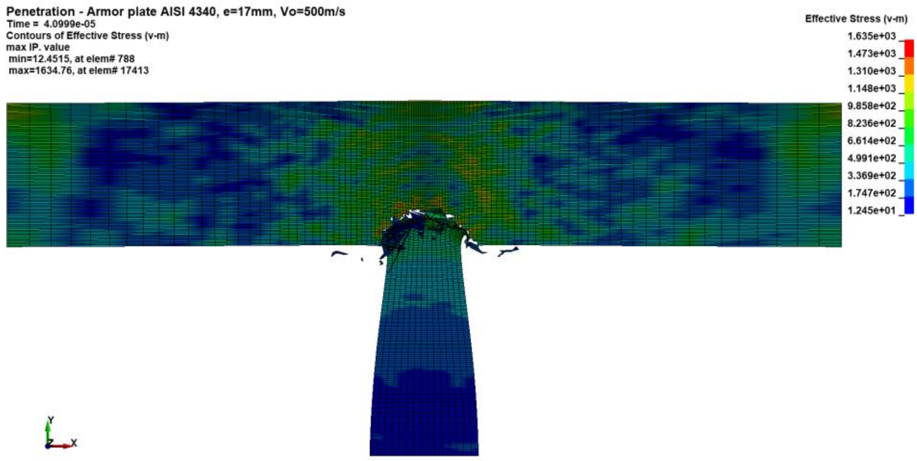

Figure 18 – Von Misses equivalent stress, time of analysis 0.04 ms – Model 3
Рис. 18 – Von Misses эквивалентное напряжение, время анализа 0.04 ms – Модель 3
Слика 18 – Von Misses-ов еквивалентни напон, време анализе 0,04 ms – модел 3

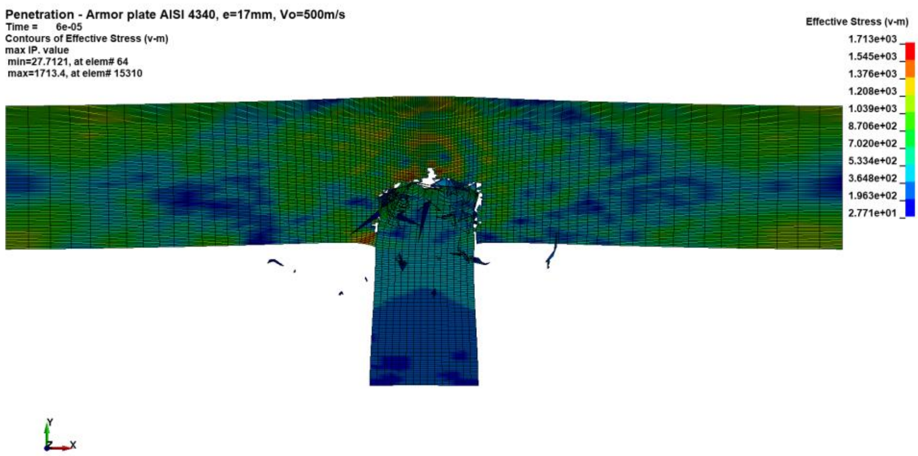

Figure 19 – Von Misses equivalent stress, time of analysis 0.06 ms – Model 3
Рис. 19 – Von Misses эквивалентное напряжение, время анализа 0.06 ms – Модель 3
Слика 19 – Von Misses-ов еквивалентни напон, време анализе 0,06 ms – модел 3

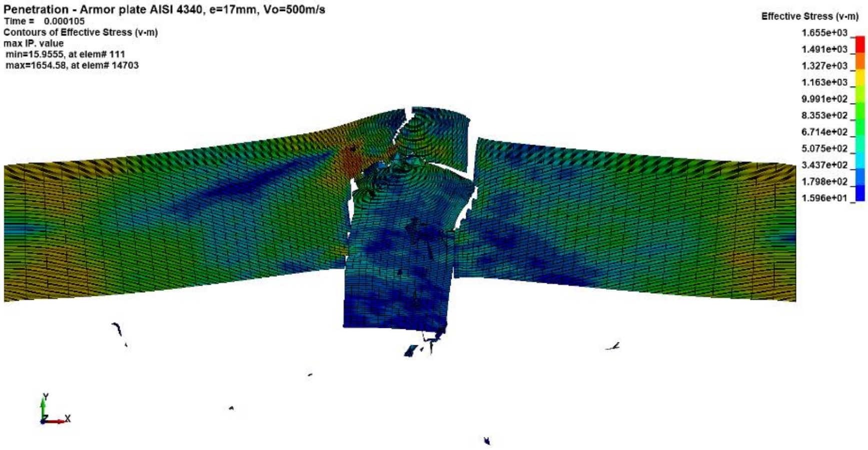

Figure 20 – Von Misses equivalent stress, time of analysis 0.1 ms – Model 3
Рис. 20 – Von Misses эквивалентное напряжение, время анализа 0.1 ms – Модель 3
Слика 20 – Von Misses-ов еквивалентни напон, време анализе 0,1 ms – модел 3

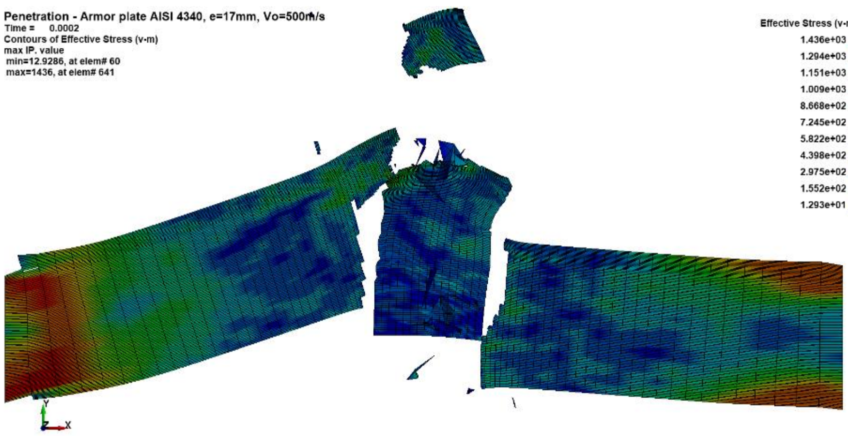

Figure 21– Von Misses equivalent stress, time of analysis 0.2 ms – Model 3
Рис. 21 – Von Misses эквивалентное напряжение, время анализа 0.2 ms – Модель 3
Слика 21 – Von Misses-ов еквивалентни напон, време анализе 0,2 ms – модел 3

As it can be seen from the previous figures, for the case when the thickness of the armor plate is 17 mm, the penetration of the projectile occurs.

The projectile velocity after the impact and penetration is shown on the diagram in Figure 22.

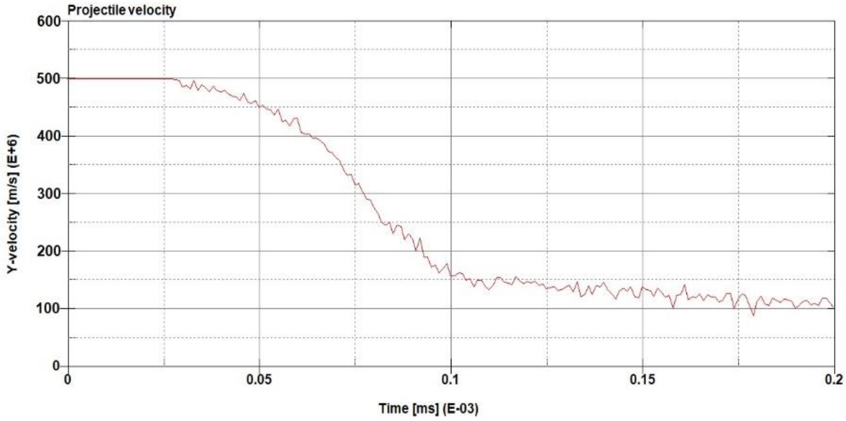

Figure 22 – Velocity of the projectile in relation to time
Рис. 22 – Скорость снаряда в зависимости от времени
Слика 22 – Брзина пројектила у зависности од времена

From the diagram in Figure 22, it can be seen that the limit perforation occurs after 0.03 ms. It can be noticed that the projectile velocity decreases between 0.03 ms and 0.1 ms, for the limit perforation required time. The velocity of projectile fragments after 0.1 ms is 140 m/s.

The displacement of the armor plate after the impact and limit perforation is shown on the diagram in Figure 23. The first displacements occur after 0.03 ms.

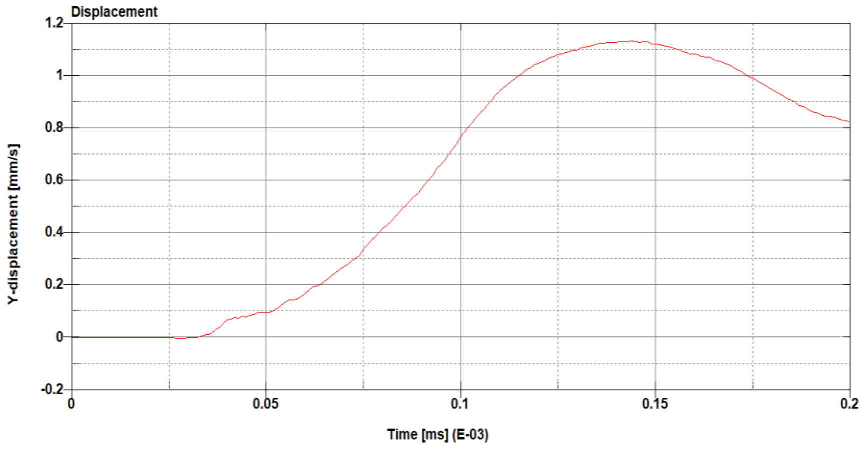

Figure 23 – Displacement of the armor plate in relation to time
Рис. 23 – Смещение в зависимости от времени
Слика 23 – Померање у зависности од времена

Model 4

Figures 24-28 show the field of distribution of the von Misses equivalent stress for model 4. For this case, the impact projectile velocity was 500 m/s, the armor plate thickness 18 mm, and the simulation time 0.2 ms.

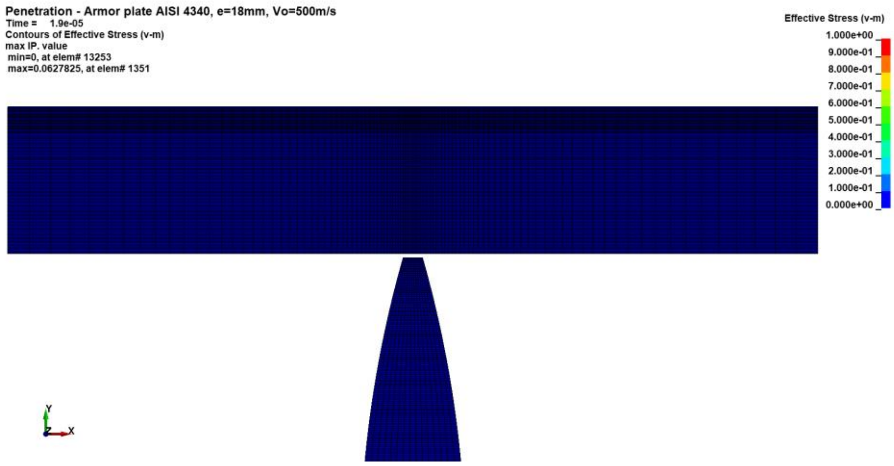

Figure 24 – Von Misses equivalent stress, time of analysis 0.01 ms – Model 4
Рис. 24 – Von Misses эквивалентное напряжение, время анализа 0.01 ms – Модель 4
Слика 24 – Von Misses-ов еквивалентни напон, време анализе 0,01 ms – модел 4

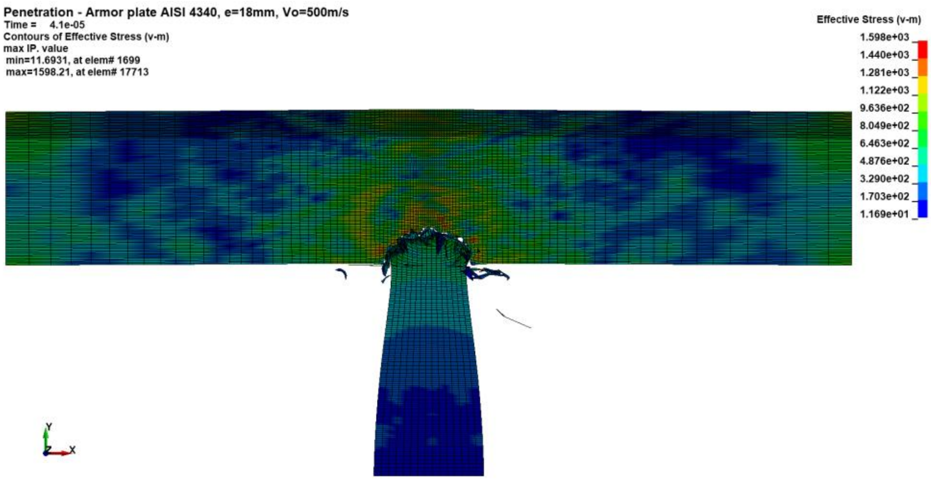

Figure 25 – Von Misses equivalent stress, time of analysis 0.04 ms – Model 4
Рис. 25 – Von Misses эквивалентное напряжение, время анализа 0.04 ms – Модель 4
Слика 25 – Von Misses-ов еквивалентни напон, време анализе 0,04 ms – модел 4

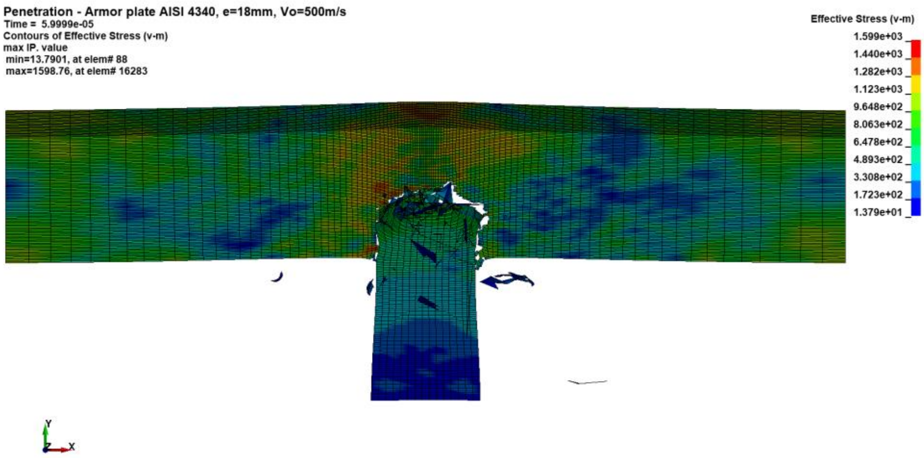

Figure 26 – Von Misses equivalent stress, time of analysis 0.06 ms – Model 4
Рис. 26 – Von Misses эквивалентное напряжение, время анализа 0.06 ms – Модель 4
Слика 26 – Von Misses-ов еквивалентни напон, време анализе 0,06 ms – модел 4

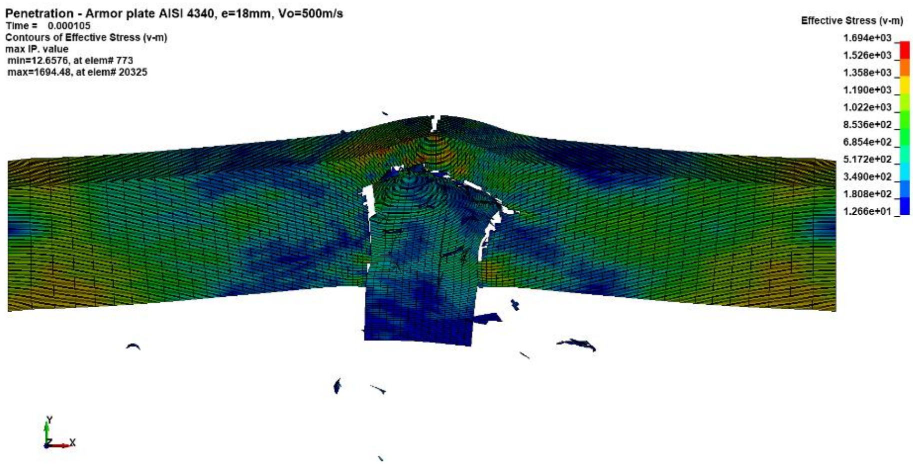

Figure 27 – Von Misses equivalent stress, time of analysis 0.1 ms – Model 4
Рис. 27 – Von Misses эквивалентное напряжение, время анализа 0.1 ms – Модель 4
Слика 27 – Von Misses-ов еквивалентни напон, време анализе 0,1 ms – модел 4

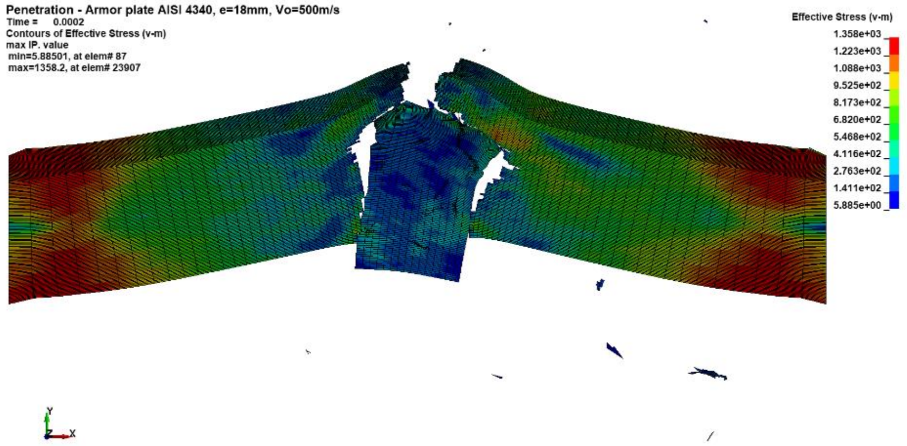

Figure 28 – Von Misses equivalent stress, time of analysis 0.2 ms – Model 4
Рис. 28– Von Misses эквивалентное напряжение, время анализа 0.2 ms – Модель 4
Слика 28 – Von Misses-ов еквивалентни напон, време анализе 0,2 ms – модел 4

As it can be seen from the previous figures, for the case when the thickness of the armor plate is 18 mm, the armor plate is splitting but the penetration of the projectile does not occur.

The projectile velocity after the impact and semi penetration is shown on the diagram in Figure 29.

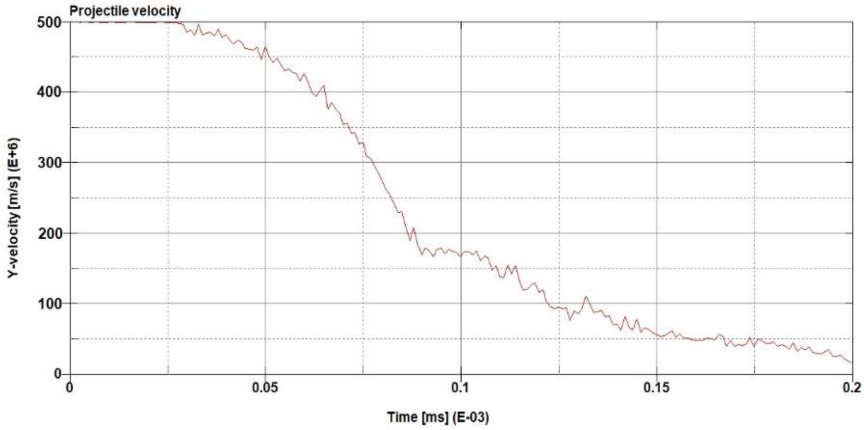

Figure 29 – Velocity of the projectile in relation to time
Рис. 29 – Скорость снаряда в зависимости от времени
Слика 29 – Брзина пројектила у зависности од времена

From the diagram in Figure 29, it can be seen that semi-penetration occurs. It can also be noticed that the 18 mm thickness of the armor plate does not provide complete ballistic protection.

The displacement of the armor plate after the impact and semi-penetration is shown on the diagram in Figure 30. The first displacements occur after 0.03 ms. After 0.2 ms of the analysis, the maximum values of the displacements are achieved.

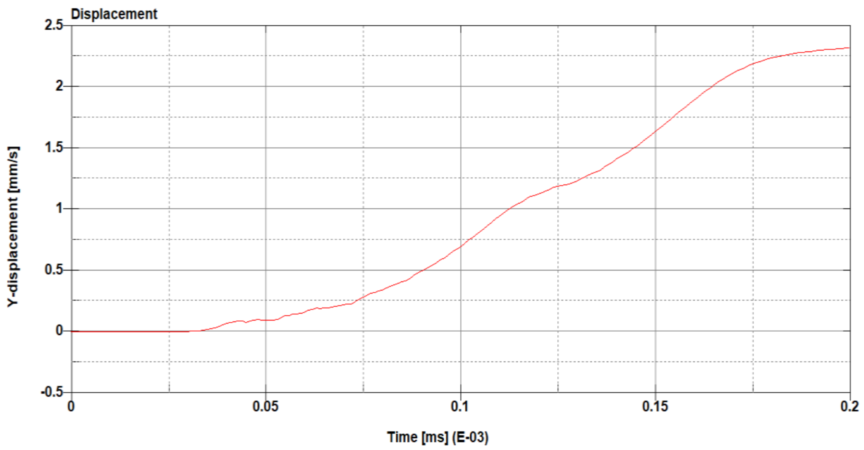

Figure 30 – Displacement of the armor plate in relation to time
Рис. 30 – Смещение в зависимости от времени
Слика 30 – Померање у зависности од времена

Conclusion

Armored projectiles are intended to destroy armored targets. They penetrate armor plates thanks to enormous kinetic energy they have at the moment of collision with an obstacle and the great endurance of their body. Impact modeling for armor obstacles is very complex, extensive, and demanding, and the formed models in a very successful way approximate the real problem of projectile penetration.

It was determined that dynamic phenomena that occur during the process of ballistic penetration largely depend on deformation, strain rate, temperature, and pressure. In order to describe these phenomena in a correct way, it is necessary to define the models of material behavior. The Johnson-Cook material model and the material damage model proved to be the most suitable models for this study.

In this paper, a numerical simulation of the process of a 12.7 mm projectile penetration into armored plates of different thicknesses made of

AISI 4340 alloy was performed. In all 4 models, there is a contact between the bullet and the armor plate after 0.03 ms of the analysis. It is clear that when the thickness of the armor plate is 10 mm, there is perforation, and when the armor plate is 23 mm thick, there is semi-penetration.

In models 1 and 3, the armor plate destruction occurs. The velocity of the bullet after perforation through the armored plate in model 1 is 350 m/s, while in model 3 the velocity of the bullet fragments is 140 m/s.

In models 2 and 4, there is no destruction of the armored plate. In model 2, the semi-penetration of the bullet is after 0.13 ms, and in model 4 after 0.2 ms.

In all 4 models, the first displacements occur after 0.03 ms of the analysis.

However, what was also very important in this paper is to determine the limit values of the thickness of obstacles/armor plates in which penetration occurs.

The semi thickness of the armor plate at which the limit penetration occurs is 18mm. With a thickness of 23 mm, the armor plate deforms but withstands the impact of projectiles without splitting, which provides complete ballistic protection.

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ЧИСЛЕННЫЙ АНАЛИЗ ЛОБОВОГО УДАРА СНАРЯДА 12,7-ММ ПО БРОНЕЛИСТУ

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РУБРИКА ГРНТИ: 78.25.00 Вооружение и военная техника
ВИД СТАТЬИ: оригинальная научная статья

Резюме:

Введение/цель: В данной статье представлено численное моделирование удара снаряда 12,7-мм по бронелисту со скоростью 500 м/с на расстоянии 900 м. Численное моделирование позволяет значительно сократить время, необходимое для получения результатов, по сравнению со временем, необходимым для планирования, организации и проведения экспериментов. Численное моделирование проводилось на пластинах разной толщины, их толщина составляла: 10 мм, 17 мм, 18 мм и 23 мм. Упомянутые толщины бронелиста были выбраны на основании полученных результатов с целью определения предельной толщины бронелиста и бронепробиваемости снаряда, а также для полной баллистической защиты.

Методы: Конечно-элементное моделирование используется для анализа напряжений и деформаций бронированных пластин при *пробитии снарядом. Упомянутый метод вычисляет удар снаряда о препятствие, а именно столкновение снаряда с бронелистом.*

Результаты: Для сравнительного анализа использовались параметры, представляющие значения напряжения и смещения. Для каждой из вышеупомянутых толщин бронированной стальной пластины были определены значения напряжений и смещений при ударе снаряда. Результаты данного исследования показывают, как толщина бронелиста влияет на взаимодействие снаряда и броневой плиты.

Выводы: Если физические и химические характеристики бронеплиты остаются неизменными, то по мере увеличения толщины бронеплиты вероятность пробития снарядом уменьшается, и наоборот. Данное исследование имеет особую значимость, поскольку в нем анализируются напряжения и деформации бронелистов, основной ролью которых является защита личного состава и оборудования от проникновения снаряда. В связи с этим определяется толщина бронелиста для предотвращения пробития снарядом.

Ключевые слова: бронелист, снаряд, удар, метод конечных элементов.

НУМЕРИЧКА АНАЛИЗА ФРОНТАЛНОГ УДАРА ПРОЈЕКТИЛА 12,7 mm У ПАНЦИРНУ ПЛОЧУ

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ОБЛАСТ: машинско инжењерство, материјали
КАТЕГОРИЈА (ТИП) ЧЛАНКА: оригинални научни рад

Сажетак:

Увод: У овом раду представљена је нумеричка симулација удара пројектила 12,7 mm у панцирну плочу брзином од 500 m/s на растојању од 900 m. Нумеричке симулације нуде могућност *драстичног смањења времена потребног за добијање резултата у поређењу са временом потребним за планирање, организацију и извођење експеримената. Нумеричка симулација је урађена за различите дебљине плоча: 10 mm, 17 mm, 18 mm и 23 mm. Поменуте дебљине панцирних плоча изабране су на основу резултата, а ради одређивања граничне дебљине панцирне плоче за продор пројектила, као и за потпуну балистичку заштиту.*

Методе: Метода коначних елемената примењена је како би се анализирали напони и деформације панцирних плоча приликом удара пројектила. Помоћу наведене методе рачуна се удар пројектила у препреку, односно колизија пројектила и панцирне плоче.

Резултати: За упоредну анализу коришћени су параметри: вредност напона и апсолутног померања. За сваку од наведених дебљина панцирне металне плоче одређене су вредности напона и апсолутног померања при удару пројектила. Резултати овог истраживања показују како дебљина плоче утиче на интеракцију пројектила и панцирне плоче.

Закључак: Уколико физичко-хемијске карактеристике панцирне плоче остану непромењене, са повећањем дебљине панцирне плоче смањује се могућност пробоја пројектила, и обрнуто. Ово истраживање је од суштинског значаја, јер анализира напоне и деформације панцирних плоча, чија је основна намена заштита људства и опреме од дејства пројектила. С тим у вези, одређена је дебљина панцирне плоче за задор пројектила.

Кључне речи: панцирна плоча, пројектил, удар, метода коначних елемената.

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APPLICATION OF PROBABILITY BASED MULTI - OBJECTIVE OPTIMIZATION IN THE PREPARATION OF DRUG ENCAPSULATION WITH A DESIGNED EXPERIMENT

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ARTICLE TYPE: Original scientific paper

Abstract:

Introduction/purpose: In this paper, probability based multi – objective optimization (PMOO) is employed to objectively study the optimization problems of the drug encapsulation of water-soluble chitosan (WSC) / poly – gama - glutamic acid (γ-PGA) - tanshinone IIA (TA) with a response surface design and glycerosome - triptolide with an orthogonal experimental design.

Methods: In PMOO, a concept of preferable probability has been introduced to describe a preference degree of the performance utility. Each beneficial and unbeneficial utility index contributes a partial preferable probability in a linear manner, positively and negatively, respectively and all the performance utility indicators are simultaneously and equally treated. The total preferable probability of a candidate is the product of all partial preferable probabilities, which thus transfers a multi-objective problem into a single-objective one.

Results: 1. The optimal encapsulation of WSC / γ-PGA - TA is for WSC of 5.755 $mg \cdot ml^{-1}$, TA of 1.0275 $mg \cdot ml^{-1}$, when the ratio of TA to the carrier material is 1: 4.9, and the reaction time is 1.302h. 2. The optimal preparation conditions of glycerosomes – triptolide are a glycerol concentration of 20%, the phospholipid to cholesterol mass ratio of 30:1 and the phospholipid to triptolide mass ratio of 5:1.

Conclusion: The results show the applicability of PMOO in the optimization of encapsulation composites with designed tests.

Key words: probability theory, multi-objective optimization, preferable probability, test design, drug encapsulation.

Introduction

Currently, probability based multi – objective optimization (PMOO) has been proposed from the viewpoint of probability theory (Zheng et al, 2021). A concept of preferable probability has been introduced to describe a preference degree of the performance utility where each beneficial and unbeneficial utility index contributes a partial preferable probability in a linear manner, positively and negatively, respectively. All the performance utility indicators are simultaneously and equally treated and the total preferable probability of a candidate is the product of all partial preferable probabilities, which thus transfers a multi-objective problem into a single-objective one. PMOO attempts to solve the intrinsic problems of artificial factors in other previous multi – objective optimizations. The new multi – objective optimization method was successfully extended to material selection applications with the multi – objective orthogonal test design method (OTDM), the response surface methodology (RSM) and the uniform test design method (UTDM) as well (Zheng et al, 2021; Zheng et al, 2022a; Zheng et al, 2022b).

Most actual optimality problems in the medical field are multi - objective optimization problems (MOOP). The main feature of multi - objective optimal problems is the contradiction and non - commutability between attributes, but they need to be optimized simultaneously (Mandal et al, 2018; Mirjalili & Dong, 2020; Mankowski & Moshkov, 2021). Besides, there is no uniform metric between theses attributes in general; therefore, they cannot be compared directly. The previous approaches give a set of optimal solutions, called the non-inferior solution set, such as the commonly used Pareto solution set. Take the preparation of a drug encapsulation composite with biopolymer as an example - it is necessary to consider the encapsulation efficiency and the drug loading efficiency to be optimal objectives at the same time (Yu et al, 2020). On the other hand, in the research of Chinese herbal compound drugs, the dose-effect relationship of Chinese herbal compound has non-linear characteristics - there may be differences in the efficacy of different doses of prescriptions, and the efficacy of Chinese herbal medicines has multiple paths, points, and multiple targets. The selection of different efficacy indicators and index weights, the ratio of the components of the compound and the interaction mechanism between the components are also different. Therefore, it is necessary to seek a proper combination of drugs that can improve the efficacy of the compound and maximize the dose of multiple efficacy indicators (Chen et al, 2021; Wu et al, 2013; Song et al, 1992).

Since probability based multi – objective optimization (PMOO) was proposed from the viewpoint of probability theory, which has the advantages of excluding inherent problems of artificial factors in other multi – objective optimizations, it has had successful applications in many practical examples. In this paper, PMOO is used to objectively perform the overall optimal preparation of the drug encapsulation of water-soluble chitosan/poly – gama - glutamic acid - tanshinone IIA with a response surface design and glycerosome - triptolide with an orthogonal experimental design, so as to open a new application field.

Principle and Method of Probability Based Multi – Objective Optimization (PMOO)

In PMOO (Zheng et al, 2021), all indices of the performance utility of candidates are divided into both beneficial or unbeneficial types according to the practical requirement or preference preliminarily; a beneficial utility index contributes a partial preferable probability in a linear manner positively, i.e.,

$$P_{ij} = \gamma_j U_{ij},\ i = 1,2,\dots,\ n;\ j = 1,2,\dots,\ m. \tag{1}$$

In Eq. (1), U_{ij} is the j^{th} performance utility index of the i^{th} candidate scheme; P_{ij} represents the partial preferable probability of the beneficial performance utility indicator U_{ij}; n is the total number of candidate schemes in the scheme group involved; m is the total number of performance utility indicators of each candidate scheme in the group; and γ_j is the normalized factor of the j^{th} performance indicator.

Furthermore, according to the general principle in probability theory (Zheng et al, 2021), the normalization of partial preferable probability P_{ij} for the index i in the j^{th} performance indicator leads to the following result naturally

$$\gamma_j = \frac{1}{n\overline{U}_j}, \tag{2}$$

$\overline{U}_j$ is the arithmetic mean value of the j^{th} performance indicator in the scheme group involved.

Similarly, an unbeneficial utility index contributes a partial preferable probability in a linear manner negatively, i.e.,

$$P_{ij} = \eta_j(U_{jmax} + U_{jmin} - U_{ij}),\ i = 1,2,\ldots, n;\ j = 1, 2, \ldots, m. \tag{3}$$

In Eq. (3), $U_{j\max}$ and $U_{j\min}$ represent the maximum and minimum values of the performance utility index U_j in the j^{th} group, respectively. Furthermore, the normalized factor η_j of the j^{th} group of performance indicator is

$$\eta_j = \frac{1}{[n(U_{j\min} + U_{j\max}) - n\overline{U}_j]}. \tag{4}$$

Moreover, according to probability theory (Zheng et al, 2021), the total / comprehensive preferable probability of the i^{th} candidate scheme is the product of its partial preferable probability P_{ij} of each performance utility indicator in the optimization, i.e.,

$$P_i = P_{i1} \cdot P_{i2} \cdots P_{im} = \prod_{j=1}^{m} P_{ij}. \tag{5}$$

Thus, by using the total preferable probability of a candidate alternative being the product of all partial preferable probabilities, it naturally transfers a multi-objective problem into a single-objective one.

The total preferable probability P_i of a candidate is the unique decisive index in the competitive optimization process. The main characteristic of PMOO is that the treatment for both the beneficial performance utility index and the unbeneficial performance utility index is equal without any artificial or subjective scaling factors and the requirements of simultaneous optimization for multi - objectives are met from the viewpoint of probability theory.

Applications in Drug Encapsulation with a Designed Experiment

Optimal preparation of drug encapsulation has been one of important issues in recent years. In this paper, the optimization problems of water-soluble chitosan / poly – gama - glutamic acid - tanshinone IIA with a response surface design and glycerosome - triptolide with an orthogonal experimental design are restudied by employing PMOO objectively.

1) Application of PMOO in the optimal preparation of the encapsulation composite of water-soluble chitosan / poly - gama - glutamic acid - tanshinone IIA with a response surface design

Yu et al (2020) conducted the optimal preparation of the encapsulation composite of water-soluble chitosan/poly-gama-glutamic acid - tanshinone IIA with a response surface design, based on the traditional treatment of a response surface design with the "additive" algorithm multi - attribute utility theory. As it was pointed in (Zheng et al, 2021), there exist intrinsic problems of artificial and subjective factors in the "additive" algorithm of the previous multi-attribute utility theory (Zheng et al, 2021). Here, the optimal preparation of the encapsulation composite of water-soluble chitosan / poly – gama - glutamic acid - tanshinone IIA with a response surface design is reanalyzed by PMOO once more.

Table 1 cited the analysis results of utility in the optimal preparation of the encapsulation composite of water-soluble chitosan (WSC) / poly-gama-glutamic acid (γ-PGA) - tanshinone IIA (TA) with a response surface design (Yu et al, 2020). The input variables include x_1, x_2, x_3 and x_4, in which x_1 is the WSC concentration (mg·ml^{-1}), x_2 represents the TA concentration (mg·ml^{-1}), x_3 is the ratio of TA to the carrier material (in weight), and x_4 indicates the reaction time (h). The encapsulation efficiency Y_e and the drug loading efficiency Y_c are the optimal objectives, which belong to the beneficial type index. Table 2 shows the evaluation results for the preferable probability in the spirit of a response surface design.

Table 2 indicates that experiments 2 and 25 are the appropriate schemes with the highest total partial probability for the preparation of the encapsulation composite of water-soluble chitosan / poly-gama-glutamic acid - tanshinone IIA with a response surface design comparatively.

Furthermore, the regression of the data in Table 2 can be used to conduct profound optimization. Eq. (6) is the regressed formula of the total preferable probability P_t *vs* the input variables, x_1, x_2, x_3, and x_4.

$$P_t\times10^3 = 1.9644 - 0.1787x_1+0.0254x_2 - 7.2\times10^{-5}x_3+0.1717x_4 - 0.4411x_1^2 - 0.7445x_2^2 -0.5602x_3^2 -0.1494x_4^2 +0.3381\, x_1x_3 -0.0329x_1x_4 - 0.0172x_2x_3 + 0.0913x_2x_4+0.0443x_3x_4$$

$$R^2 = 0.8620. \tag{6}$$

Table 1 – Details of the Box-Behnken test design and the results

Таблица 1 – Подробная информация о конструкции теста Бокса-Бенкена и результатах

Табела 1 – Детаљи теста Box-Behnken дизајна и његови резултати

Test No.	x_1	x_2	x_3	x_4	Encapsulation efficiency Y_e (%)	Drug loading efficiency Y_c (%)
1	1	0	1	1	79.31	5.25
2	0	0	0	0	93.25	11.22
3	0	0	0	0	94.31	9.92
4	0	0	−1	1	85.22	6.38
5	0	1	1	0	72.51	4.38
6	0	0	−1	−1	75.87	5.18
7	1	0	0	−1	84.56	6.97
8	0	0	0	0	90.34	10.09
9	0	−1	0	1	85.69	6.38
10	1	1	0	0	79.84	6.39
11	0	0	1	1	87.21	7.89
12	−1	0	0	−1	92.80	8.73
13	1	0	0	1	89.96	6.34
14	0	0	1	−1	79.65	6.05
15	0	−1	0	−1	80.79	5.08
16	−1	1	0	0	66.73	3.96
17	0	1	0	−1	78.62	4.26
18	−1	0	1	0	76.97	6.32
19	−1	0	0	1	90.73	9.58
20	0	−1	1	0	78.22	4.73
21	1	0	−1	0	78.34	6.21
22	1	0	0	0	84.97	5.07
23	−1	−1	0	0	84.46	6.01
24	0	−1	0	0	83.36	6.32
25	0	0	0	1	95.02	11.03
26	0	−1	−1	0	80.33	4.98
27	0	1	−1	0	70.67	5.38
28	0	0	0	0	92.73	9.89
29	−1	0	−1	0	80.39	6.54

Table 2 – Evaluation results of the preferable probability of utility in the preparation of the encapsulation composite of WSC / γ-PGA-TA in the spirit of a response surface design

Таблица 2 – Результаты оценки предпочтительной вероятности полезности при приготовлении герметизирующего композита WSC / γ-PGA-TA в духе конструкции поверхности отклика

Табела 2 – Резултати евалуације пожељне вероватноће корисности у припреми композита WSC / γ-PGA-TA за енкапсулацију у складу са дизајном површине одговора

Test No.	*Partial preferable probability*		*Total preferable probability*
	P_e	P_c	P_t×10^3
1	0.0329	0.0267	0.8781
2	0.0386	0.0571	2.2064
3	0.0391	0.0505	1.9729
4	0.0353	0.0325	1.1466
5	0.0301	0.0223	0.6698
6	0.0314	0.0264	0.8288
7	0.0350	0.0355	1.2429
8	0.0374	0.0513	1.9223
9	0.0355	0.0325	1.1529
10	0.0331	0.0325	1.0759
11	0.0361	0.0401	1.4511
12	0.0385	0.0444	1.7085
13	0.0373	0.0323	1.2028
14	0.0330	0.0308	1.0162
15	0.0335	0.0258	0.8655
16	0.0277	0.0202	0.5573
17	0.0326	0.0217	0.7063
18	0.03190	0.0322	1.0258
19	0.0376	0.0487	1.8330
20	0.0324	0.0241	0.7802
21	0.0325	0.0316	1.0259
22	0.0352	0.0258	0.9085
23	0.0350	0.0306	1.0705
24	0.0345	0.0322	1.1110
25	0.0394	0.0561	2.2102
26	0.0333	0.0253	0.8436
27	0.0293	0.0274	0.8018
28	0.0384	0.0503	1.9340
29	0.0333	0.0333	1.1087

The total preferable probability P_t gets its maximum $P_{tmax} \times 10^3 = 2.0394$ at $x_1 = 5.755$ mg·ml⁻¹, $x_2 = 1.0275$ mg·ml⁻¹, $x_3 = 1{:}\ 4.9$, and $x_4 = 1.302$ h.

Simultaneously, the encapsulation efficiency Y_e (%) and the drug loading efficiency Y_c (%) of the preparation can be fitted, and are given as follows

$$Y_e = 92.4514 - 0.8660x_1 - 2.6375x_2 + 0.1389x_3 + 2.2449x_4 - 4.8949x_1^2 - 10.3491x_2^2 - 8.7025x_3^2 - 0.1760x_4^2 + 6.1970x_1x_3 + 0.7516x_1x_4 + 0.9875x_2x_3 - 0.6621x_2x_4 - 0.6855x_3x_4$$

$$R^2 = 0.9060. \quad (7)$$

Y_e gets its optimal $Y_{eopt} = 93.43\%$ at $x_1 = 5.755$ mg·ml⁻¹, $x_2 = 1.0275$ mg·ml⁻¹, $x_3 = 1{:}\ 4.9$, and $x_4 = 1.302$ h.

$$Y_c\ (\%) = 10.0612 - 0.8559x_1 + 0.2232x_2 - 0.0247x_3 + 0.7516x_4 - 1.9370x_1^2 - 3.3194x_2^2 - 2.3077x_3^2 - 0.7432x_4^2 + 1.5714x_1x_3 - 0.2467x_1x_4 - 0.1875x_2x_3 + 0.4711x_2x_4 + 0.2385x_3x_4$$

$$R^2 = 0.8420. \quad (8)$$

Y_c gets its optimal $Y_{copt} = 10.40\%$ at $x_1 = 5.755$ mg·ml⁻¹, $x_2 = 1.0275$ mg·ml⁻¹, $x_3 = 1{:}\ 4.9$, and $x_4 = 1.302$ h.

The predicted values for Y_e and Y_c are close to the averaged encapsulation efficiency and the drug loading average, so their values of the tested encapsulated composite were 91.89% and 10.29%, respectively (Yu et al, 2020). This indicates that this is a reasonable method for the optimal conditions of an encapsulation composite with a response surface design.

2) Application of PMOO in the optimal preparation of the encapsulation composite of glycerosomes – triptolide with an orthogonal experimental design

Zhu et al (2022) conducted optimizing glycerosome formulations via an orthogonal experimental design to enhance transdermal triptolide delivery. The entrapment efficiency (*EE*) of the nanocarriers and the drug loading (*DL*) are taken as evaluated attribute indexes. The glycerol concentration (*A*, %), the phospholipid to cholesterol mass ratio (*B*, m/m) and the phospholipid to triptolide mass ratio (*C*, m/m) were set as independent variables with three levels of *A* (10, 20, 30 %), *B* (10:1, 20:1, 30:1 m/m) and *C* (5:1, 15:1, 30:1 m/m). Thereafter, the three-level orthogonal table [L9(3^4)] was employed in the study.

Here, the optimal preparation of the encapsulation composite of glycerosomes – triptolide with an orthogonal experimental design is restudied by PMOO again. Table 3 cited the experimental arrangement and the results based on the L9(3^4) orthogonal design (Zhu et al, 2022).

The encapsulation efficiency and the drug loading efficiency belong to the beneficial type index. Table 4 shows the evaluation results of the preferable probability of the experimental data; Table 5 represents the evaluation results of the range analysis for total preferable probability.

From Table 5, the optimal composite is *C1A2B3*, which is the same as the first glanced rank 1 of test No. 6 in Table 4 luckily.

Table 3 – Experimental arrangement and the results based on the L9(34) orthogonal design

Таблица 3 – Экспериментальная схема и результаты, основанные на ортогональной конструкции L9(34)

Табела 3 – Уређеност експеримента и резултати засновани на ортогоналном дизајну Л9(34)

Test No.	A	B	C	EE (%)	DL (%)
1	1	1	1	65.67	15.41
2	1	2	2	61.87	5.97
3	1	3	3	55.79	3.12
4	2	1	2	65.56	5.71
5	2	2	3	54.64	3.07
6	2	3	1	77.40	16.19
7	3	1	3	43.25	2.93
8	3	2	1	67.37	15.97
9	3	3	2	54.85	6.06

Table 4 – Evaluation results of the preferable probability of the experimental data
Таблица 4 – Результаты оценки предпочтительной вероятности экспериментальных данных
Табела 4 – Резултати евалуације пожељне вероватноће експерименталних података

Test No.	Partial *preferable probability*		Total *preferable probability and rank*	
	EE	DL	P_t×10^2	Rank
1	0.1202	0.2070	2.4883	3
2	0.1132	0.0802	0.9082	5
3	0.1021	0.0419	0.4280	7
4	0.1200	0.0767	0.9205	4
5	0.1000	0.0412	0.4125	8
6	0.1417	0.2175	3.0813	1
7	0.0792	0.0394	0.3116	9
8	0.1233	0.2146	2.6455	2
9	0.1004	0.0814	0.8173	6

Table 5 – Evaluation results of the range analysis for total preferable probability
Таблица 5 – Результаты оценки анализа диапазона предпочтительной вероятности
Табела 5 – Резултати евалуације анализе рангирања за пожељну вероватноћу

Level	A	B	C
1	1.2749	1.2401	2.7384
2	1.4714	1.3221	0.8820
3	1.2581	1.4422	0.3840
Range	0.2133	0.2021	2.3544
Order	2	3	1

Discussion

Since many problems involved in drug research are multi-objective optimization ones such as encapsulation efficiency and drug loading efficiency being optimal objectives in the preparation of drug encapsulation composites with biopolymer, it is necessary to reach the optimal status at the same time. In the investigation of Chinese herbal compound drugs, the dose-effect relationship of Chinese herbal compounds has non-linear characteristics, and there may be differences in the efficacy of different doses of prescriptions. Furthermore, the efficacy of Chinese herbal medicines has multiple paths, points, and multiple targets. The PMOO method attempted to deal with the problem of simultaneous optimization of multiple objectives and to exclude the intrinsic problems of previous optimization methods due to subjective factors, so it might be an appropriate assessment for drug research.

The above results indicate that probability based multi-objective optimization is applicable in the preparation of encapsulation composites with a designed test.

Conclusion

The newly developed probability based multi-objective optimization method has been successfully applied for the appropriate optimal preparation of the drug encapsulation composite with a designed test, which includes the water-soluble chitosan / poly - gama - glutamic acid - tanshinone IIA with a response surface design and glycerosome - triptolide with an orthogonal experimental design. The main features of the new probability theory are: the treatment for both the beneficial performance utility index and the unbeneficial performance utility index being equal and simultaneous; no artificial or subjective scaling factors involved in the assessment process; and fulfilling the requirements of simultaneous optimization for a multi – objective problem from the viewpoint of probability theory. The potential future direction for the application of the probability theory based multi-objective optimization method is to explore more cases with complexity.

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ПРИМЕНЕНИЕ МНОГОКРИТЕРИАЛЬНОЙ ОПТИМИЗАЦИИ, ОСНОВАННОЙ НА ВЕРОЯТНОСТИ, ПРИ ПОДГОТОВКЕ ИНКАПСУЛЯЦИИ ЛЕКАРСТВЕННЫХ СРЕДСТВ С ПОМОЩЬЮ СПРОЕКТИРОВАННОГО ЭКСПЕРИМЕНТА

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РУБРИКА ГРНТИ: 27.47.00 Математическая кибернетика,
27.47.19 Исследование операций,
81.09.00 Материаловедение,
45.09.00 Электротехнические материалы

ВИД СТАТЬИ: оригинальная научная статья

Резюме:

Введение/цель: В данной статье представлена многокритериальная оптимизация, основанная на вероятности (probability based multi – objective optimization (PMOO)), с целью объективного изучения проблем оптимизации инкапсуляции лекарственных средств из водорастворимого хитозана (WSC) / поли – гама - глутаминовой кислоты ((γ-PGA) - таншинона IIA (TA) с дизайном поверхности отклика и глицеросомы - триптолида с помощью ортогональной экспериментальной конструкции.

Методы: В многокритериальной оптимизации, основанной на вероятности, было введено понятие предпочтительной вероятности для описания степени полезности производительности. Каждый полезный или бесполезный индекс производительности линейно влияет на частичную предпочтительную вероятность в положительном или отрицательном смысле, а все показатели полезности одновременно рассматриваются в одинаковом порядке. Общая предпочтительная вероятность кандидата является произведением всех отдельных предпочтительных вероятностей, что переводит многокритериальную проблему в однокритериальную.

Результаты: 1. Оптимальная WSC / γ-PGA-TA составляет для WSC 5,755 мг · мл*$^{-1}$, TA 1.0275* мг · мл*$^{-1}$, когда соотношение TA к материалу-носителю составляет 1: 4,9, а время реакции − 1,302 ч. 2. Оптимальные условия приготовление глицеросомы – триптолида при концентрации глицерина 20%, при массовом отношении фосфолипида к холестерину 30:1 и массовом соотношение фосфолипидов к триптолидам 5:1.*

Выводы: Результаты показывают применимость многокритериальной оптимизации, основанной на вероятности, в оптимизации герметизирующих композитов с помощью разработанных тестов.

Ключевые слова: теория вероятностей, многокритериальная оптимизация, предпочтительная вероятность, разработка тестов, инкапсуляция лекарственных средств.

ПРИМЕНА ВИШЕКРИТЕРИЈУМСКЕ ОПТИМИЗАЦИЈЕ НА БАЗИ ВЕРОВАТНОЋЕ У ПРИПРЕМИ ЕНКАПСУЛАЦИЈЕ ЛЕКОВА ПОМОЋУ ДИЗАЈНИРАНОГ ЕКСПЕРИМЕНТА

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Сажетак:

Увод/циљ: У раду је представљена вишекритеријумска оптимизација заснована на вероватноћи (probability based multi objective optimization –PMOO) за објективно проучавање проблема оптимизације енкапсулације лекова помоћу хитозана растворљивог у води (water-soluble chitosan – WSC)/гама полиглутаминске киселине ((γ-PGA) – таншинона IIA (TA) помоћу дизајна површине одговора и глицерозома – триптолида помоћу ортогоналног дизајна експеримента.

Методе: У вишекритеријумској оптимизацији, заснованој на вероватноћи, уведен је концепт пожељне вероватноће како би се описао степен пожељности корисности неке перформансе. Сваки корисни или некорисни индекс корисности линеарно доприноси делимичној пожељној вероватноћи у позитивном, односно у негативном смислу, а сви показатељи корисности перформанси третирају се подједнако и једновремено. Укупна пожељна вероватноћа кандидата производ је свих парцијалних пожељних вероватноћа, чиме се вишекритеријумски проблем преводи у једнокритеријумски.

Резултати: 1. До оптималне енкапсулације WSC / γ-PGA-TA долази када је WSC 5.755 mg·ml^{-1}, TA 1.0275 mg·ml^{-1}, однос TA и носећег материјала 1:4.9, а време реакције 1.302h. 2. Оптимални услови припреме глицерозома – триптолида су при концентрацији

глицерина од 20%, масеном односу фосфолипида и холестерола 30:1 и масеном односу фосфолипида и триптолида 5:1.

Закључак: Резултати показују применљивост вишекритеријумске оптимизације засноване на вероватноћи у оптимизацији енкапсулације композита помоћу дизајнираних тестова.

Кључне речи: теорија вероватноће, вишекритеријумска оптимизација, пожељна вероватноћа, дизајн теста, енкапсулација лекова.

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DEFINING RISKS ON ROAD SECTIONS DURING THE TRANSPORT OF DANGEROUS GOODS IN THE SERBIAN ARMY USING THE LINEAR MATHEMATICAL PROGRAMMING MODEL

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FIELD: Mathematics, Transport, Logistics
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Summary:

Introduction/purpose: The paper presents a model for the selection of a route for the transport of dangerous goods using DEA (Data Envelopment Analysis) models and fuzzy logic systems. The presented model is used to define the risk on road sections during the transport of dangerous goods as well as to select the optimal route for the realization of the transport task.

Methods: The model consists of two phases. The first phase includes the application of DEA models in which formed input and output models are connected in the output DEA final form which shows routes with a satisfactory level of traffic safety and at the same time eliminates routes with low traffic safety. The second phase involves the application of fuzzy logic systems, and as a way out of the fuzzy system, preference is given to one route. Route evaluation is based on six criteria, namely: route length, number of access points, AADT (annual average daily traffic), the number of traffic accidents with fatalities, the number of traffic accidents with the injured and the number of traffic accidents with material damage. When the values of the input criteria are entered, a calculation and evaluation is performed, and, as an exit from the fuzzy system, preference is given to one of the entered routes (the route with the lowest level of risk). The criteria used were defined on the basis of expert assessments.

Results: A user program that represents decision support in traffic service.

Conclusion: The user platform was created for the Matlab R2015a software package with the ability to be adapted to specific problems.

Keywords: DEA model, fuzzy logic system, dangerous goods, risk, route preference.

Introduction

One of the daily activities in the Ministry of Defense and the Serbian Army is the transport of dangerous goods, which is one of constant tasks of traffic service. In the process of managing the transport of dangerous goods, the emphasis is placed on reducing the negative consequences of transport, especially those that affect the safety of population, but also those that cause harmful effects on the environment. One of the crucial factors influencing the safe management of the transport of dangerous goods is the choice of a route on which motor vehicles will move during the transport of dangerous goods.The goal of managing the risks that may occur during the transport of dangerous goods is to minimize the negative consequences for the environment and avoid incidents. With technological development, there is a mass application of modern software systems based on artificial intelligence, while solving the problem of transporting dangerous goods.

This paper presents a model for selecting a route for the transport of dangerous goods using DEA (Data Envelopment Analysis) models and fuzzy logic systems as a type of artificial intelligence systems. The system presented in the paper is a system to help the decision-making bodies of the transport service in choosing one of several possible routes in a particular situation when transporting dangerous goods based on potential risks.

Creating a model involves two phases. In the first phase, the DEA model was applied, and in the second, the fuzzy logic system. The route is evaluated on the basis of six criteria, namely: route length, number of access points, AADT (annual average daily traffic), number of traffic accidents with fatalities, number of traffic accidents with the injured and number of traffic accidents with material damage.

A user platform was created for the practical application of the created model. After the values of the input parameters are entered, calculation and evaluation are performed, and, as an exit from the system, a preference is given to one of the entered routes, according to the minimum risk.

The creation of this and similar models helps in the development of modern systems for solving complex problems in many industries, affecting the ongoing increase of interest in scientific fields that deal with optimizing the process of technical and organizational systems. Wide possibilities in the application of optimization and prediction models enable their constant development and improvement.

The model created in this research facilitates the work of traffic service authorities and reduces the risk of incidental situations by choosing the route that is the safest in relation to the available alternatives. Reducing the possibility of undesirable situations during the transport of dangerous goods increases the level of safety of traffic participants and environmental protection. Since the created model is adaptive in nature, by pre-tuning, it is possible to apply it to solve other problems.

Review of risk assessment models and the definition of the basic concepts for risk modeling

Literature review

Transport and routing of dangerous goods is an area of interest for many authors. Improving the efficiency of the transport process affects the improvement of the efficiency of the system in which it is located. By applying modern software and information systems, it is possible to find the optimal solution not only for the transport of dangerous goods, but also for other problems in various processes.

The subject of interest of many scientific papers is the optimization of the transport process that makes up the subsystem of both military and civilian structures of the logistics system. By using optimization models, it is possible to improve the functioning of many technical and organizational systems. Models that have found application in the optimization of the transport process are also applied in other logistics structures, which shows numerous possibilities of their use. Based on the opinion of Cavaignac et al (2021), by applying the DEA (data envelopment analysis) method, it is possible to improve the business performance of the logistics system. Their study deals with the analysis of the performance of French markets as logistics systems. Comparing the business of several different companies and processing the results using the DEA method give the results of business success of companies, indicating positive and negative influencing factors and potential corrective measures in order to improve the process.

The DEA model has also found application in storage systems, which Karande et al (2019), present in their work. Warehouse management is an extremely challenging task due to a number of activities realized within the warehousing process including internal and external transport processes. Through a study presented by Karande et al, a data envelopment analysis (DEA) model was developed to calculate relative efficiencies. The model was used to rank warehouses and identify the warehouse with the most successful business.

The development of the transport sector promotes rapid economic growth in many countries, but also has a negative impact on the society and the environment, such as pollution and irrational energy consumption. In their study, Tian et al (2020), propose an improved model of DEA based on super-efficiency (SBM-DEA) with weighting to assess the efficiency of regional transport sustainability. An empirical application for measuring the sustainability of transport in Shanxi Province in China, in the period 2000-2015, is presented to demonstrate the effectiveness of the proposed model. Compared to the result obtained by the SBM-DEA model without weighting, the number of effective decision-making units was reduced by the proposed DEA model. The calculated results show that the evaluation result is generally in line with reality, which reflects the relevance of the model.

The research conducted by Vesković et al (2020) presents the definition and evaluation of the criteria that affect the efficiency of railway companies, increase their competitiveness and propose an approach based on the DEA method. The assessment of the efficiency of railway undertakings was considered using the DEA approach. The results show that the proposed approach successfully enables the consolidation of a set of criteria (resource, operational, financial, quality and safety) into a single assessment of the efficiency of railway companies, providing information on corrective actions that can improve the efficiency of railway companies.

In the paper Mitrović Simić et al (2020), the DEA model was applied to define the route with the highest level of traffic safety, in combination with other models such as fuzzy logic systems. Through this paper, the possibilities of combining DEA models with other models are presented, and a model was created representing support in the decision - making of transport service authorities regarding the choice of the safest route. As already shown, the DEA model has an extremely wide application, and the combination with other models further expands the possibilities of its use.

Multi-criteria models are often used in the field of traffic to define the level of road safety in order to protect road users. The process of making decisions about dangerous sections of the road using the model of multi-criteria decision-making involves defining quantitative and qualitative criteria of traffic safety, which Nenadić, D. (2019) explains in his paper. The model used in this paper consists of five quantitative and two qualitative traffic safety criteria. Based on these criteria, the ranking of alternative sections was performed.The analysis of the total number of traffic accidents, by their categories, the analysis of the current state of traffic infrastructure and the annual average daily traffic (AADT) defined seven traffic safety criteria that were assessed and ranked in the first phase of the model by their importance. Using the full consistency method (FUCOM), Pamučar et al (2018), determined the weighted coefficients of the defined criteria and ranked dangerous sections of the road using the weighted aggregate product assessment method (WASPAS) (Baykasoglu & Golcuk, 2020). The obtained results show which section of the road is the safest.

Logistics process management has a significant role in ensuring the competitive growth of the industry. Pamučar et al (2021) in their study propose a new multicriteria decision-making framework (MCDM) to assess the operational efficiency of logistics service providers (LSPs). They present a case study of a comparative analysis of six leading LSPs in India using the proposed framework. Three operational metrics such as annual overheads, annual fuel consumption and delay cost were considered.The result shows that the final ranking is a combined effect of all criteria. It was determined that the method is more stable, gives consistent results, and does not suffer from the problem of changing rank.

In addition to the activities undertaken in order to optimize the process in logistics systems, it is necessary to take measures that relate specifically to the security aspect, which is also one of the elements for improving logistics processes. Risks that appear in the field of logistics, especially in the field of transport, are an area of interest for many authors. Milovanović (2012) in his dissertation pays special attention to the risks that occur during the transport of dangerous goods, which is one of the subjects of this research. In order to safely implement the transport process of dangerous goods, it is necessary to manage the risk, which is a very complex process, and one of the steps in this process is the choice of routes for vehicles transporting dangerous goods.Selecting routes for the movement of vehicles transporting dangerous goods without quantifying the level of risk within each route would lead to the possibility of wrong choice and, consequently, to potential serious consequences that can be caused by dangerous goods transported. In order to make an adequate choice of routes from the aspect of risk management, this paper presents the process of risk management in the transport of dangerous goods, i.e. the phases of which the risk management process consists and a detailed description of each phase separately. The place and importance of the choice of routes for the movement of vehicles transporting dangerous goods was determined, which is one of the initial activities within the analysis of the danger of an incident situation, i.e. the first phase of the mentioned process.Special attention is paid to determining the level of social and individual risk due to the great importance of these types of risks in the choice of routes for the movement of vehicles transporting dangerous goods from the aspect of risk management. By applying optimization methods, it is possible to influence risk management processes, which is also presented in this paper.

Transport of hazardous materials by road poses a risk to the entire environment along the route (traffic participants and the surrounding population). Cassini (1998), in his paper presents the activities most often undertaken in the process of road risk management. In order to minimize the risks, the question arises as to which route the vehicle should use in order to protect the environment (people and the environment). Is it safer to transport through an urban area or less populated areas? Choosing a route is not always an easy task. Various problems are possible with vehicles passing through tunnels and similar areas, which could cause catastrophic consequences in the event of an accident, while the passage of vehicles on the open road in the event of undesirable situations would leave fewer negative consequences. In order to adequately choose the route for the movement of a vehicle carrying dangerous goods, it is necessary to perform risk assessments using an approach that deals with the scenarios of accidents that are likely to occur, their probability and possible consequences. One of the useful steps is the identification of a class of dangerous goods that has a great influence on the choice of route, as stated by Cassini (1998).

Bubbico et al (2004) propose an approach to the transport risk analysis for road and rail transport of hazardous materials based on the use of geographic information systems (GIS) for territorial information management, together with a database of products in the tool for risk assessment. Such an approach makes it possible to accurately take into account local data affecting risk analysis, such as population, accident rate and weather conditions along the entire route, using a system that can be easily updated.

Fabiano et al (2002), in their paper present the conflict of risks that arise when transporting dangerous goods and strategies for choosing the route to move, developing an original location-oriented framework and general applicability at the local level. The realistic assessment must take into account on the one hand the inherent factors (e.g. tunnels, railway bridges, radii of curvature, slope, neighborhood characteristics, etc.) and on the other hand the factors that correlate with traffic conditions. The field data used in this study were collected on the selected highway, by systematic research, providing input data for the database at a given location. The developed technique was applied to the pilot area, taking into account individual and social risk and referring to flammable and explosive scenarios. In this way, risk assessment, sensitive to route characteristics and population exposure, is proposed in order to reduce overall uncertainties in risk analysis.

Based on the review of the literature that considers different methods for optimizing logistics processes and the choice of route for the transport of dangerous goods, it can be seen that the authors used different criteria. The analysis of the criteria used by many authors in their research defined the criteria used to create the model presented in this study.

The concept of risk

A large number of authors define risk as a combination of the probability of occurrence of incident situations and the magnitude of potential consequences of the incident situation (Lavell, 2000; Fabiano et al, 2002; Cassini, 1998). This definition of risk is the starting definition of the concept of risk. There are many different factors in different spheres that can cause an incident situation. One of the biggest difficulties in defining the level of risk is defining the parameters that determine the probability of occurrence of incident situations and the magnitude of potential consequences. Risk can also be defined as the probability of harmful outcomes resulting from the interaction between two hazards[1], vulnerabilities of the community and the natural environment. In 1994, the National Assembly of the Republic of Serbia defined risk as the expected number of lost persons, damage to private property and disruption of economic activities due to hazards. Based on the definitions of risk, it is concluded that there are three elements that define risk, and they are:

1) type of hazard (Lavell, 2000),
2) vulnerability, and
3) the risk element.

When defining risk, one can talk about individual and social risk. First of all, it is necessary to terminologically define these two terms, describe the differences between them and define their limit values. Individual risk represents the annual mortality rate of the average individual who is constantly exposed to danger without protective equipment within the zone of influence (Milovanović, 2012). Social risk is the cumulative value of the probability of an accident with several victims within the impact zone

[1] Hazard-potential harmful physical events that can occur in the context of vulnerability of population, infrastructure and production, and that lead to economic and social losses and can reach disaster levels.

(Ormsby & Le, 1988; Castillo, 2004). The frequency of accidents depends on:

1) types and quantities of hazards,
2) processes that cause accidents, and
3) levels of applied security measures.

The magnitude of the consequences depends on:

1) population structure,
2) effects caused by the danger during the occurrence of the incident situation, and
3) the ability of participants to avoid the consequences of the accident and apply available measures to control the consequences.

In order to effectively manage the risk of an incident during the transport of dangerous goods, it is necessary to carry out activities that are interdependent and to have relevant data on the characteristics of dangerous goods, the number of incidents from the previous period by dangerous goods classes, population and environmental consequences, incident situations in the previous period and the capacities of the response services in case of an incident situation (ambulance, firefighters and police). As already mentioned, risk is the product of the probability of an accident involving a vehicle transporting dangerous goods and the number of inhabitants who are potentially exposed to dangerous goods due to the accident (Milovanović, 2012):

$$\text{Risk} = \text{probability} * \text{consequences} \tag{1}$$

Probability is the rate of accidents involving vehicles transporting dangerous goods in units of the number of accidents per vehicle per kilometer, and the consequences of the degree of population potentially exposed to accidents involving vehicles transporting dangerous goods along the road intended for transport of dangerous goods. In order to determine the probability of an accident with the participation of vehicles transporting dangerous goods, it is necessary to collect all data on traffic accidents and traffic flows for all road sections that are analyzed. The consequences of traffic accidents with dangerous goods are expressed through the population that is in the zone of influence of dangerous goods.Transport of dangerous cargo represents the type of transport for which the greatest risks and potential magnitude of damage to the population and the environment are associated, so that in accordance with that it is necessary to influence the reduction of risk, i.e. manage risk when transporting dangerous goods. By starting to deal with the risk of occurrence of incidental urbanization in transport of dangerous goods, the probability of occurrence of an incident situation and the magnitude of the consequences for the environment are reduced. There are two types of activities aimed at reducing primary risk, namely (Milovanović, 2012):

1) reduction of the subsequent risk, and
2) control of the expected risk.

In addition to standard physical and chemical properties, dangerous goods also have dangerous properties, which is why they are classified in the category of dangerous goods. These characteristics are reflected in the types and degree of danger of dangerous goods and have a considerable influence on the choice of the section for the movement of vehicles transporting this type of cargo. In addition to the characteristics of dangerous cargo, the important factors are the amount of cargo transported by a given road section, current meteorological conditions, the characteristics of the terrain on which the road section is located, etc. In order to effectively manage the risks on road sections during the transport of dangerous goods, it is necessary to analyze the dangers of incidents. A hazard analysis is realized through three phases (Milovanović, 2012):

1) hazard identification,
2) consequence analysis, and
3) risk assessment.

The choice of the route for the movement of vehicles transporting dangerous goods based on the size of the risk is an important factor in the risk management process for the transport of dangerous goods. The size of the risk is one of the crucial factors for choosing the route when transporting dangerous goods. Consequently, new methods are being developed on a daily basis to select optimal routes for the transport of dangerous goods, with the aim of managing road risks. Based on the analysis of influential factors on potential routes for the transport of dangerous goods, the route for the transport of dangerous goods is selected. This is the first phase in the process of risk management during the transport of dangerous goods and is a key factor for effective risk management from incidents (Milovanović, 2012).

The concept of linear programming

The mathematical methodology for modeling and solving the problem of finding the maximum or minimum of a linear function, under conditions expressed as linear equations or inequalities, is called linear programming. The term "programming" is used as a synonym for planning, i.e. determining the optimal solution. This means determining the values of variables such that they give the optimal value of the objective function, while satisfying the conditions (constraints).

Linear programming is a special case of mathematical programming where the function of the goal and the constraints are linear. Such mathematical models can present the problems of resource allocation, vehicle movement planning, various problems of production in industry, problems in the economy, etc. The problem of linear programming was formulated by the Soviet mathematician Leonid Kantorovich in 1939 (Kantorovich, 1960). The first models were used in wood production, and during the Second World War Kantorović worked for the army to optimize military operations. The method of solving the problem was not publicly known until 1947 when George B.Danzig published the simplex algorithm (Dantzig, 1947).

An overview of current models for road risk assessment

The choice of a route for the transport of dangerous goods is one of the key activities during the planning and organization of transport. Given that the route used by vehicles during the transport of dangerous goods is an extremely important part of the transport process, a number of different methodologies have been developed for selecting routes for vehicles transporting dangerous goods: from case studies involving risk analysis, the route is performed on the basis of data obtained from statistical analyzes and research on the number of incident situations, until the choice of routes is resolved through vehicle routing algorithms (Thomson, 1999).

Based on the above, it is concluded that there are numerous methodologies developed with the aim of selecting routes for the movement of vehicles transporting dangerous goods from the aspect of risk management. The most important methodologies necessary for understanding the improved methodology related to the selection of routes for vehicles transporting dangerous goods based on the levels of individual and social risk as well as on the levels of absolute and specific risk are presented in the following section (Milovanović, 2012).

a) methodology for the selection of routes for the movement of vehicles transporting dangerous goods based on the size of individual and social risk and minimum costs

The problem that defines this methodology is better known as the "minimum cost problem of goods flows", and was developed by three Italian authors: Pierre Leonelli, Sara Bonvincini and Giulia Spadoni. In order to define and select the optimal route for the movement of vehicles transporting dangerous goods, it is necessary to define the values of the level of individual and social risk. To determine these two types of risk, there are two software programs defined by the same authors: TransIn (for determining the level of individual risk) and TransSoc (for determining the level of social risk). For the optimization of routes within this methodology, the software package OPTHIPATH selects the optimal route based on the minimum total costs and the minimum size of individual and social risk between the source and the destination point (Milovanović, 2012).

b) methodology for the selection of routes for road and rail transport using social risk curves

This methodology presents a simplified risk analysis for the transport of dangerous goods by road and rail. It is based on the use of data from a large database, taking into account the size of impact zones for several predefined incident scenarios, and on the choice of several average values of selected parameters relevant to the type of transport activity (i.e. for the type of transport and the type of dangerous goods) and the category of road (roads), (Bubbico et al, 2004).

c) methodology for determining the level of risk and decision-making strategies in the transport of dangerous goods

This methodology is based on determining the individual and social level of risk in order to choose routes for the transport of dangerous goods. It was developed by four Italian authors. Decisions made within the defined methodology are made based on the level of individual and social risk (Fabiano et al, 2002).

d) methodology for the selection of routes for the movement of vehicles transporting dangerous goods on the basis of specific and absolute risk

This methodology, based on the selection of routes for the movement of vehicles transporting dangerous goods on the basis of specific risk and developed by the American Road Institute, consists of eight steps. All steps within the defined methodology are interdependent, so that it is only possible to choose the route for the transport of dangerous goods by implementing all eight steps (Milovanović, 2012).

One of the most important aspects of the methodology for optimizing the routes for vehicles transporting dangerous goods is a high level of flexibility sufficient to make certain changes within the methodology and / or supplement it with certain goals defined by various stakeholders.The methodology for selecting routes for vehicles based on absolute risk is defined by the Institute for Geological Research in the Netherlands and is based on defining the optimal route for vehicles transporting dangerous goods for each phase separately, based on the objectives and scenarios defined within that phase and the previously completed phases. For each of the phases, special mathematical models were defined individually, on the basis of which optimizations were performed. The limitations defined within the methodology apply and they apply to all phases (Castillo, 2004).

Proposal of a model for risk assessment on road sections during the transport of dangerous goods in the Serbian Army

The whole process includes a series of activities that need to be implemented and consists of two phases. The first phase involves the application of the DEA model which defines routes that have a minimum level of risk. At the same time, DEA models eliminate routes that have an extremely high level of risk. In this step, two DEA models are formed to determine the state of traffic safety on the observed routes: an input-oriented model and an output-oriented model. Both models are connected in the output DEA-final form in order to, on the one hand, obtain routes that enter the further processing process and, on the other hand, to eliminate routes that do not meet the set conditions sufficiently (routes that do not have the required level of security traffic). In the second phase, after identifying the routes with a minimum degree of risk, route evaluation is performed using the fuzzy logic system (FLS). The FLS was created through several steps shown in Figure 1. The presented FLS gives the final ranking of the routes for the transport of dangerous goods, i.e. it defines the optimal route for the transport of dangerous goods. A user platform has been created for the created model, through which the practical application of the model is realized.

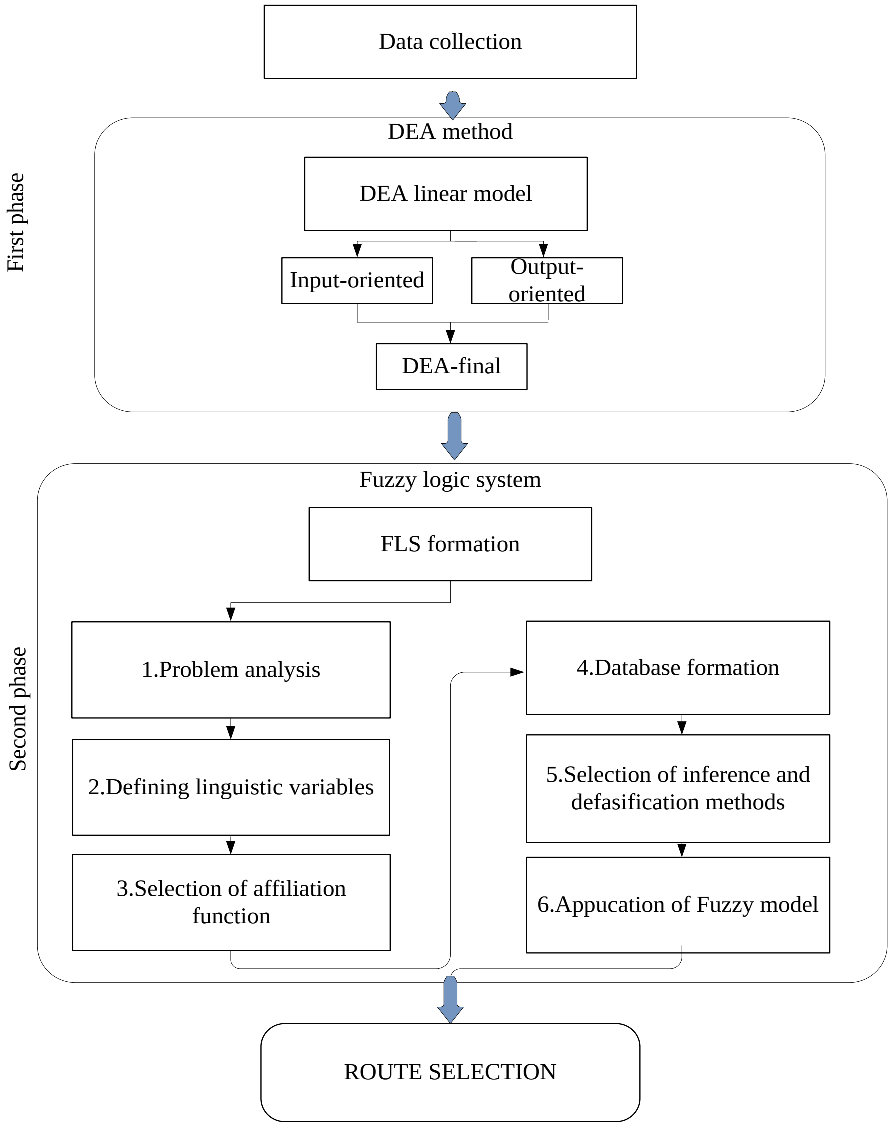

Figure 1– Research realization process
Рис. 1 – Процесс проведения исследования
Слика 1 – Процес реализације истраживања

Phase I - DEA model

The DEA method is one of the most widely used tools for measuring efficiency. It was used to compare organizations, companies, regions and countries. It has made significant contribution to logistics (Cavaignac et al, 2021), warehousing systems (Karande et al, 2019) and transport (Tian et al, 2020), as well as to logistics and supply chain subsystems (Amirteimoori & Khoshandam, 2011), which could be seen in the literature review at the end of the paper.

In addition to the assessment of efficiency, this method was used in defining the degree of safety and risk in transport and traffic systems (Ranković et al, 2011), which was applied in this paper. The specific problem that is the core of this research is the choice of the optimal route for the transport of dangerous goods from the aspect of traffic safety, using the model of linear mathematical programming. The research considered six alternative routes for which data related to eight parameters were collected. The database of the Traffic Safety Agency was used for the realization of the research.

The DEA-Data Envelope Analysis is a numerical approach based on the evaluation of the performance of a set of similar entities (organizational or production units), i.e .units to be decided (DMU - Decision Making Unit) (Cooper, 2014).The DEA is based on an extension of Farrell's 1957 efficiency measure (Farrell, 1957), upgraded with the method of linear programming by Charnes, Cooper, and Rhodes in 1978, resulting in a new method for calculating efficiency called the Data Envelope Analysis (DEA).

Creating virtual inputs (outputs) involves determining the weight coefficients associated with the inputs and outputs that participate in the analysis, in accordance with the objectives of the DMU. After calculating the efficiency measure for each DMU, the DEA analysis provides information on whether the DMU, based on its inputs and outputs, is relatively efficient or not, compared to other DMUs included in the analysis. The algorithm is basically reflected in the iterative solution of a linear program for each point in the data set. It is the standard basic algorithm of the DEA methodology and is widely used in numerical terms. This algorithm must solve as many linear programs as there are points in the data set (Dawson et al, 2000). In the process of applying the DEA model, two models were formed to determine the state of traffic safety during the transport of dangerous goods on the observed routes: input-oriented and output-oriented models. After that, based on the obtained results, the alternatives whose values deviate from 1 are eliminated from further calculations. In the process of traffic safety management, it is necessary to know the existing situation, which may include dangerous parts of the road (Nenadić, 2019).

There are two basic types of DEA models: the CCR model and the BCC model. In this case, the CCR (Charns, Cooper and Rhodes modeling model) model was used (Vesković et al, 2020). The CCR model is also known as the DEA model with constant volume yield, and the BCC model is known as the DEA model with variable volume yield. CCR models measure the overall technical efficiency of a unit, which includes pure technical efficiency and volume efficiency. It is assumed that the units operate with a constant return on volume, i.e. that the increase in the input must result in a proportional increase in the output (Jeremić, 2012).

Mathematically, the DEA method can be described by models that consist of a goal function and two types of constraints. In this case, two DEA CCR models were formed according to the input-oriented model (max) and according to the output-oriented model (min) (Srdjevic et al, 2002).

The input-oriented CCR model is obtained by converting a nonlinear task into a linear one which is solved by the method of linear programming. This gives an optimal solution, where the optimization criterion is the maximization of the value of the objective function under given constraints (system of linear inequalities) (Mitrović Simić et al, 2020). The DEA CCR input-oriented model (max) is:

$$DEA_{input} = \max \sum_{i=1}^{m} W_i X_{i-input} \quad (2)$$

St:

$$\sum_{i=1}^{m} W_i X_{ij} - \sum_{i=m+1}^{m+s} W_i Y_{ij} \leq 0, \quad j=1,\ldots\ldots\ldots\ldots, n \quad (3)$$

$$\sum_{i=m+1}^{m+s} W_i Y_{i-output} = 1 \quad (4)$$

$$Wi \geq 0, \quad i=1,\ldots\ldots\ldots., m+s \quad (5)$$

The DEA consists of m input parameters for each alternative Xij, while S shows the output parameters for each alternative Yij, taking into account the weights of the parameters denoted by wi, n represents the total number of DMUs. The DEA CCR output-oriented model (min) is:

$$DEA_{output} = \min \sum_{i=m+1}^{m+s} W_i Y_{i-output} \quad (6)$$

St:

$$-\left(\sum_{i=1}^{m} W_i X_{ij} \right) + \sum_{i=m+1}^{m+s} W_i Y_{ij} \geq 0, \quad j=1,\ldots\ldots\ldots\ldots, n \quad (7)$$

$$\sum_{i=1}^{m} W_i X_{i-input} = 1 \quad (8)$$

$$Wi \geq 0, \quad i=1,\ldots\ldots\ldots, m+s \tag{9}$$

Finally, to obtain the efficiency index for each DMU, the equation is applied:

$$DEA_{safety} = \frac{\min output}{\max input} \tag{10}$$

Phase II - Fuzzy logic system

Artificial intelligence is one of the computer fields. The main goal of artificial intelligence is to enable computers to behave like humans. This effect is achieved by using a fuzzy system. The fuzzy theory is designed to represent human knowledge and reason in such a way that it can be easily presented on a computer. The creator of fuzzy logic is Lotfi Zadeh (Zadeh, 1965). The fuzzy system has the property of "quantification of uncertainty". Unlike formal logic, in which reasoning is performed with two values(true-false, 0-1), fuzzy logic uses numbers from the interval (0.1).This interval is much closer to reality, human thinking and expression. During the modeling of the fuzzy logic system, it is necessary to go through all six steps listed in Figure 1 and finally apply the created model. An explanation of these steps is provided below.

1) Problem analysis

In order to determine the number of variables and their interdependence, a detailed analysis of the problem is performed when modeling a fuzzy logic system. If the problem is complex, the system can be divided into several smaller subsystems, the goal and purpose of each subsystem is determined, after which the way of connecting these subsystems and priorities among them is determined.

2) Defining linguistic values

As already mentioned, linguistic variables take values from a spoken language or are artificially synthesized and represented by fuzzy sets. In this case, it was assumed that the designed fuzzy system, for the selection of the route for transport of dangerous goods, contains six input linguistic variables (two input variables (out of eight) used in creating the previously presented DEA model were not taken into account) and system load (a large number of rules) to facilitate the modeling of fuzzy systems and perform simpler manipulation of parameters, as follows:

- ❖ Route length,
- ❖ Number of access points,
- ❖ AADT (annual average daily traffic),

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- Number of traffic accidents (TA) with fatalities,
- Number of traffic accidents with the injured, and
- Number of traffic accidents with material damage,

as well as the output linguistic variable Route preference.

For all inputs and outputs, it is necessary to determine the number and type of membership functions. A larger number of membership functions increase the total number of rules, which makes it difficult to set up the system, so it is recommended, in accordance with the nature of the variable, to start with the smallest number of membership functions. Reducing the number of membership functions must not be done to the detriment of the quality of the description of the variable. Starting from the above settings, it is defined that in the model each input variable has three linguistic values, and the output variable has six. The linguistic values assigned to all input variables were: Route length (Small, Medium, Large), Number of access points, AADT, Number of TA with fatalities, Number of TA with the injured and Number of TA with material damage (Small, Medium, Large). The output variable preference by route has the values: Very small, Small, Medium small, Medium, Large and Very large preference.

A large number of linguistic values were not needed because this is a decision support system, so it does not require enormous precision. With three linguistic values, a satisfactory gradual change in output values was achieved, which limits the number of rules to 729, a domain that an expert can control. By the way, most fuzzy systems contain 3, 5 or 7 variables, and practice has shown that people manipulate well with a maximum of 7 values.

3) Selection of affiliation functions

The choice of affiliation functions and their range in the confidence interval is one of the most important phases.The fuzzy system used Gaussian curves for input and output variables. In this way, a satisfactory sensitivity of the system is achieved. Figures 2 and 3 show the membership functions of the input variables and the output variable.

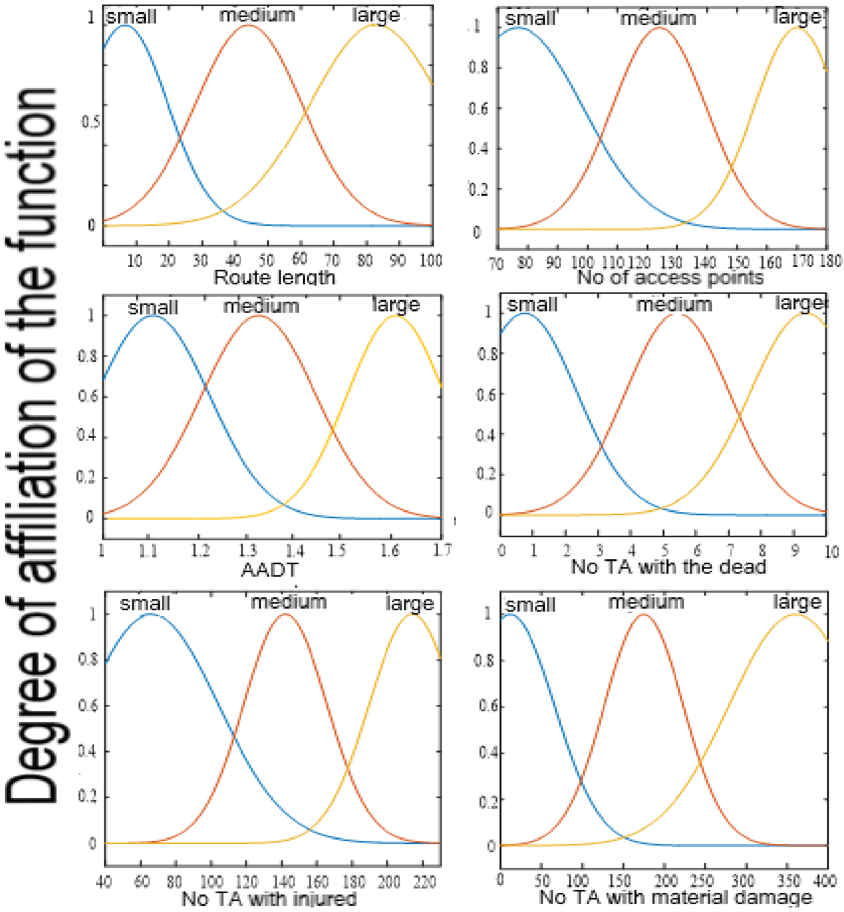

Figure 2 – Function of belonging of linguistic variables: Length of the route, Number of access points, AADT, Number of TA with fatalities, Number of TA with the injured and Number of TA with material damage

Рис. 2 – Функция принадлежности лингвистических переменных: Длина маршрута, количество точек доступа, СДТ, количество ДТП со смертельным исходом, количество ДТП с пострадавшими и количество ДТП с материальным ущербом

Слика 2 – Функција припадности лингвистичких промењивих: дужина руте, број приступних тачака, ПГДС, број СБН са погинулим лицима, број СБН са повређеним лицима и број СБН са материјалном штетом

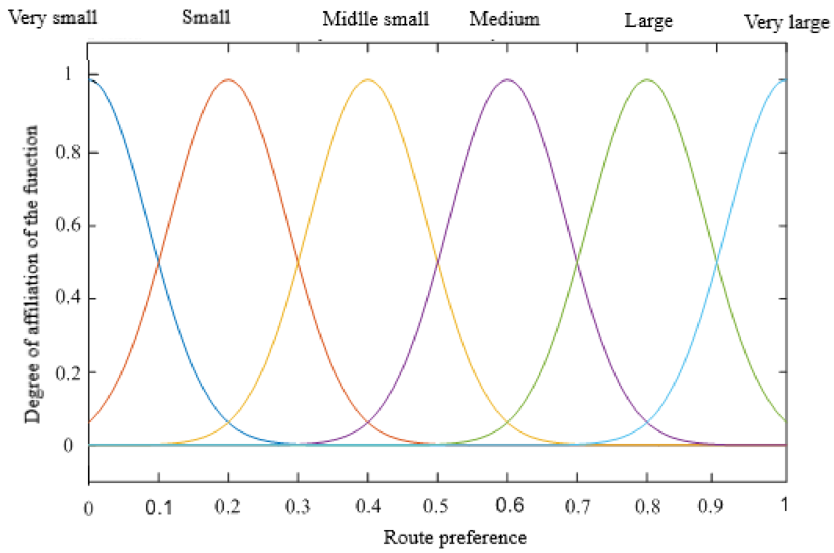

Figure 3 – Output variable membership functions
Рис. 3 – Функции принадлежности выходных переменных
Слика 3 – Функције припадности излазне промењиве

4) Forming a database of rules

Linguistic rules are the connection between the inputs and outputs of a fuzzy system. Expert knowledge of the process can be expressed through a number of linguistic rules consisting of spoken or artificial language words. When it comes to complex systems, one of the big problems is the lack of standard and systematic methods for transforming engineering knowledge or experience into fuzzy rules. There is also no general procedure for selecting the optimal number of rules, as many factors influence such a decision, which is very important for the speed of the system (Božanić & Pamučar, 2014).

The domain expert enters his knowledge primarily through production rules. At the beginning, it is important that for each combination of the input values of the linguistic variables, the expert suggests appropriate output values. In this case, there are six input linguistic variables (n = 6), with three linguistic values each (M = 3) and they can be combined in the database with a total of $M^n = 3^6 = 729$ rules. The weighting coefficients of the criteria were determined using the LMAW method. The method of aggregation of the weight of the premise of the rule is presented in (Pamučar et al, 2021).

5) Choice of the inference and defasification method

The methods most commonly used in direct inference are MIN-MAX and PROD-SUM (Mamdani method). In the initial phase of the system development, the MIN-MAX method was used. This method is a common choice when it is not important to manage the entire confidence interval of the output variable. However, a large number of system simulations have shown that the MIN-MAX method is inappropriate. One of the basic requirements was to achieve a satisfactory level of system sensitivity, which means that with certain small changes in the input, the output from the fuzzy system must also have small changes, which could not be achieved using the MIN-MAX method.The Matlab R2015b software package was used to construct the fuzzy logic system. In order to increase the sensitivity of the system, the PROD-SUM method of direct inference was used, as the best one offered by the Matlab software package.

Application of the model

To test the model, a transport task was used to transport dangerous goods from the Vasa Čarapić barracks (Banjica) to the Rastko Nemanjić barracks (Pančevo). Six routes were defined and taken into account for the realization of transport. Taking into account the fact that it is an urban zone where the vehicle moves, the safety of transport is the primary task during planning, primarily due to the increased concentration of pedestrians who represent the most endangered category of traffic participants, and then environmental protection.

Phase I - application of the DEA model

Based on expert consultations, eight input and output parameters in total have been defined, the validity of which has been confirmed in research work (Mitrović Simić et al, 2020). A list of four criteria was formed for the inputs: the route length expressed in kilometers, the number of access points, the AADT and the road slope. The following were taken as the outputs: the number of traffic accidents with fatalities, the number of traffic accidents with severely injured persons, the number of traffic accidents with lightly injured persons and the number of traffic accidents with material damage. The approximate numerical values are taken for the used criteria, presented in Tables 1 and 2.

Table 1 – Route parameters in relation to the formed list of inputs
Таблица 1 – Параметры маршрута по отношению к сформированному списку входных данных
Табела 1 – Параметри рута у односу на формирану листу импута

ROUTE	Route lenght (KM)	Number of access points	AADT (veh/day)	Road slope(%)
1.	73.37	92	16340	0.281
2.	33.28	88	14592	0.232
3.	52.27	123	15466	0.199
4.	34.6	140	10019	0.321
5.	32.23	132	11521	0.359
6.	53.36	161	10103	0.253

Table 2 shows the values of the observed routes in relation to the output parameters.

Table 2 – Route parameters in relation to the formed output list
Таблица 2 – Параметры маршрута относительно сформированного списка выходных данных
Табела 2 – Параметри рута у односу на формирану листу аутпута

Route	No.TA with fatalities	No. TA with seriously injured persons	No. TA with slightly injured persons	No. TA with material damage
1.	5	73	77	168
2.	3	101	104	255
3.	4	93	98	247
4.	4	202	105	254
5.	2	121	123	367
6.	4	51	54	73

In the following section, two DEA models are presented: input and output oriented models, in order to show the routes that had a satisfactory level of safety and which were taken into further ranking. The formed linear programming algorithms were solved using Lingo 18. The given examples of the input-oriented model were solved using equation (2), and the output-oriented one was solved using equation (6), for route 1 whose parameters are shown in Tables 1 and 2. All inputs belong to the benefit group, because they need to be maximized, and the outputs belong to the cost group and need to be minimized.

DEA_{INPUT}=MAX =73.37*w1+92*w2+16340*w3+0.281*w4;
73.37*w1+92*w2+16340*w3+0.281*w4-
(5*w5+73*w6+77*w7+168*w8)<=0;
33.28*w1+88*w2+14592*w3+0.232*w4-
(3*w5+101*w6+104*w7+255*w8)<=0;
52.27*w1+123*w2+15466*w3+0.199*w4-
(4*w5+93*w6+98*w7+247*w8)<=0;
34.6*w1+140*w2+10019*w3+0.321*w4-
(4*w5+202*w6+105*w7+254*w8)<=0;
32.23*w1+132*w2+11521*w3+0.359*w4-
(2*w5+121*w6+123*w7+367*w8)<=0;
53.36*w1+161*w2+10103*w3+0.253*w4-
(4*w5+51*w6+54*w7+73*w8)<=0;
5*w5+73*w6+77*w7+168*w8=1;
w1>0;w2>0;w3>0;w4>0;w5>0;w6>0;w7>0;w8>0;

The obtained results show that the objective function is equal to 1.000.

DEA_{outout} =MIN=5*w5+73*w6+77*w7+168*w8;
-73.37*w1-92*w2-16340*w3-
0.281*w4+(5*w5+73*w6+77*w7+168*w8)>=0;
-33.28*w1-88*w2-14592*w3-
0.232*w4+(3*w5+101*w6+104*w7+255*w8)>=0;
-52.27*w1-123*w2-15466*w3-
0.199*w4+(4*w5+93*w6+98*w7+247*w8)>=0;
-34.5*w1-140*w2-10019*w3-
0.321*w4+(4*w5+202*w6+105*w7+254*w8)>=0;
-32.23*w1-132*w2-11521*w3-
0.359*w4+(2*w5+121*w6+123*w7+367*w8)>=0;
-53.36*w1-161*w2-10103*w3-
0.253*w4+(4*w2+51*w6+54*w7+73*w9)>=0;
73.37*w1+92*w2+16340*w3+0.281*w4=1;
w1>0;w2>0;w3>0;w4>0;w5>0;w6>0;w7>0;w8>0;

The obtained results for output oriented route 1 also show that the objective function is equal to 1.000. The algorithms of other routes were solved in the same way and the final results of the DEA model are shown in Table 3. The DEA-final values were obtained applying equation (10).

Table 3 – Traffic safety situation on the analyzed routes after the application of the DEA model
Таблица 3 – Ситуация с безопасностью движения на анализируемых маршрутах после применения модели DEA
Табела 3 – Стање безбедности саобраћаја на анализираним рутама након примене модела ДЕА

ROUTE	DEA-INPUT	DEA-OUTPUT	DEA-FINAL
1.	1.000	1.000	1.000
2.	1.000	1.000	1.000
3.	1.000	1.000	1.000
4.	0.852	0.999	1.172
5.	1.000	1.000	1.000
6.	1.000	0.249	0.249

Routes 4 and 6 are eliminated from the further processing process due to poor traffic safety characteristics. On these routes, it is necessary to take certain corrective measures in order to improve traffic safety. Routes with a value of DEA-final=1.000 are taken into further consideration.

Phase II - Application of the fuzzy logic system

The application of the system is an integral step in the life cycle of the system. The model needs to be applied and, if necessary, certain corrective measures, changes and improvements need to be taken, which is relatively easy in a fuzzy system because it is characterized by the property of adaptability. When designing a fuzzy system, the interval of (0.1) was taken as the confidence interval of the output variable route preference. During the practical application of the system, the interval (0.1) can be adjusted depending on the obtained preference according to the route. For transport, six routes - alternatives were considered, and after the application of the DEA model, further consideration is given, i.e. in the second phase of the work, four alternatives were taken. The fuzzy logic system was applied to routes 1, 2, 3 and 5, which were mentioned earlier, because after the application of the DEA model, these alternatives proved to be routes that meet the necessary conditions and were taken into further processing.

Table 4 shows the values of the input variables in the fuzzy logic system for each route (alternative). The database of the Traffic Safety Agency was used for the realization of the research.

Table 4 – Values of the input variables in the fuzzy logic system
Таблица 4 – Значения входных переменных в системе нечеткой логики
Табела 4 – Вредности улазних промењивих у fuzzy логички систем

Route	Route length(KM)	No.of access point	AADT	No. TA with fatalities	No. TA with injured persons	No. TA with material damage
A1	73.37	92	16340	5	75	168
A2	33.28	88	14592	3	102.5	255
A3	52.27	123	15466	4	95.5	247
A4	32.23	132	11521	2	122	367

After entering the input parameters into the fuzzy logic system, the calculation and evaluation of these alternatives is performed. The results of the calculation, i.e. the output values of the fuzzy logic system are shown in Table 5.

Table 5 – Evaluation of the alternatives
Таблица 5 – Оценка альтернатив
Табела 5 – Евалуација алтернатива

Alternatives	Route preference	
	Numerical value	Linguistic description
A1	0.567	Medium small
A2	0.680	Medium
A3	0.519	Small
A4	0.668	Medium

After the evaluation of the alternatives, it was found that there was the greatest preference for alternative A2. The ranking of the alternatives can be displayed as A2> A4> A1> A3.

For the developed fuzzy logic system, a user program was created for the practical realization of the model. The user platform was created in the Matlab R2015b software package. By entering the term "PROGRAM" in the command line of the Matlab software package, a program is launched to select the route for the transport of dangerous goods based on the minimum degree of risk. The user form is shown in Figure 4.

Figure 4 – User form of the program
Рис 4. – Форма пользовательской программы
Слика 4 – Корисничка форма програма

Clicking on the "Run program" button opens a fuzzy logic model in which the user enters the desired values of input variables. The values of the input and output variable criteria are shown in Tables 4 and 5. The values are entered by typing in the window of the specified criteria located on the user form, which is shown in Figure 5.

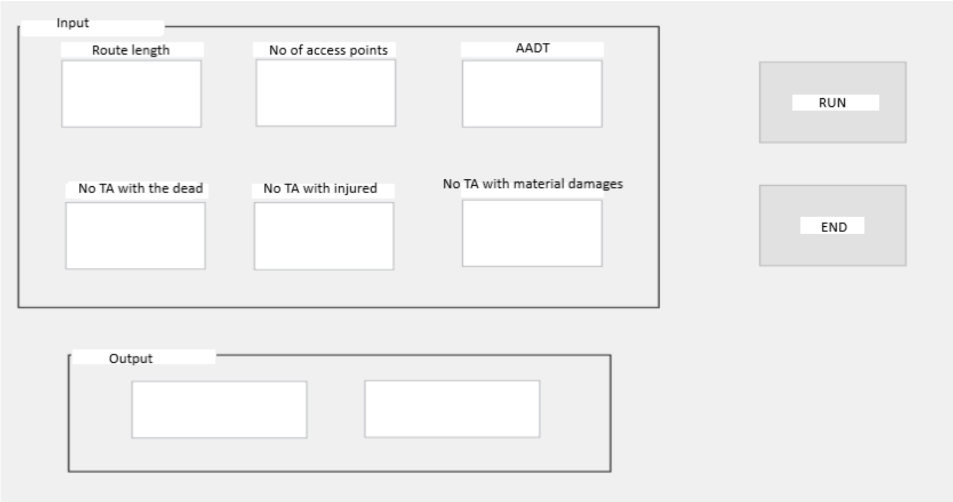

Figure 5 – Window for entering the input values into the system and displaying the output values from the FLS
Рис. 5 – Окно для ввода входных значений в систему и отображения выходных значений из ФЛС
Слика 5 – Прозор за унос улазних вредности у систем и приказ излазних вредности из FLS-a

Activating the "RUN" button starts the developed fuzzy system in which the calculation and evaluation of routes is performed. The presented model expands the theoretical framework of knowledge in the field of choosing the route for the transport of dangerous goods. The existing problem can be considered with a new methodology, which provides a basis for further theoretical and practical upgrades.

Conclusion

The model used to select the route for the transport of dangerous goods using the DEA model and fuzzy logic systems is based on the minimum level of risk. Fuzzy logic systems belong to a group of models that are based on artificial intelligence and can be used to support the decision-making bodies of the transport service in the Ministry of Defense and the Serbian Army. Technological development leads to the development of new models for the selection and optimization of the route by which dangerous goods are transported. The primary goal is to reduce the risks and consequences of incidents.

Based on the data collected from the database of the Traffic Safety Agency, eight criteria for creating DEA models have been defined. For the input values (inputs), the following criteria were taken: the route length, the number of access points, the AADT and the road slope. For the output values(outputs), the following criteria were taken: the number of traffic accidents with fatalities, the number of traffic accidents with seriously injured persons, the number of traffic accidents with slightly injured persons and the number of traffic accidents with material damage. The algorithms for the inputs and the outputs were solved using the Lingo18 software package.The last step in the implementation phase of the DEA model was the definition of DEA final results that eliminate routes with extremely low levels of safety and on which appropriate measures need to be taken in order to improve traffic safety. In this case, two routes were eliminated out of the total of six alternatives, and these routes were not considered in the fuzzy logic system that includes the second phase of work. The following criteria were taken as the input values into the fuzzy logic system: the route length, the number of access points, the AADT, the number of traffic accidents with fatalities, the number of traffic accidents with injured persons and the number of traffic accidents with material damage. In the phase of the application of the fuzzy logic system, the total number of the input criteria was reduced compared to the DEA model for simplifying system modeling and for easier manipulation of the total number of rules, 729, with the six input criteria.

All input variables are represented by three membership functions, and the output variable is defined by six membership functions. One of the basic requirements when modeling the system was the existence of a certain degree of sensitivity of the system, so Gaussian bell functions were used as functions for the input and output variables. All rules in the fuzzy logic system are determined by applying the method of aggregation of weight premise rules (ATPP) (Božanić & Pamučar, 2014), which allows the formation of a database based on experience and intuition. Based on the number of input variables and the number of their membership functions, a basic base of 729 rules was defined. The values of the weight coefficients were determined using the LMAW method (Pamučar et al, 2021). In order to increase the sensitivity of the system, the PROD-SUM method of direct inference was used.

The presented model was tested on the example of choosing a route for the transport of dangerous goods. Four possible routes were considered and evaluated on the basis of the six previously mentioned criteria. After the calculation and evaluation of the individual routes, the values of the output variable of the fuzzy system were obtained in the user form, that is, the preference for the route was obtained in the form of a numerical value and a linguistic descriptor.

Based on the output values of the system for each route, the observed routes were compared and ranked. In this case, route 2 proved to be optimal for the realization of the transport task. The advantage of using this system is the presence of the adaptability feature which gives the possibility to adjust the base of the fuzzy rule. Fuzzy inference rules are essential for the management of dangerous goods transport, due to the descriptive approach and the heuristic solution of the problem. The fuzzy logical model transcends the limitations of conventional evaluation methods. This system is implemented as a user program within the Matlab software package. As such, it is suitable for application in a dynamic environment and real-time decision making. The presented system leaves room for further research that should move in the direction of identifying additional parameters that may affect the choice of a route for the transport of dangerous goods and the implementation of additional decision criteria in the presented model. The presented software can be additionally tested by solving problems from the real environment in the units of the Serbian Army and adjusted to the needs and requirements of users.

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ВЫЯВЛЕНИЕ РИСКОВ НА УЧАСТКАХ ДОРОГ ПРИ ПЕРЕВОЗКЕ ОПАСНЫХ ГРУЗОВ СЕРБСКИХ ВООРУЖЕННЫХ СИЛ С ИСПОЛЬЗОВАНИЕМ МОДЕЛИ ЛИНЕЙНОГО МАТЕМАТИЧЕСКОГО ПРОГРАММИРОВАНИЯ

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РУБРИКА ГРНТИ: 27.47.19 Исследование операций,
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ВИДСТАТЬИ: оригинальная научная статья

Резюме:

Введение/цель: В статье представлена модель по выбору маршрута перевозки опасных грузов с использованием моделей DEA (Data Envelopment Analysis) и систем нечеткой логики. Представленная модель используется для выявления рисков на участках дорог при перевозке опасных грузов, а также для выбора оптимального маршрута для реализации транспортной задачи.

Методы: Модель состоит из двух этапов. Первый этап включает в себя применение моделей DEA, в которых сформированные входные и выходные данные объединяются в конечной форме выходной DEA модели, которая показывает маршруты с удовлетворительным уровнем безопасности дорожного движения и в то же время устраняет маршруты с низкой безопасностью дорожного движения. Второй этап предполагает применение систем нечеткой логики, при выходе из которой рекомендуется один из маршрутов. Оценка маршрута основана на шести критериях, а именно: расстояние маршрута, количество точек доступа, СДТ (среднегодовой дневной трафик), количество дорожно-транспортных происшествий со смертельным исходом, количество дорожно-транспортных происшествий с пострадавшими и количество дорожно-транспортных происшествий с материальным ущербом. При вводе значений входных критериев выполняется вычисление и оценка, а при выходе из нечеткой системы рекомендуется один из введенных маршрутов (маршрут с наименьшим уровнем риска). Используемые критерии были определены на основании проведенной экспертизы.

Результаты: Пользовательская программа используется в качестве поддержки при принятии решений органами управления дорожным движением.

Выводы: Пользовательская платформа была разработана в рамках пакета прикладных программ Matlab R2015a с возможностью адаптации к конкретным задачам.

Ключевые слова: модель DEA, система нечеткой логики, опасные грузы, риск, предпочтительный маршрут.

ДЕФИНИСАЊЕ РИЗИКА НА ПУТНИМ ДЕОНИЦАМА ПРИ ТРАНСПОРТУ ОПАСНОГ ТЕРЕТА У ВОЈСЦИ СРБИЈЕ ПРИМЕНОМ МОДЕЛА ЛИНЕАРНОГ МАТЕМАТИЧКОГ ПРОГРАМИРАЊА

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ОБЛАСТ: математика, саобраћај, логистика
КАТЕГОРИЈА (ТИП) ЧЛАНКА: оригинални научни рад

Сажетак:

Увод/циљ: У раду је представљен модел за избор руте, за транспорт опасног терета, употребом модела DEA (Data Envelopment Analysis) и fuzzy логичких система. Приказани модел се користи за дефинисање ризика на путним деоницама при транспорту опасног терета, као и за избор оптималне руте за реализацију транспортног задатка.

Методе: Модел се састоји од две фазе. У првој фази је примењена метода DEA која се састоји од два модела: импута и аутпута, повезана у излазни DEA final модел, који показује руте са задовољавајућим степеном безбедности саобраћаја и истовремено елиминише руте са ниским степеном безбедности саобраћаја. Друга фаза укључује примену fuzzy логичких система, а као излаз из fuzzy система дата је преференција према рути. Евалуација руте врши се на основу шест критеријума, а то су: дужина руте, број приступних тачака, ПГДС (просечан годишњи дневни саобраћај), број саобраћајних незгода са погинулим лицима, број саобраћајних незгода са повређеним лицима и број саобраћајних незгода са материјалном штетом. Када се унесу вредности улазних критеријума, врши се прорачун и евалуација, при чему се, као излаз из развијеног система, добија преференција према унетој рути (рута са најмањим нивоом ризика). Коришћени критеријуми дефинисани су на основу експертских процена.

Резултати: Кориснички програм користи се као подршка у одлучивању органима саобраћајне службе.

Закључак: Корисничка платформа креирана је у програмском пакету Matlab R2015a и пружа могућност надоградње и прилагођавања конкретном проблему.

Кључне речи: модел DEA, fuzzy логички систем, опасан терет, ризик, преференција према рути.

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EVACUATION OF AIRCRAFT ON LAND

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Abstract:

Introduction/Purpose: Aircraft present one of the safest and most frequently used means of transport. However, despite taking many security measures, accidents happen. No matter a damage level, if the aircraft somehow manages to land, the most important is to evacuate passengers from the aircraft, fast and safely. Evacuation of aircraft is very complex and depends on many different factors such as a damage degree, presence of fire, speed of passengers, presence of panic and fear, etc. So, it is important to, somehow, as much as possible, predict potential ways of evacuation and potential evacuation strategies and routes. Landed aircraft can be in different conditions so fast and safe evacuation of passengers is very important. The only way to predict safe evacuation routes, to determine proper evacuation strategies and to calculate potential evacuation times needed to leave the aircraft is to use some adequate simulation software.

Methods: In this paper, for calculating needed evacuation times and potential evacuation routes, the simulation method was used. Simulations of evacuation scenarios and calculations of evacuation times were realized in Pathfinder software. The simulation model created in Pathfinder was a model of the A 321 aircraft related to its real dimensions.

Results: The results of this paper, obtained on an appropriate simulation model of the aircraft with stairs and emergency slides, have shown the evacuation times for two different evacuation scenarios with different speeds of passengers/occupants.

Conclusion: A proper evacuation strategy and the fastest evacuation of occupants are crucial for saving lives. Simulation software use in evacuation problems presents a very effective way in terms of safety, cost-effectiveness and prediction. This kind of software presents an obligatory engineering tool for more effective and more precise dealing with evacuation and similar problems. This paper was written to show how simulation software can be used for calculating evacuation times from an airplane on land.

Key words: evacuation, aircraft, simulation, passenger.

Introduction

Evacuation of people, animals and material resources from endangered places or buildings to a safe location in a fast and safe way always presents a complex task.

Generally, traffic presents a very important social activity where potential situations and occasions demand very effective evacuation strategies. According to reports and statistics, air traffic presents one of the safest ways of traffic. Most accidents in air traffic occurred in the air during taking off and landing. A diagram of all accidents in civil air traffic from 1919 till October 1921 is presented in Figure 1 (Planecrach, 2021).

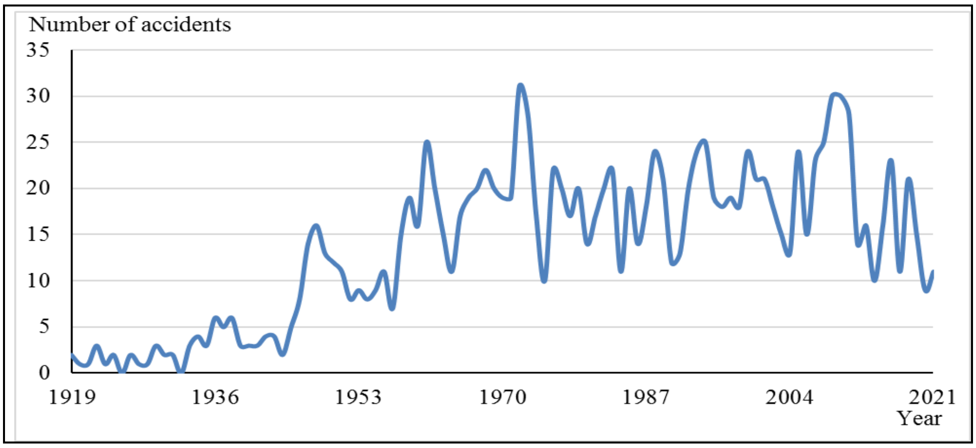

Figure 1 – Aircraft accidents from 1919 till October 2021
Рис. 1 – Авиакатастрофы с 1919 по октябрь 2021 год
Слика 1 – Авионске несреће од 1919. до октобра 2021. године

One of the most important advantages for air traffic in terms of safety are rigid procedures and rules for aircraft and high scientific and quality levels used in manufacturing aircraft. But accidents do occur. Sometimes, it is possible to avoid accidents or to reduce consequences of accidents. Of course, the most important course of action in any case of damage, malfunction or any problem, is to land the aircraft immediately. Many accidents were avoided by forced landings, very often out of the runway. A well-known case called „Miracle on the Hudson flight“ occurred in January 15, 2009, when a commercial airplane landed on the Hudson river and all 155 passengers and the crew members survived (CN Traveler, 2022). However, even when an aircraft somehow lands, a very important moment is to evacuate the passengers from the airplane to the safety. Potential scenarios can be different: an engine failure while taking off, missed runway while landing, wheals failure while taking off or landing, outing of the runway, etc. This task can be very hard, depending on many factors. In many cases, passengers try to evacuate the aircraft without any kind of organisation, tactics or plans; the presence of fear and panic is huge and injuries and even deaths are very likely to happen. Also, very often, the cabin crew do not have precise and continual connection and communication both between themselves and with the passengers which implies a delayed or considerably longer evacuation. The fact that the aircraft must be evacuated immediately is disturbing enough in its own right, but in the presence of other phenomena such as fire, smoke, sparking, etc., the whole situation becomes extremely tense and likelihood for accidents significantly increases. Generally, evacuation from a landed aircraft can be realized by emergency exits which can be equipped with stairs, emergency slides and emergence ropes (SKY Bary, 2022).

Therefore, for safe and effective evacuation, it is very important to know which evacuation strategy to apply in different situations: in which direction to go, which exit to choose, how fast to go in order to avoid the jams and crowds, etc. In a situation such as force landing or a bomb threat, these tasks can be very hard to realise because of the presence of many different factors such as fear, stress, panic, lack of time, etc. It would be thus best to somehow predict as many potential scenarios as possible and calculate potential evacuation times. All of this is possible with the use of simulation software. So, this paper was written to show the advantages of simulation software use in the prediction of evacuation. The main motive and the aim of evacuation is to save human lives. The most important advantages of simulation software use are to calculate times needed for evacuation from aircraft in different situations and scenarios and predict a lot of potential situations and scenarios in a safe, inexpensive and effective way. The number of all potential scenarios is huge and it is almost impossible to predict all potential scenarios, but with use of simulation software a lot of scenarios can be realized and analysed so that the knowledge and results gained in that way generally significantly improve a degree and efficiency of evacuation. The main contribution of this work is in prediction - in determining the best evacuation scenarios for occupants in some real situations by using simulation software. The use of the simulation software method for predicting evacuation is still a novelty and a method not used enough for determining evacuation scenarios and calculating evacuation times. The use of simulation software is safe for human lives because real occupants are not involved. It is also very cost-effective because it does not require the use of different material resources. And, of course, it is very precise because simulation software uses checked algorithms for calculations with the influences of many different factors (speed of occupants, stairs dimensions, number of occupants, etc.).

Pathfinder simulation software

The main method used in this paper was the simulation method. Although computer-based simulations have been used in last 20-25 years, it is still a method not often used (Galea et al, 1998). Simulation is possible by the use of proper simulation software, called Pathfinder software, version 2021. There are very important reasons for simulation software use in evacuation. Safety is above all. It would be almost impossible in reality to design and test evacuation scenarios without accidents and even victims and material resources destruction. Then comes accuracy. The software of this kind shows a great degree in precision and accuracy. In addition, the time needed for calculating the whole simulation is significantly shorter than the time needed for a simulation without simulation software.

This powerful simulation software provides simulations of people moving along various passageways such as stairs, elevators, or ramps with different speeds of occupants. There are two different simulation modes: a so-called SFPE mode and a steering mode. One of very important characteristics of this software is to „import“ files from other programs which significantly increases the compete simulation process because in that case it is not necessary to draw the environment (Thunderhead, 2017; Jevtić, 2021).

Simulation model

The aircraft simulation model was designed in the Pathfinder simulation software based on the aircraft real dimensions. The simulated aircraft was A 321. There are different versions of this aircraft. The simulated version was based on the following dimensions: overall length 44.51 m, cabin length 34.44 m, maximum seating places 220, fuselage width 3.95 m, maximum cabin width 3.7 m and wing span 35.8 m. The complete interior was designed to imitate the real aircraft (seats, compartments, etc.). The complete number of persons/occupants in the aircraft was 228. This number also included three members of the pilot crew and five members of the flight attendant crew (Airbus Aircraft, 2022).

The evacuation from the aircraft was simulated with two different scenarios: using ordinary stairs for passengers and using emergency slides. For the first scenario, there were three different simulations realized: with one front door opened, with two doors opened (the front and the back ones) and with three doors opened. All used doors in simulations for this scenario were on the same side of the aircraft. Although there are four doors on both sides of the aircraft, in real situations at airports, mostly one or two entry/exit doors with stairs are used. The speeds of passengers/occupants were 0.2 m/s, 0.3 m/s, 0.4 m/s, 0.5 m/s, 0.75 m/s, 0.95 m/s and 1.15 m/s. These speeds were chosen because of the lack of space in the aircraft for bigger speeds and the existence of seats and a narrow aisle in the middle of the aircraft.

For the second scenario, there were eight different cases simulated. The first case involved all eight doors opened; the second case involved one door closed and seven doors opened; the third case involved two doors closed and six doors opened; the fourth case involved three doors closed and five doors opened; the fifth case involved four doors closed and four doors opened; the sixth case involved five doors closed and three doors opened; the seventh case involved six doors closed and two doors opened, and the eighth case involved seven doors closed and one door opened. For each of seven cases, the emergency slides were used. The speeds of passengers/occupants were 0.2 m/s, 0.3 m/s, 0.4 m/s, 0.5 m/s, 0.75 m/s, 0.95 m/s and 1.15 m/s in the plane, while the emergency slide speed was 1 m/s.

The first case involved $\binom{8}{0}$=1 possibility for every speed of passengers/occupants. The second case involved $\binom{8}{1}$=8 different possibilities for every speed of passengers/occupants. The third case involved $\binom{8}{2}$=28 different possibilities for every speed of passengers/occupants. The fourth case involved $\binom{8}{3}$=56 different possibilities for every speed of passengers/occupants. The fifth case involved $\binom{8}{4}$=70 different possibilities for every speed of passengers/occupants. The sixth case involved $\binom{8}{5}$=56 different possibilities for every speed of passengers/occupants. The seventh case involved $\binom{8}{6}$=28 different possibilities for every speed of passengers/occupants. The eighth case involved $\binom{8}{7}$=8 different possibilities for every speed of passengers/occupants. The doors with emergency slides were marked with numbers from 1 to 8. The exit doors also have the same marks.

The simulation model of the A 321 aircraft with its interior is presented in Figure 2 in Pathfinder software while the emergency slides of the aircraft marked with numbers from 1 to 8 are presented in Figure 3. Many details in figures could be visible if the HIDE function were not activated.

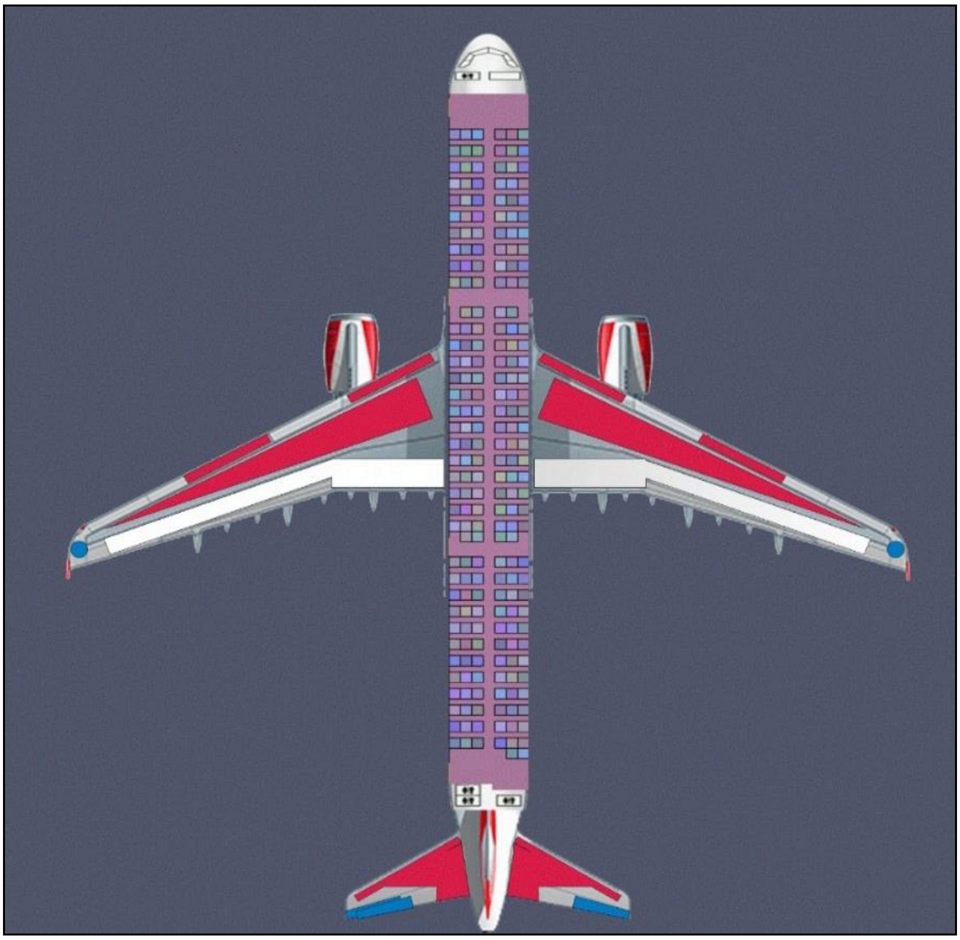

Figure 2 – The simulation model of the A 321 aircraft with its interior, Pathfinder presentation
Рис. 2 – Имитационная модель самолета А 321 с его интерьером, презентация Pathfinder
Слика 2 – Симулациони модел авиона А 321 са унутрашњошћу у презентацији Pathfinder

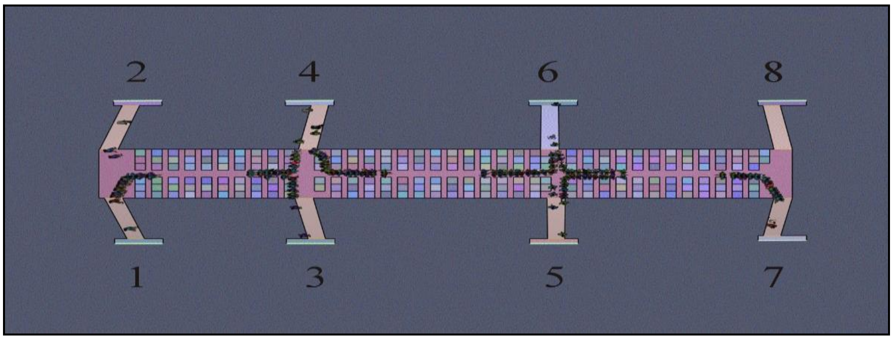

Figure 3 – Emergency slides of the aircraft marked with numbers from 1 to 8
Рис. 3 – Аварийные выходы самолета обозначены цифрами от 1 до 8
Слика 3 – Излази у случају евакуације из авиона, означени бројевима од 1 до 8

Simulation results

All simulation results for this paper were realized on a laptop Dell Latitude, with Intel® Core™ i7-1185G7 (4 Core, 12M cache, base 3.0GHz, up to 4.8GHz, vPro) processor and 16 GB of RAM memory. It is recommended to use a computer with „strong“ hardware support for work with simulations.

Some simulation moments are presented in Figures from 4 to 7, while the simulation results are given in Figures 8 to 25.

Because of a huge number of realised simulations and the paper limitations, only the results for the fastest and the slowest possibilities for every case of the second scenario were presented.

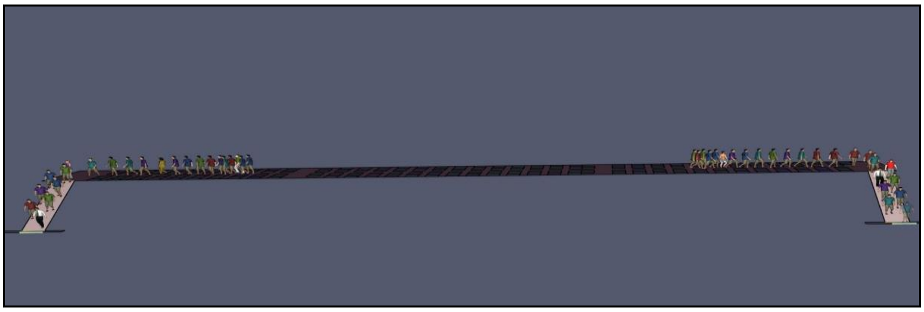

Figure 4 – Simulation moment for the second case of the first scenario at 170 seconds after the start of the simulation
Рис. 4 – Момент имитации другого случая по первому сценарию через 170 секунд после начала имитации
Слика 4 – Тренутак симулације за други случај првог сценарија после 170 секунди од почетка симулације

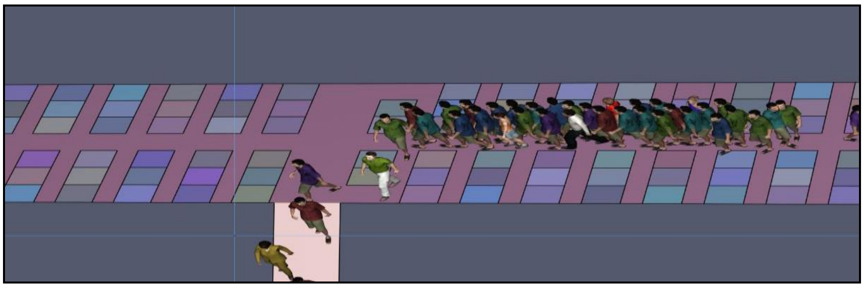

Figure 5 – Simulation moment for the third case of the first scenario at 88 seconds after the start of the simulation

Рис. 5 – Момент имитации третьего случая по первому сценарию через 88 секунд после начала имитации

Слика 5 – Тренутак симулације за трећи случај првог сценарија после 88 секунди од почетка симулације

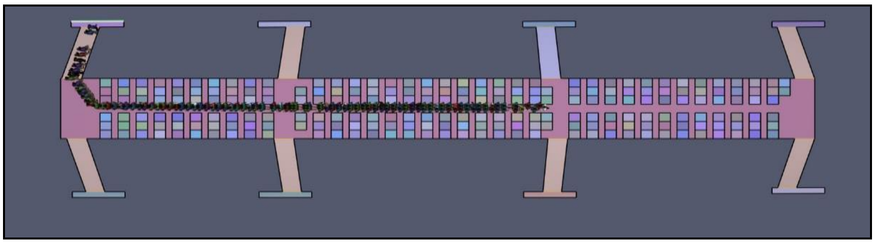

Figure 6 – Simulation moment for the seventh case of the second scenario after 130 seconds after the start of the simulation

Рис. 6 – Момент имитации седьмого случая по второму сценарию через 130 секунд после начала имитации

Слика 6 – Тренутак симулације за седми случај другог сценарија после 130 секунди од почетка симулације

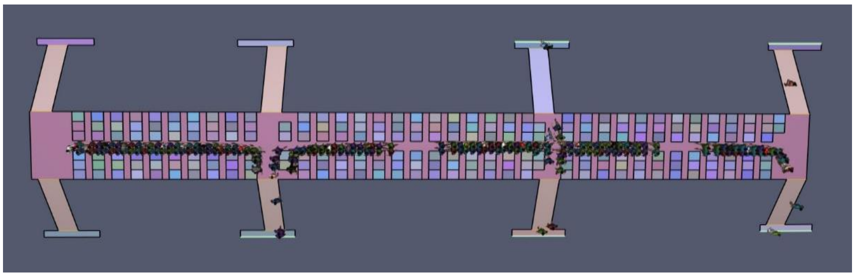

Figure 7 – Simulation moment for the fourth case of the second scenario after 28 seconds after the start of the simulation

Рис. 7 – Момент имитации четвертого случая по второму сценарию через 28 секунд после начала имитации

Слика 7 –Тренутак симулације за четврти случај другог сценарија после 28 секунди од почетка симулације

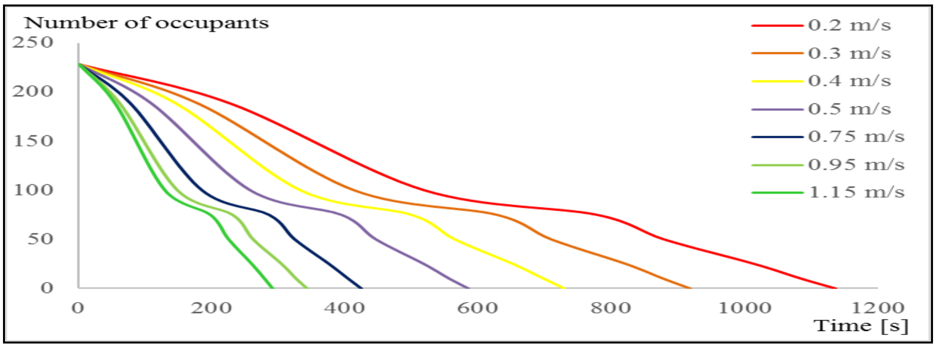

Figure 8 – Simulation results for the first case of the first scenario
Рис. 8 – Результаты моделирования первого случая по первому сценарию
Слика 8 – Симулациони резултати за први случај првог сценарија

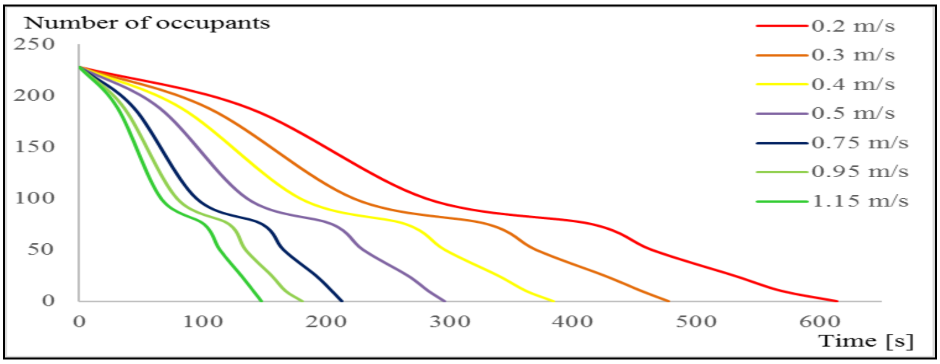

Figure 9 – Simulation results for the second case of the first scenario
Рис. 9 – Результаты моделирования второго случая по первому сценарию
Слика 9 – Симулациони резултати за други случај првог сценарија

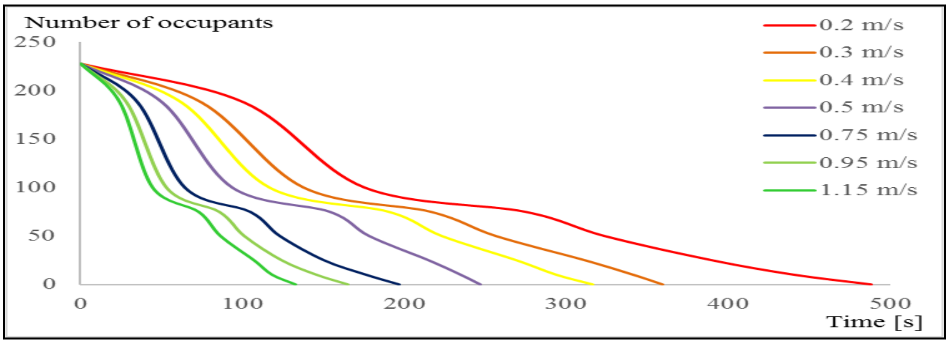

Figure 10 – Simulation results for the third case of the first scenario
Рис. 10 – Результаты моделирования третьего случая по первому сценарию
Слика 10 – Симулациони резултати за трећи случај првог сценарија

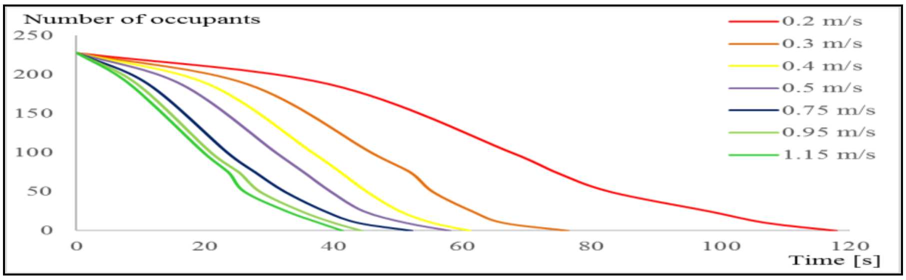

Figure 11 – Simulation results for the first case of the second scenario
Рис. 11 – Результаты моделирования первого случая по второму сценарию
Слика 11 – Симулациони резултати за први случај другог сценарија

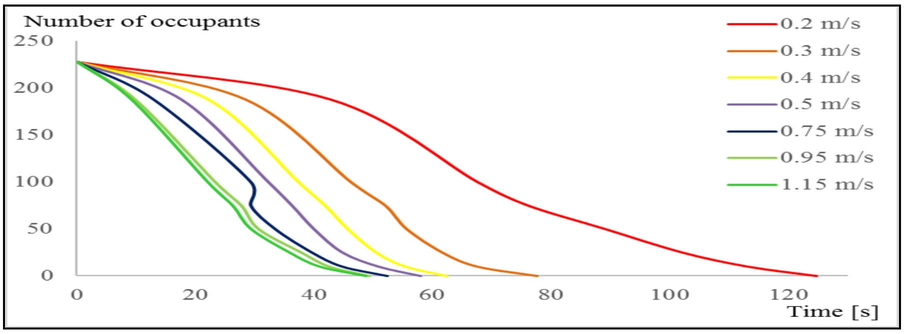

Figure 12 – Simulation results for the fastest times of the second case of the second scenario
Рис. 12 – Результаты моделирования скорейшего времени второго случая по второму сценарию
Слика 12 – Симулациони резултати за најбржа времена другог случаја другог сценарија

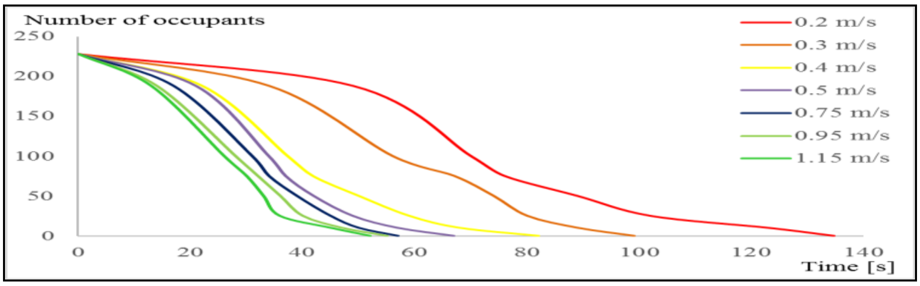

Figure 13 – Simulation results for the slowest times of the second case of the second scenario
Рис. 13 – Результаты моделирования самого медленного времени второго случая по второму сценарию
Слика 13 – Симулациони резултати за најспорија времена другог случаја другог сценарија

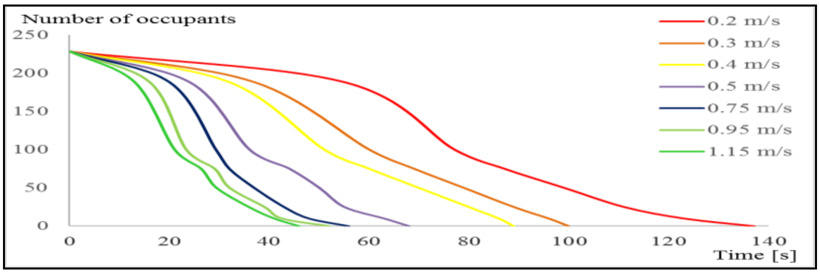

Figure 14 – Simulation results for the fastest times of the third case of the second scenario
Рис. 14 – Результаты моделирования скорейшего времени третьего случая по второму сценарию
Слика 14 – Симулациони резултати за најбржа времена трећег случаја другог сценарија

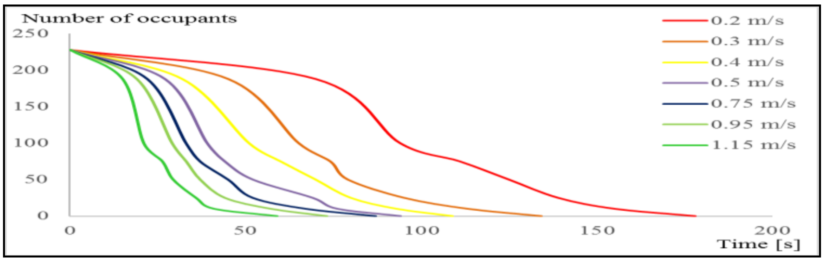

Figure 15 – Simulation results for the slowest times of the third case of the second scenario
Рис. 15 – Результаты моделирования самого медленного времени второго случая по второму сценарию
Слика 15 – Симулациони резултати за најспорија времена другог случаја другог сценарија

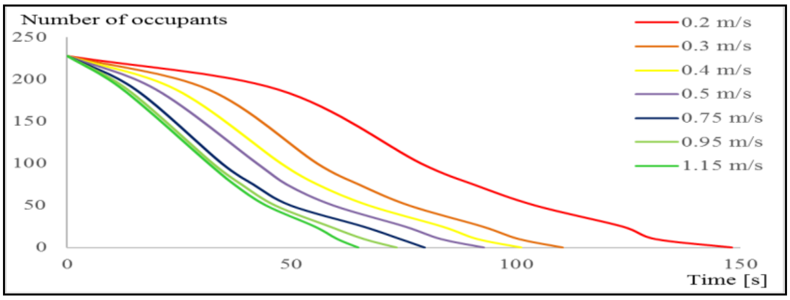

Figure 16 – Simulation results for the fastest times of the fourth case of the second scenario
Рис. 16 – Результаты моделирования скорейшего времени четвертого случая по второму сценарию
Слика 16 – Симулациони резултати за најбржа времена четвртог случаја другог сценарија

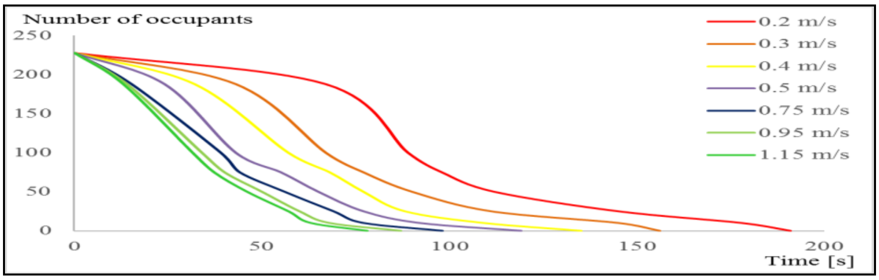

Figure 17 – Simulation results for the slowest times of the fourth case of the second scenario

Рис. 17 – Результаты моделирования самого медленного времени четвертого случая по второму сценарию

Слика 17 – Симулациони резултати за најспорија времена четвртог случаја другог сценарија

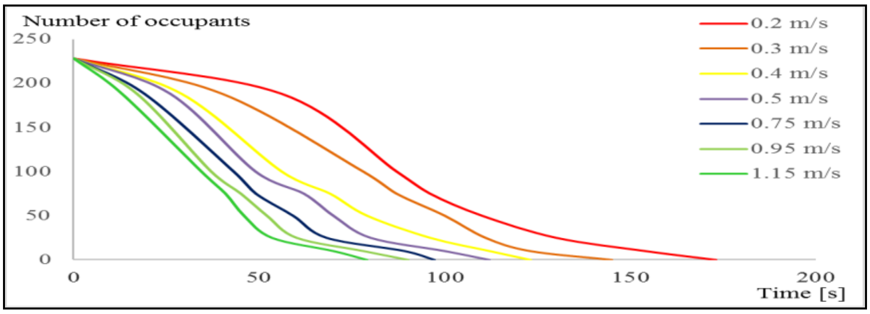

Figure 18 – Simulation results for the fastest times of the fifth case of the second scenario

Рис. 18 – Результаты моделирования скорейшего времени пятого случая по второму сценарию

Слика 18 – Симулациони резултати за најбржа времена петог случаја другог сценарија

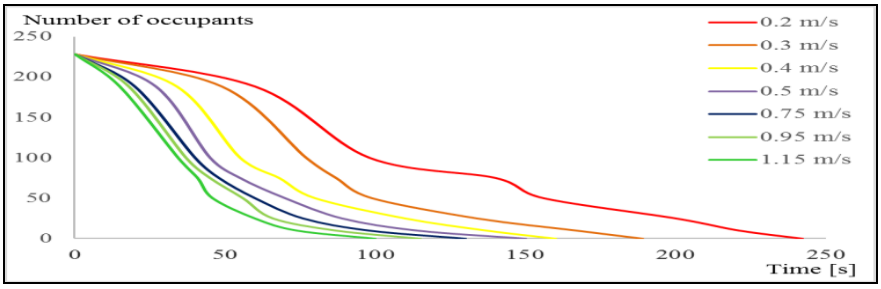

Figure 19 – Simulation results for the slowest times of the fifth case of the second scenario

Рис. 19 – Результаты моделирования самого медленного времени пятого случая по второму сценарию

Слика 19 – Симулациони резултати за најспорија времена петог случаја другог сценарија

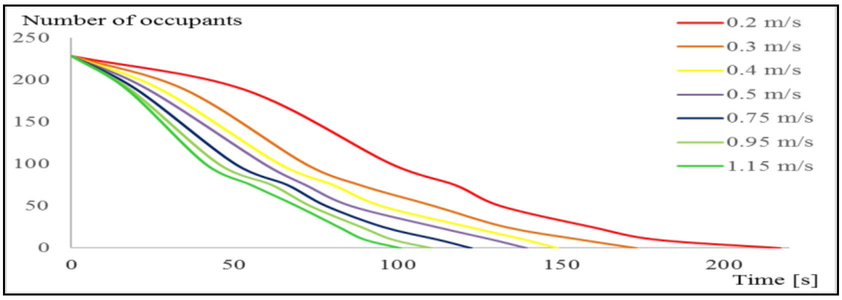

Figure 20 – Simulation results for the fastest times of the sixth case of the second scenario
Рис. 20 – Результаты моделирования скорейшего времени шестого случая по второму сценарию
Слика 20 – Симулациони резултати за најбржа времена шестог случаја другог сценарија

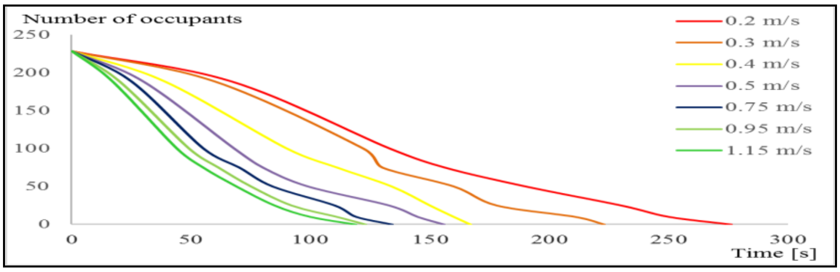

Figure 21 – Simulation results for the slowest times of the sixth case of the second scenario
Рис. 21 – Результаты моделирования самого медленного времени шестого случая по второму сценарию
Слика 21 – Симулациони резултати за најспорија времена шестог случаја другог сценарија

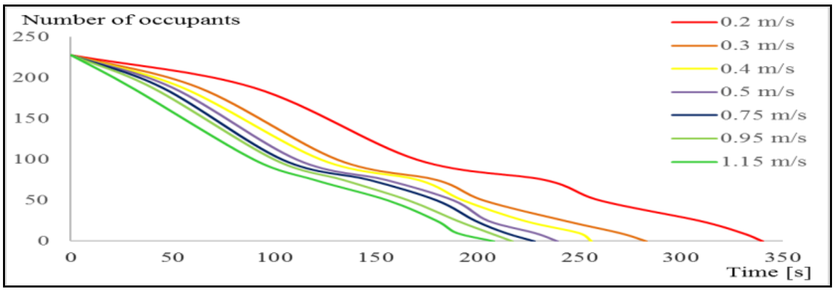

Figure 22 – Simulation results for the fastest times of the seventh case of the second scenario
Рис. 22 – Результаты моделирования скорейшего времени седьмого случая по второму сценарию
Слика 22 – Симулациони резултати за најбржа времена седмог случаја другог сценарија

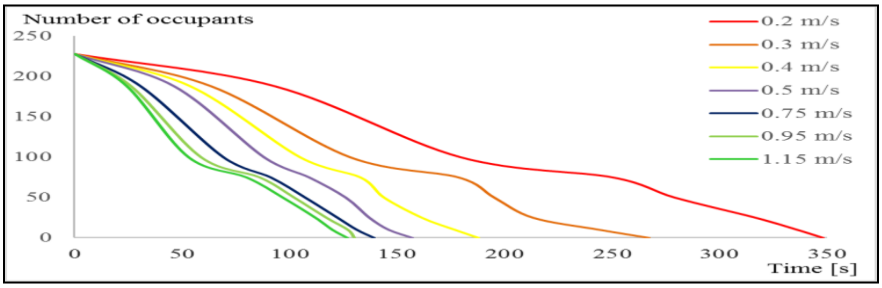

Figure 23 – Simulation results for the slowest times of the seventh case of the second scenario

Рис. 23 – Результаты моделирования самого медленного времени седьмого случая по второму сценарию

Слика 23 – Симулациони резултати за најспорија времена седмог случаја другог сценарија

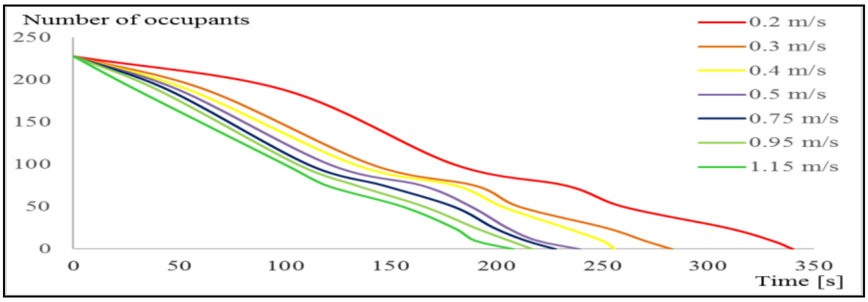

Figure 24 – Simulation results for the fastest times of the eighth case of the second scenario

Рис. 24 – Результаты моделирования скорейшего времени восьмого случая по второму сценарию

Слика 24 – Симулациони резултати за најбржа времена осмог случаја другог сценарија

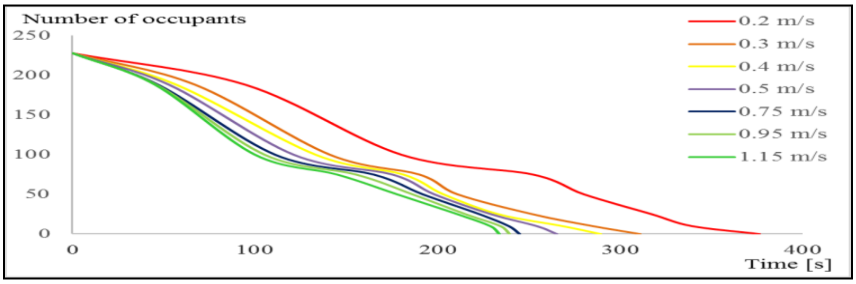

Figure 25 – Simulation results for the slowest times of the eighth case of the second scenario

Рис. 25 – Результаты моделирования самого медленного времени второго случая по второму сценарию

Слика 25 –– Симулациони резултати за најспорија времена осмог случаја другог сценарија

Analysis of the results

The total number of realised simulations was 255. The first scenario involved evacuation from the aircraft but with the use of ordinary stairs, for noted speeds of passengers/occupants. The second scenario involved evacuation with the use of emergency slides. Because of eight doors with eight emergency slides, every potential combination of opened/closed doors was simulated. Of course, due to the paper limitations, only the fastest and the slowest possibilities of seven cases of the second scenario were presented (the first case of the second scenario had only one possibility).

The realized results for the first scenario showed that the longest time needed for a complete evacuation of the aircraft was for the case where one door was opened and one staircase in use, for the passengers/occupants' speed of 0.2 m/s and it was 1137.4 seconds (Figure 8). The shortest time needed for a complete evacuation of the aircraft was for the case where three doors were opened and three stairs in use, for the passengers/occupants' speed of 1.15 m/s and it was 132.4 seconds (Figure 10). These times are in line with similar calculated and realised times on other similar aircraft types.

There were a lot of potential possibilities needed for some cases from the second scenario, such as the second, the third, the fourth, the fifth, the sixth and the seventh possibility. Because of the paper size limitations, only the fastest and the slowest possibility for every of the noted cases of the second scenario would be presented. It was necessary to analyse this huge number of potential possibilities in order to comprise all potential cases.

There was only one possibility for the first case of the second scenario where all available doors/exits were opened. The shortest time needed for a complete evacuation of the aircraft was for the passengers/occupants' speed of 1.15 m/s and it was 42.9 seconds. The longest time needed for a complete evacuation of the aircraft was for the passengers/occupants' speed of 0.2 m/s and it was 122.7 seconds (Figure 11).

There were eight possibilities for the second case of the second scenario. The shortest time needed for a complete evacuation of the aircraft was for the passengers/occupants' speed of 1.15 m/s and it was 46.6 seconds, when the third or the fourth or the fifth or the sixth door was closed (Figure 12). The longest time needed for a complete evacuation of the aircraft was for the passengers/occupants' speed of 0.2

m/s and it was 134.7 seconds, when the first or the second doors were closed (Figure 13).

There were twenty-eight possibilities for the third case of the second scenario. The shortest time needed for a complete evacuation of the aircraft was for the passengers/occupants' speed of 1.15 m/s and it was 48.8 seconds, when the first and the seventh doors were closed (Figure 14). The longest time needed for a complete evacuation of the aircraft was for the passengers/occupants' speed of 0.2 m/s and it was 178.9 seconds, when the third and the sixth doors were closed (Figure 15).

There were fifty-six possibilities for the fourth case of the second scenario. The shortest time needed for a complete evacuation of the aircraft was for the passengers/occupants' speed of 1.15 m/s and it was 64.8 seconds, when the second, the fourth and the seventh doors were closed (Figure 16). The longest time needed for a complete evacuation of the aircraft was for the passengers/occupants' speed of 0.2 m/s and it was 191.3 seconds, when the third, the fourth and sixth doors were closed (Figure 17).

There were seventy possibilities for the fifth case of the second scenario. The shortest time needed for a complete evacuation of the aircraft was for the passengers/occupants' speed of 1.15 m/s and it was 79.4 seconds, when the second, the fourth, the sixth and the seventh doors were closed (Figure 18). The longest time needed for a complete evacuation of the aircraft was for the passengers/occupants' speed of 0.2 m/s and it was 221.6 seconds, when the third, the fourth, the fifth and the sixth doors were closed (Figure 19).

There were fifty-six possibilities for the sixth case of the second scenario. The shortest time needed for a complete evacuation of the aircraft was for the passengers/occupants' speed of 1.15 m/s and it was 100.8 seconds, when the second, the fourth, the sixth, the seventh and the eighth doors were closed (Figure 20). The longest time needed for a complete evacuation of the aircraft was for the passengers/occupants' speed of 0.2 m/s and it was 276.2 seconds, when the third, the fourth, the fifth, the sixth and the seventh doors were closed (Figure 21).

There were twenty-eight possibilities for the seventh case of the second scenario. The shortest time needed for a complete evacuation of the aircraft was for the passengers/occupants' speed of 1.15 m/s and it was 127.5 seconds, when the second and the fourth doors were opened (Figure 22). The longest time needed for a complete evacuation of the aircraft was for the passengers/occupants' speed of 0.2 m/s and it was 348.25 seconds, when the first and the second doors were opened (Figure 23).

There were eight possibilities for the eighth case of the second scenario. The shortest time needed for a complete evacuation of the aircraft was for the passengers/occupants' speed of 1.15 m/s and it was 208.5 seconds, when the third door was opened (Figure 24). The longest time needed for a complete evacuation of the aircraft was for the passengers/occupants' speed of 0.2 m/s and it was 376.65 seconds, when the second door was opened (Figure 25).

It is very important to note that the speeds of passengers/occupants were the same for all of the realised simulations, which is almost impossible in real situations because of a lot of factors such as fear and panic, different anatomy, positions, group and individual behaviour and many others. As an example, fear and panic can have a significant influence on human behaviour in terms of disorder and confusion, which directly implies an increase in evacuation time, accidents and even death epilogues (Kady & Davis, 2009; Deng, 2016).

In comparison to other similar papers, it is important to add that the realised simulation results were in line with similar realised simulation results, although the jam effect on a complete evacuation times has not been taken into account. In comparison with similar simulations realised in the simulation model of the Airbus A330-300 aircraft, the evacuation times were shorter in cases when several doors/exits were available (42.9 to 48.8 seconds against 50 to 59 seconds). These results were expected because the Airbus A330-300 has 285 passengers (Choochart & Thipyopas, 2020). Also, in the comparison with similar scenarios realised on the SSJ-100 aircraft, similar results were expected and realised, taking into account all potential differences between scenarios and aircraft with slight differences (79.4 seconds against 92 seconds) (Suharev et al, 2020). Moreover, it is very important for an evacuation strategy and evacuation times to know the cause for an evacuation (fire, bomb threat, forced landing, fumes, etc.).

Conclusion

The results realised in this paper showed the evacuation times for two different scenarios with the use of stairs and emergency slides. The use of simulation software in the solutions of evacuation problems is of great significance. The most important advantages of simulation software use in evacuations are efficiency, cost-effectivenesss and safety. For some situations, of course, it is almost impossible to determine all potential evacuation routes and to calculate all evacuation times, but, with the use of this software, it is possible to calculate evacuation times for different evacuation routes and for different speeds of occupants. Also, it would be almost an impossible task to create a real model of the aircraft and test all potential scenarios with real humans as passengers with different speeds in a safe, precise and inexpensive way.

The main contributions of this paper are in calculating evacuation times for different speeds of passengers/occupants and in determining which combinations of opened/closed doors/exits are the most effective. In case that an aircraft must land immediately, many failures can occur so passengers cannot use all predicted exits. Therefore, it would be very useful in some real situations for passengers and crew members to know how to behave, organise and where and how fast to go in order to leave the aircraft in the fastest and safest way.

Future investigations would be directed towards calculations of evacuation times and predictions of evacuation routes with the presence of children, immobile or hard mobile persons, crowd and jams potentials and consequences, etc inside the aircraft. These calculations and simulations will provide proper evacuation strategies and precisely calculate potential evacuation times even in these cases, taking into account all potential factors (speed of hard mobile or immobile persons, dimensions of wheelchairs, etc).

Simulation software in evacuation presents a very important engineering tool with important advantages and its use for complex evacuation problems is mandatory.

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ЭВАКУАЦИЯ ПАССАЖИРОВ ИЗ СОВЕРШИВШЕГО ПОСАДКУ САМОЛЕТА

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РУБРИКА ГРНТИ 73.37.17 Безопасность полетов воздушных судов
73.01.92 Пожарная безопасность
ВИД СТАТЬИ: оригинальная научная статья

Резюме:

Введение/цель: Самолеты являются одним из самых популярных и безопасных видов транспорта. Но несмотря на то, что регулярно предпринимаются различные меры безопасности, несчастные случаи все же случаются. Если самолет каким-то образом произвел посадку, независимо от степени его повреждения, главной задачей является быстрая и безопасная эвакуация пассажиров из самолета. Эвакуация пассажиров является очень сложной и ответственной задачей в силу различных факторов, таких как степень повреждения, наличие огня, скорость пассажиров, паника, страх и пр. Поэтому *чрезвычайно важно предусмотреть возможные способы эвакуации, возможные стратегии и маршруты эвакуации. Состояние приземлившегося самолета может быть разным, поэтому необходимо произвести быструю и безопасную эвакуацию пассажиров.*

Методы: В данной статье для расчета необходимого времени эвакуации и возможных маршрутов эвакуации применен метод имитационного моделирования. Моделирование проводилось в программе для моделирования эвакуации в чрезвычайных ситуациях Pathfinder.

Результаты: Исследование проведено на соответствующей имитационной модели самолета с трапами и эвакуационными горками. Результаты исследования показали время эвакуации по двум разным сценариям, с учетом различной скорости реагирования пассажиров.

Выводы: Использование программного обеспечения для моделирования эвакуации является весьма эффективным способом с точки зрения безопасности, стоимости и предсказуемости. Этот тип программного обеспечения является обязательным инженерным инструментом для эффективного и точного решения эвакуационных и аналогичных задач. Цель данной статьи заключается в представлении возможных сценариев эвакуации пассажиров из совершившего посадку самолета.

Ключевые слова: эвакуация, самолет, моделирование, пассажир.

ЕВАКУАЦИЈА ПУТНИКА ИЗ АВИОНА НА ЗЕМЉИ

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ОБЛАСТ: авијација, безбедност и здравље људи
КАТЕГОРИЈА (ТИП) ЧЛАНКА: оригинални научни рад

Сажетак:

Увод: Авиони представљају једна од најкоришћенијих и најбезбеднијих саобраћајних средстава. Ипак, и поред предузимања многих сигурносних мера, несреће се дешавају. Без обзира на степен оштећења, уколико авион ипак слети, најважније је да се путници брзо и безбедно евакуишу. То је веома комплексан и захтеван задатак због неколико различитих фактора, као што су степен оштећења, присуство ватре, брзина путника, присуство панике и страха итд. Зато је веома важно да се предвиде могући начини евакуације, као и евакуационе стратегије и руте. Авион *који је слетео може бити у различитим стањима, тако да је брза и сигурна евакуација путника веома важна.*

Методе: За прорачун потребних времена евакуације и могућих евакуационих рута коришћен је симулациони метод. Симулације су реализоване у симулационом софтверу Pathfinder.

Резултати: Резултати овог рада реализовани су на одговарајућем симулационом моделу авиона са степеницама и тобоганима и показали су евакуациона времена за два различита сценарија, за различите брзине путника – окупаната.

Закључак: Употреба симулационог софтвера у евакуационим ситуацијама врло је ефикасна. Ова врста софтвера је сигурна, јефтина и има могућност предвиђања. Представља обавезан инжењерски алат за ефикасније и прецизније решавање евакуационих и сличних проблема.

Кључне речи: евакуација, авион, симулација, путник.

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ПРЕГЛЕДНИ РАДОВИ
ОБЗОРНЫЕ СТАТЬИ
REVIEW PAPERS

PATH INTEGRAL IN QUANTUM FIELD THEORIES

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FIELD: Mathematics
ARTICLE TYPE: Review paper

Abstract:

Introduction/purpose: Starting from the Hamiltonian an alternative description of quantum mechanics has been given, based on the sum of all possible paths between an initial and a final point.

Methods: Theoretical methods of mathematical physics. Integral method based on the path integral.

Results: The method and concepts of the path integral could be applied to other branches of physics, not limited to quantum mechanics.

Conclusions: The Path Integral approach gives a global description of fields, unlike the usual Lagrangian approach which is a local description of fields.

Key words: path integral, quantum mechanics, quantum field theory.

Path integral

The standard formulations of quantum mechanics, developed by Schrödinger, Heisenberg and others in the 20-ies, have shown to be equivalent to one another soon thereafter.

In 1933, Dirac (Dirac, 1933) made the observation that the action plays a central role in classical mechanics – he considered the Lagrangian formulation of classical mechanics to be more fundamental than the Hamiltonian one, but it seemed to have no important role in quantum mechanics as it was known at the time. He arrived at the conclusion that the propagator in quantum mechanics “corresponds to” $e^{(i/\hbar)S}$, where S is the classical action evaluated along the classical path.

In 1948, Feynman developed Dirac's suggestion (Feynman, 1948), and succeeded in deriving the third formulation of quantum mechanics, based on the fact that the propagator can be written as a sum over all possible paths, not just the classical one, between the initial and final points (Feynman, 1950, 1951). Each path contributes $e^{(i/\hbar)S}$ to the propagator. So while Dirac considered only the classical path, Feynman showed that all paths contribute: in a sense, the quantum particle takes all paths and the amplitudes for each path add according to the usual quantum mechanical rule for combining amplitudes.

This discovery remains valid even for relativistic quantum mechanics, represented by quantum field theory. While the usual Lagrangian approach is a local description, the path integral approach corresponds to a global description of fields, being integrated over all possible configurations.

Young's experiment

Suppose to create a *Gedankenexperiment* inspired by the original Young's two slit diffraction experiment (Feynman & Hibbs, 1965). A source S emits non classical particles (for instance, electrons) that end on a detector sited in O. In between, there is a screen with two slits, A_1 and A_2. The source emits particles at the time $t = 0$ that are detected at the time $t = T$. From quantum mechanics, we know that because of the superposition principle, the amplitude of particle detection is obtained by summing over all possible amplitudes, that is, the amplitude of travelling through the slit A_1 to O, and the amplitude of travelling through the slit A_2, namely

$$\mathcal{A}(\text{Starting from } S, \text{ detected at } \mathcal{O}) = \sum_{i=1}^{2} \mathcal{A}(S \to A_i \to O) \ , \tag{1}$$

and of course one sums over different A_is when having more slits than two.

Add now another screen between A and O, with slits B_i. Then another one between B and O with slits C_i and so on. We have to add all these intermediate steps, so in the limit of infinite screens with the infinite number of slits we have the relation

$$\mathcal{A}(\text{From } S \text{ detected at } \mathcal{O} \text{ travelling in the time } T) =$$

$$\sum_{path} \mathcal{A}(S \to O \text{ in the time } T \text{ for a particular path}) \,, \tag{2}$$

so we have to sum over all possible paths that start from S and end in O in the time T.

We shall now fully translate eq. (2) in the quantum mechanics language. Remember that the Hamiltonian H is the generator of time translations, so the amplitude to propagate from an initial point q_I to a final point q_F in a time T is given by

$$\langle q_F \left| e^{-iTH} \right| q_I \rangle \,. \tag{3}$$

Dirac suggested, and Feynman first used eq. (3) to obtain an expression for eq. (2) by splitting each path into infinitesimal elements and then taking the continuum limit.

Divide the time T in N parts each lasting $\delta t = T/N$, then eq. (3) could be rewritten as

$$\langle q_F \left| e^{-iTH} \right| q_I \rangle = \langle q_F \left| e^{-i\delta tH} e^{-i\delta tH} \ldots e^{-i\delta tH} \right| q_I \rangle \,, \tag{4}$$

the term $e^{-i\delta tH}$ being repeated N times. Now use the fact that $|q\rangle$ is a complete set of states, that is, $\int \mathrm{d}q/(2\pi)^{1/2} \, |q\rangle\langle q| = 1$, and insert 1 between every exponential factor $\exp(-i\delta tH)$:

$$\begin{aligned} &\langle q_F \left| e^{-iTH} \right| q_I \rangle = \\ &\left(\prod_{j=1}^{N-1} \int \frac{\mathrm{d}q_j}{\sqrt{2\pi}} \right) \langle q_F \left| e^{-i\delta tH} \right| q_{N-1} \rangle \langle q_{N-1} \left| e^{-i\delta tH} \right| q_{N-2} \rangle \ldots \\ &\langle q_2 \left| e^{-i\delta tH} \right| q_1 \rangle \langle q_1 \left| e^{-i\delta tH} \right| q_I \rangle \,. \end{aligned} \tag{5}$$

Feynman's formulation of quantum mechanics

The key ingredient of eq. (5) is the factor $\langle q_{j+1} \left| e^{-i\delta tH} \right| q_j \rangle$. From quantum mechanics we know the explicit form of the Hamiltonian function,

$$H = \frac{\hat{p}^2}{2m} + V(\hat{q}) \,, \tag{6}$$

where $\hat{p}, \hat{q}$ are the usual operators with eigenspace $\hat{p}|p\rangle = p\,|p\rangle$, $\hat{q}|q\rangle = q|q\rangle$. Since the spaces q and p are connected via a Fourier transformation, they have the property that $\langle q|p\rangle = e^{ipq}$, $\langle p|q\rangle = e^{-ipq}$, and the p space is complete as well as the q space: $\int \mathrm{d}p/(2\pi)^{1/2}\ |p\rangle\langle p| = 1$. From the explicit form of the Hamiltonian (6),

$$e^{-i\delta tH} = e^{-i\delta t\hat{p}^2/2m} e^{-i\delta tV(\hat{q})} \ , \tag{7}$$

and by a judicious insertion of factors 1 coming from the completeness of the q and p spaces we find

$$\langle q_{j+1} \left| e^{-i\delta tH} \right| q_j\rangle = \frac{1}{2\pi}\int \mathrm{d}q \int \mathrm{d}p\ \langle q_{j+1} \left| e^{-i\delta t\hat{p}^2/2m} \right| p\rangle\langle p \left| e^{-i\delta tV(\hat{q})} \right| q\rangle\langle q \mid q_j\rangle\ . \tag{8}$$

It is clear that for any function f, $f(\hat{q})|q\rangle = f(q)|q\rangle$ and $f(\hat{p})|p\rangle = f(p)|p\rangle$, because it is acting on eigenstates. Therefore, we could drop the symbol of the operator in eq. (8) and write

$$\begin{aligned}\langle q_{j+1} \left| e^{-i\delta tH} \right| q_j\rangle &= \\ \frac{1}{2\pi}\int \mathrm{d}q \int \mathrm{d}p\ e^{-i\delta tp^2/2m} e^{-i\delta tV(q)}\langle q_{j+1} \mid p\rangle\langle p \mid q\rangle\langle q \mid q_j\rangle &= \\ \int \mathrm{d}q \int \mathrm{d}p\ e^{-i\delta tp^2/2m} e^{-i\delta tV(q)} e^{ipq_{j+1}} e^{-ipq}\delta(q-q_j) &= \\ e^{-i\delta tV(q_j)}\int \mathrm{d}p\ e^{-i\delta tp^2/2m} e^{ip(q_{j+1}-q_j)}\ .\end{aligned} \tag{9}$$

We could readily recognise that the last integral over p is Gaussian and can be solved with the aid of eq. (57) of (Fabiano, 2021a):

$$\begin{aligned}\langle q_{j+1} \left| e^{-i\delta tH} \right| q_j\rangle &= \\ e^{-i\delta tV(q_j)}\int \mathrm{d}p\ e^{-i\delta tp^2/2m} e^{ip(q_{j+1}-q_j)} &= e^{-i\delta tV(q_j)}\left(\frac{-2\pi im}{\delta t}\right)^{1/2} \times \\ e^{[im(q_{j+1}-q_j)^2]/2\delta t} &= e^{-i\delta tV(q_j)}\left(\frac{-2\pi im}{\delta t}\right)^{1/2} e^{i\delta t(m/2)[(q_{j+1}-q_j)/\delta t]^2}\ .\end{aligned} \tag{10}$$

Putting this result into eq. (5) gives us

$$\langle q_F \left| e^{-iTH} \right| q_I\rangle =$$

$$\left(\frac{-2\pi i m}{\delta t}\right)^{N/2} \prod_{j=0}^{N-1} \int \mathrm{d}q_j \; e^{i\delta t\left\{(m/2)\sum_{j=0}^{N-1}[(q_{j+1}-q_j)/\delta t]^2 - V(q_j)\right\}} \; , \qquad (11)$$

where $q_0 \equiv q_I$ and $q_N \equiv q_F$. We can now go to the continuum limit, that is, $\delta t \to 0$ or $N \to +\infty$, so we can replace $[(q_{j+1} - q_j)/\delta t]^2$ with $\dot{q}^2$ and sums with integrals.

A very important definition is the integral over paths:

$$\int \mathcal{D}q(t) \;\equiv \lim_{N\to+\infty} \left(\frac{-2\pi i m}{\delta t}\right)^{N/2} \prod_{j=0}^{N-1} \int \mathrm{d}q_j \; , \qquad (12)$$

where the $\mathcal{D}$ symbol means that one has to integrate over all possible paths $q(t)$ with fixed start and ending points, $q(0) = q_I$ and $q(T) = q_F$. It is a functional integration.

We have thus obtained the so called *path integral* representation for the amplitude:

$$\langle q_F \left| e^{-iTH} \right| q_I \rangle = \int \mathcal{D}q(t) \; e^{i\int_0^T \mathrm{d}t \; \frac{1}{2}m\dot{q}^2 - V(q)} = \int \mathcal{D}q(t) \; e^{i\int_0^T \mathrm{d}t \; \mathcal{L}(q,\dot{q})} \; . \qquad (13)$$

Comparing both sides of eq. (13), one could notice that starting from the Hamiltonian we have naturally ended up with the Lagrangian. In classical mechanics, the action S is defined starting from the Lagrangian as $S(q) = \int_0^T \mathrm{d}t \; \mathcal{L}(q,\dot{q})$, and is a functional of $q(t)$. By restoring Planck's constant $\hbar$ and by dropping the explicit t notation for the functional measure, we could rewrite eq. (13) as

$$\langle q_F \left| e^{-(i/\hbar)TH} \right| q_I \rangle = \int \mathcal{D}q \; e^{(i/\hbar)S(q)} \; . \qquad (14)$$

It is worth noticing that the quantum mechanical amplitude of eq. (14) involves the explicit calculation of the classical action S. The path integral is the only occurrence where the action is explicitly needed, where in all other cases only the extremisation of the action, that is, the equations of motion, are required.

Schrödinger equation

Our next step is to derive the Schrödinger equation by means of path integral formalism. Since it is a differential equation we need only to find out

Fabiano, N., Path integral in quantum field theories, pp.993–1016

the infinitesimal evolution of the wave function in time and space. Setting the initial conditions as $t_I = 0$, $q_I = q'$, $t_F = t$, $q_F = q$; $\delta t = t$ and $\eta = q' - q$ are infinitesimal. The time and space evolution for the wave equation from the point $(0, q')$ to the point (q, t) is given by

$$\psi(q,t) = \int_{-\infty}^{+\infty} \mathrm{d}q' \ K(q,t;q',0)\psi(q',0) \ , \tag{15}$$

where K is the evolution amplitude with proper normalisation, as $|\psi|^2 = 1$. From eq. (10), we have the explicit form for a propagation amplitude between two points, so restoring $\hbar$ we can write

$$K(q,\delta t;q',0) = \left(\frac{m}{2\pi i\hbar\delta t}\right)^{1/2} e^{i\delta t/\hbar\left\{(m/2)[(q-q')/\delta t]^2 - V(q')\right\}} \ . \tag{16}$$

By changing the integration variable to $\eta = q' - q$ and reinserting eq. (16) into eq. (15), we obtain

$$\psi(q,\delta t) = \left(\frac{m}{2\pi i\hbar\delta t}\right)^{1/2} \int_{-\infty}^{+\infty} \mathrm{d}\eta e^{i\delta t/\hbar\left\{(m/2)[\eta/\delta t]^2 - V(q+\eta)\right\}} \psi(q+\eta,0) \ . \tag{17}$$

Now, we have two infinitesimal quantities, η and δt. Because of the speed of light, we have the limit $\eta/\delta t < 1$ and both are infinitesimals of the same order. So we can expand the potential and the wave function at the same time

$$\begin{aligned} e^{-i\delta t/\hbar V(q+\eta)} &= 1 - i\frac{\delta t}{\hbar}\left[V(q) + \eta V'(q) + \mathcal{O}(\eta^2)\right] = \\ & 1 - i\frac{\delta t}{\hbar}V(q) - i\frac{\delta t}{\hbar}\eta V'(q) + \mathcal{O}(\delta t^2, \eta^2) \ , \end{aligned} \tag{18}$$

and

$$\psi(q+\eta) = \psi(q,0) + \eta\psi'(q,0) + \frac{1}{2}\eta^2\psi''(q,0) + \mathcal{O}(\eta^3) \ . \tag{19}$$

Plugging Taylor expansions back in eq. (17) yields

$$\begin{aligned} \psi(q,\delta t) &= \left(\frac{m}{2\pi i\hbar\delta t}\right)^{1/2} \int_{-\infty}^{+\infty} \mathrm{d}\eta e^{im\eta^2/(2\hbar\delta t)} \times \\ & \left[\psi(q,0) - i\frac{\delta t}{\hbar}V(q)\psi(q,0) + \eta\psi'(q,0) + \frac{1}{2}\eta^2\psi''(q,0) + \mathcal{O}(\delta t^2,\eta^3)\right] \ . \end{aligned} \tag{20}$$

By inspection, the integral in η is reduced to Gaussian momenta given in eq. (58) of (Fabiano, 2021a), where linear terms vanish because of symmetry. By resolving integrals, we obtain

$$\psi(q,\delta t) = \left(\frac{m}{2\pi i\hbar\delta t}\right)^{1/2}\left[\left(\frac{2\pi i\hbar\delta t}{m}\right)^{1/2}\left(\psi(q,0) - i\frac{\delta t}{\hbar}V(q)\psi(q,0)\right) + \left(\frac{2\pi i\hbar\delta t}{m}\right)^{1/2}\frac{i\hbar\delta t}{2m}\psi''(q,0) + \mathcal{O}(\delta t^3)\right] = \psi(q,0) + \delta t\left[\frac{i\hbar}{2m}\psi''(q,0) - i\frac{1}{\hbar}V(q)\psi(q,0)\right] + \mathcal{O}(\delta t^2)\ . \tag{21}$$

After moving the first term $\psi(q,0)$ to lhs and dividing it by δt, we obtain

$$\frac{\psi(q,\delta t) - \psi(q,0)}{\delta t} = -\frac{i}{\hbar}\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial q^2} + V(q)\right]\psi(q,0) + \mathcal{O}(\delta t^2)\ , \tag{22}$$

and by taking the limit $\delta t \to 0$ we obtain the time dependent Schrödinger equation

$$i\hbar\frac{\partial\psi(q,t)}{\partial t} = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial q^2} + V(q)\right]\psi(q,t)\ . \tag{23}$$

Relativistic field theory

Instead of dealing with fixed initial and final positions q_I and q_F we are often faced with specifying more general initial and final states $|I\rangle$ and $|F\rangle$. Then we are interested in calculating $\langle F|e^{-iTH}|I\rangle$, that can be obtained from eq. (13) by inserting two complete sets of states

$$\langle F|e^{-iTH}|I\rangle = \int\int \mathrm{d}q_F\, \mathrm{d}q_I\, \langle F|q_F\rangle\langle q_F\left|e^{-iTH}\right|q_I\rangle\langle q_I|I\rangle\ . \tag{24}$$

Almost always initial and final states are the same, that is, the ground state $|0\rangle$. The amplitude $\langle 0|e^{-iTH}|0\rangle$ is denoted by Z,

$$Z \equiv \langle 0|e^{-iTH}|0\rangle\ , \tag{25}$$

because *Zustandssumme*, that is, the "sum over states" was the original German term for the *partition function*.

The path integral formalism can be extended from quantum mechanics to continuum field theories that describe physical systems with an infinite numbers of degrees of freedom. Starting from $q(t)$, a $0+1$ dimensional case for quantum mechanics (we have just discretised the time coordinate in section Young's experiment) to a field theory in $1+1$ dimensions for simplicity, $\phi(x,t)$, the procedure is completely analogue. The new step is the space discretisation - the length L of space has to be divided in infinitesimal parts δx such that

$$\delta x = \frac{L}{N'} \; , \tag{26}$$

and by denoting the coordinate as $x_m = m\delta x$, with $\phi(x_m) = \phi_m$ for $0 \leq m \leq N'$ we can define the functional integral over the field ϕ like:

$$\int \mathcal{D}\phi \; \equiv \lim_{L\to+\infty} \lim_{N'\to+\infty} \prod_{m=0}^{N'} \int \mathrm{d}\phi_m \; , \tag{27}$$

in complete analogy to eq. (12). The action is now of course a function of ϕ and $\partial_\mu\phi$, $S(\phi) = \int \mathrm{d}^D x_\mu \; \mathcal{L}(\phi, \partial_\mu\phi)$.

An essential difference from the quantum mechanical case, however, is that, from a mathematically rigorous point of view, the integral just defined in eq. (27) is divergent in the continuum limit. This difficulty is obviated by absorbing the divergence into a normalisation constant N when computing quantities such as, for instance, the partition function of eq. (25):

$$Z = N \int \mathcal{D}\phi \; e^{(i/\hbar)S(\phi)} \; . \tag{28}$$

From this expression for Z, we see that the integral in the classical limit $\hbar \to 0$ is given by a phase S multiplied by a large quantity, that is, a rapidly oscillating quantity. Mathematically, it is clear that the major contribution to the path integral comes from fields that extremise the action, while other configurations tend to cancel each other by symmetry. Those fields are the ones that satisfy

$$\frac{\delta S(\phi)}{\delta \phi} = 0 \; , \tag{29}$$

and such fields are by definition classical fields ϕ_{cl} that solve Lagrange equations

$$\partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi_{cl})} = \frac{\delta \mathcal{L}}{\delta \phi_{cl}} \; . \tag{30}$$

To prove this statement, we will use the so-called *saddle point method* or *stationary phase method* that applies when the integral could be written as some exponential function. For a review on the subject, see, for instance (Fabiano & Mirkov, 2022). We can expand the action in series to read

$$S(\phi) = S(\phi_{cl}) + \frac{1}{2}\left(\frac{\delta^2 S(\phi_{cl})}{\delta\phi^2}\right)(\phi - \phi_{cl})^2 + \mathcal{O}((\phi - \phi_{cl})^3)\ , \quad (31)$$

where the linear term is missing by definition because the action is stationary. By plugging this result back into eq. (28) we have, yet another time, a Gaussian integral in infinite dimensions that can be solved with the aid of eq. (60) of (Fabiano, 2021a):

$$Z = Ne^{(i/\hbar)S(\phi_{cl})}\left[\frac{2\pi i\hbar}{\det[S''(\phi_{cl})]}\right]^{1/2}[1 + \mathcal{O}(\hbar)]\ , \quad (32)$$

and it is clear that the exponential term is the essential contribution as $\hbar \to 0$.

Free field

We begin with the Lagrangian

$$\mathcal{L} = \frac{1}{2}\left[(\partial\phi)^2 - m^2\phi^2\right]\ , \quad (33)$$

that describes the so called free or Gaussian theory. The equation of motion is the well–known Klein–Gordon equation describing a relativistic boson particle of the mass m

$$(\Box + m^2)\phi = 0\ , \quad (34)$$

where $\Box = \partial^\mu\partial_\mu$ is the d'Alembert operator, with a plane wave solution $\phi(x,t) = e^{i(\omega t - \vec{k}\cdot\vec{x})}$ and a dispersion relation $\omega^2 = \vec{k}^2 + m^2$. Before writing the amplitude, it is customary to add a term like $J(x)\phi(x)$ in the Lagrangian, where $J(x)$ is the so-called *source function* whose actual form is not relevant, provided integrals are convergent, as will be clear later. We have

$$Z = N\int \mathcal{D}\phi\ e^{i\int \mathrm{d}x^4\ \left\{\frac{1}{2}[(\partial\phi)^2 - m^2\phi^2] + J\phi\right\}}\ , \quad (35)$$

and focussing on the action integrating by parts, and provided the fields ϕ fall off sufficiently rapidly at infinity, we could rewrite it as

$$\int \mathrm{d}x^4\ \frac{1}{2}[(\partial\phi)^2 - m^2\phi^2] + J\phi = \int \mathrm{d}x^4\ \left[-\frac{1}{2}\phi(\Box^2 + m^2)\phi^2 + J\phi\right]\ . \quad (36)$$

Fabiano, N., Path integral in quantum field theories, pp.993–1016

By putting this new form back into eq. (35)

$$Z = N \int \mathcal{D}\phi \; e^{i\int \mathrm{d}x^4 \; \left\{-\frac{1}{2}\phi(\Box^2+m^2)\phi^2+J\phi\right\}} \; , \tag{37}$$

one obtain once again a Gaussian integral, quite similar to the one of eq. (57) of (Fabiano, 2021a). This time, however, a and b are not numbers, but matrices. Consider the generalisation of the Gaussian integral to matrices, then we have

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}\prod_{i=1}^{N}\mathrm{d}x_i \; e^{-\frac{1}{2}\mathbf{x}\cdot A\cdot\mathbf{x}+J\cdot\mathbf{x}} = \left[\frac{(2\pi)^N}{\det(A)}\right]^{\frac{1}{2}} e^{\frac{1}{2}J\cdot A^{-1}\cdot J} \; , \tag{38}$$

where $\mathbf{x}\cdot A\cdot\mathbf{x} = x_i A_{ij} x_j$ and $J\cdot\mathbf{x} = J_{ij}x_j$, with repeated indices summed over. To prove eq. (38), diagonalise A by an orthogonal transformation $O : A = O^{-1}\cdot D\cdot O$, where D is a diagonal matrix with elements given by all eigenvalues of A. This operation is always possible because A is a definite positive matrix, otherwise the integral would not converge. Define a new variable $\mathbf{y} = O\cdot\mathbf{x}$, that is, $y_i = O_{ij}x_j$, then the exponential will reduce to a sum of squares:

$$\mathbf{x}\cdot A\cdot\mathbf{x} = x_i A_{ij} x_j = \mathbf{y}\cdot O^{-1}\cdot A\cdot O\cdot\mathbf{y} = \mathbf{y}\cdot D\cdot\mathbf{y} = y_i D_{ii} y_i \; . \tag{39}$$

The Jacobian of such transformation is 1 by definition, so eq. (38) reduces to a product of one dimensional Gaussian integrals, which proves the formula for $J = 0$. If not a further step is needed, a variable translation defined as $\mathbf{y}' = \mathbf{y} + A^{-1}J$, which again does not change the integration measure, $\mathrm{d}y' = \mathrm{d}y$.

Coming back to eq. (37), the role of A is here played by the differential operator $-(\Box + m^2)$. Its inverse is given by the function D that obeys

$$-(\Box + m^2)D(x-y) = \delta^{(4)}(x-y) \; , \tag{40}$$

because, since we are dealing with the continuum, Kronecker's delta δ_{ij} for the definition of inverse operators A^{-1}_{jk} have to be replaced by Dirac's delta functions $\delta^{(4)}(x-y)$. The resulting function D is the well–known free propagator for a scalar relativistic particle of the mass m, here written as a less familiar function of the coordinates x instead of its more popular Fourier transform.

We end up with

$$Z(J) = Ce^{-(i/2)\int\int \mathrm{d}^4x\,\mathrm{d}^4y\; J(x)D(x-y)J(y)} \equiv Ce^{iW(J)} \;, \tag{41}$$

where $D(x-y)$ obeys eq. (40). The overall normalisation factor C clearly does not depend on J, but on the determinant of D which has no interest. Observe that $C = Z(J=0)$ so that

$$Z(J) \equiv Z(J=0)e^{iW(J)} \;, \tag{42}$$

and

$$W(J) = -\frac{1}{2}\int\int \mathrm{d}^4x\,\mathrm{d}^4y\; J(x)D(x-y)J(y) \tag{43}$$

is only quadratic in J, while $Z(J)$ depends on arbitrarily high powers of J.

Green functions

By going in momentum space, eq. (40) is easily solvable (Schwinger, 1951). Remembering the Dirac delta function in momentum space

$$\delta^{(4)}(x-y) = \int \frac{\mathrm{d}^4k}{(2\pi)^4}\; e^{ik(x-y)} \;, \tag{44}$$

one obtains

$$D(x-y) = \int \frac{\mathrm{d}^4k}{(2\pi)^4}\; \frac{e^{ik(x-y)}}{k^2 - m^2 + i\epsilon} \;. \tag{45}$$

With the help of eqs. (59) and (61) of (Fabiano, 2021a), it is possible to obtain the explicit form for $D(x)$ in Euclidean space. By rewriting the denominator with eq. (61) of (Fabiano, 2021a) and computing the Gaussian integral, we obtain

$$D(x) = \frac{1}{2^D\pi^{D/2}}\int_0^{+\infty} \mathrm{d}t\; t^{-D/2}e^{-\frac{x^2}{4t}-tm^2} = \frac{1}{(2\pi)^{(D/2)}}\frac{K_{(D-2)/2}(|x|)}{|x|^{(D-2)/2}}m^{(D-2)/2} \;, \tag{46}$$

where $K_\nu(x)$ is a Bessel function. For a half integer argument, Bessel functions reduce to elementary functions, for example in $D = 1$

$$D(x) = \frac{1}{2m} e^{-m|x|} \ , \tag{47}$$

while for $D = 3$

$$D(x) = \frac{1}{4\pi|x|} e^{-m|x|} \ . \tag{48}$$

Equation (46) can be used to obtain asymptotic behaviours for $D(x)$; one finds for $|x| \to +\infty$

$$D(x) = \left(\frac{\pi}{2}\right)^{1/2} (2\pi)^{-D/2} |x|^{(-D/2+1/2)} m^{(D/2-3/2)} e^{-m|x|} \ , \tag{49}$$

while for $|x| \to 0$ we have

$$D(x) = \frac{1}{4}\pi^{-D/2}\Gamma\left(\frac{D}{2} - 1\right) |x|^{-D+2} \ \text{ for } D > 2 \ , \tag{50}$$

and

$$D(x) = (4\pi)^{-D/2}\Gamma\left(1 - \frac{D}{2}\right) m^{D-2} \ \text{ for } D < 2 \tag{51}$$

respectively.

We shall see next the importance of the source function. From eq. (35), we see how a functional derivative in $J(x)$ will furnish us the expectation value of the field. Using the fact that

$$\frac{\delta}{\delta J(x)} J(y) = \delta^{(4)}(x - y) \ , \tag{52}$$

as per definition of a functional derivative, from $Z(J)$ we could obtain the propagator, or the time ordered two point function as

$$-iG(x - y) = -iG(x, y) = \langle 0 \,| T\phi(x)\phi(y) |\, 0\rangle = \frac{\delta}{\delta J(x)} \frac{\delta}{\delta J(y)} Z(J)\bigg|_{J=0} \ . \tag{53}$$

It is straightforward to generalise this expression to an n–point function:

$$G(x_1, x_2, \ldots, x_n) = i^n \langle 0 \,| T\phi(x_1)\phi(x_2)\ldots\phi(x_n) |\, 0\rangle = \frac{\delta^n Z(J)}{\delta J(x_1)\delta J(x_2)\ldots\delta J(x_n)}\bigg|_{J=0} \ . \tag{54}$$

Explicitly calculating the four point function yelds:

$$
\begin{aligned}
-\langle 0\,|T\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)|\,0\rangle &= -\left.\frac{\delta^4 Z(J)}{\delta J(x_1)\delta J(x_2)\delta J(x_3)\delta J(x_4)}\right|_{J=0}\\
&= G(x_1-x_2)G(x_3-x_4)+G(x_1-x_3)G(x_2-x_4)+\\
&\qquad G(x_1-x_4)G(x_2-x_3)\;, \qquad (55)
\end{aligned}
$$

the sum of all possible combinations of x_i comes out because of the functional derivative that sports also a Dirac's delta. In this manner, we have derived Wick's theorem on contractions starting purely with c–numbers expressions.

$Z(J)$ can also be written as a power series in J. Calling

$$
Z^{(n)}(x_1,x_2,\ldots,x_n) \equiv \left.\frac{\delta^n Z(J)}{\delta J(x_1)\delta J(x_2)\ldots\delta J(x_n)}\right|_{J=0}\;, \qquad (56)
$$

and noting besides the equivalence with eq. (54), we can write

$$
Z(J) = \sum_{n=0}^{+\infty}\frac{1}{n!}\int\ldots\int \mathrm{d}x_1\ldots\mathrm{d}x_n\; J(x_1)\ldots J(x_n)Z^{(n)}(x_1,\ldots,x_n)\;. \qquad (57)
$$

In this manner, we have shown that path integral formalism can rederive all the expressions earlier known of canonical formalism without using operators algebra.

Connected graphs

When analysing Feynman graphs, there are two distinct types of diagrams: *connected* and *disconnected graphs* (Coleman, 1985): the latter can be separated into two, or more, distinct parts without cutting a line; not so for the former. For instance, a propagator is a connected graph.

$Z(J)$ is also known as the generating functional, and it generates both types of Feynman diagrams described above. However, in a variety of physical problems, for example renormalisation theory, and statistical mechanics, it is useful to generate only connected graphs. Also, the scattering amplitude receives contribution only from connected diagrams. We have already defined such generating functional in eq. (42), called $W(J)$. By

Fabiano, N., Path integral in quantum field theories, pp.993–1016

neglecting the normalisation constants, we have the relation[1]

$$W(J) = -i \log Z(J) \tag{58}$$

among two generating functionals. By taking repeated derivatives with respect to J, we find

$$\frac{\delta^2 W}{\delta J(x_1)\delta J(x_2)} = \frac{i}{Z^2}\frac{\delta Z}{\delta J(x_1)}\frac{\delta Z}{\delta J(x_1)} - \frac{i}{Z^2}\frac{\delta^2 Z}{\delta J(x_1)\delta J(x_1)} \; , \tag{59}$$

and

$$\frac{\delta^4 W}{\delta J(x_1)\delta J(x_2)\delta J(x_3)\delta J(x_4)} = \left(\frac{i}{Z^2}\frac{\delta^2 Z}{\delta J(x_1)\delta J(x_2)}\frac{\delta^2 Z}{\delta J(x_3)\delta J(x_4)} + \right.$$
$$\left. \text{permutations}\right) - \frac{i}{Z}\frac{\delta^4 Z}{\delta J(x_1)\delta J(x_2)\delta J(x_3)\delta J(x_4)} \; . \tag{60}$$

Following the Taylor expansion of eq. (57), we could write an analogous series for W:

$$W(J) = \sum_{n=0}^{+\infty}\frac{1}{n!}\int\ldots\int \mathrm{d}x_1\ldots\mathrm{d}x_n \; J(x_1)\ldots J(x_n) W^{(n)}(x_1,\ldots,x_n) \; . \tag{61}$$

By taking $J = 0$ and comparing the two series, we arrive at:

$$iW^{(2)}(x_1,x_2) = Z^{(2)}(x_1,x_2) \; , \tag{62}$$

rather unsurprising as the propagator is connected. To higher orders, however, the relations becomes non–trivial:

$$W^{(4)}(x_1,x_2,x_3,x_4) = i\left[Z^{(2)}(x_1,x_2)Z^{(2)}(x_3,x_4) + \text{ permutations }\right] +$$
$$Z^{(4)}(x_1,x_2,x_3,x_4) \; . \tag{63}$$

It is possible to prove that W generates only connected graphs to all orders, that is, that $W^{(n)}$ is the n–point connected Green function.

[1]Observe the similarity of W to the free energy.

Effective action

Besides connected and disconnected diagrams, there is another important class of Feynman graphs, the *one particle irreducible* (1PI) diagrams. These diagrams cannot be disconnected by cutting any internal line. In other terms, one cannot obtain two Feynman diagrams by cutting a line of the 1PI diagram. Sometimes they are also known as *strongly connected* diagrams, because they are basically diagrams connected by more than one line.

They have a generating functional called *effective action*, defined by a Legendre transformation (Coleman, 1985)

$$\Gamma(\phi) = W(J) - \int \mathrm{d}^4x \; J(x)\phi(x) \; . \tag{64}$$

The fields ϕ and J have a duality relation among them, like p and $\dot{q}$ coordinates in Hamiltonian and Lagrangian formalism. The inverse transformation gives the relation

$$W(J) = \Gamma(\phi) + \int \mathrm{d}^4x \; J(x)\phi(x) \; . \tag{65}$$

Deriving eq. (64) with respect to ϕ gives us

$$\frac{\delta\Gamma(\phi)}{\delta\phi(x)} = -J(x) \; , \tag{66}$$

while the derivative of eq. (65) with respect to J furnishes us with the result

$$\frac{\delta W(J)}{\delta J(x)} = \phi(x) \; . \tag{67}$$

By comparing eqs. (54) and (67) we also see that

$$-i\frac{\delta \log Z(J)}{\delta J(x)} = \frac{\langle 0\,|\phi(x)|\,0\rangle}{\langle 0|0\rangle} = \phi_{cl} \; , \tag{68}$$

that is, the classical field, defined as the *vacuum expectation value* (VEV) of the quantum field, could be obtained by deriving the generator of connected graphs W with respect to the source field J.

By taking repeated differentials of eqs. (66) and (67) we find

$$G(x,y) = -\frac{\delta^2 W}{\delta J(x)\delta J(y)} = -\frac{\delta\phi(x)}{\delta J(y)} , \tag{69}$$

and

$$\Gamma(x,y) = \frac{\delta^2 \Gamma}{\delta\phi(x)\delta\phi(y)} = -\frac{\delta J(x)}{\delta\phi(y)} . \tag{70}$$

$\Gamma(x,y)$ and $G(x,y)$ are inverse of each other. Treating them as matrices with continuous indices, we could write

$$\int \mathrm{d}^4y\; G(x,y)\Gamma(y,z) = -\int \mathrm{d}^4y\; \frac{\delta^2 W}{\delta J(x)\delta J(y)}\frac{\delta^2\Gamma}{\delta\phi(y)\delta\phi(z)} = \int \mathrm{d}^4y\; \frac{\delta\phi(x)}{\delta J(y)}\frac{\delta J(y)}{\delta\phi(z)} = \frac{\delta\phi(x)}{\delta\phi(z)} = \delta^{(4)}(x-z) . \tag{71}$$

About the third derivatives of functionals, it is clear from the last line of eq. (71) that $\int \mathrm{d}^4y\; G(x,y)\Gamma(y,z)$ does not depend on J. In fact, by taking the derivative with respect to $J(u)$ we find

$$\frac{\delta}{\delta J(u)}\int \mathrm{d}^4y\; G(x,y)\Gamma(y,z) = 0 = \int \mathrm{d}^4y\; \frac{\delta^3 W}{\delta J(x)\delta J(y)\delta J(u)}\frac{\delta^2\Gamma}{\delta\phi(y)\delta\phi(z)} + \int \mathrm{d}^4y\; \frac{\delta^2 W}{\delta J(x)\delta J(y)}\frac{\delta^3\Gamma}{\delta\phi(y)\delta\phi(z)\delta J(u)} . \tag{72}$$

Now for the second term we can write

$$\int \mathrm{d}^4y\; \frac{\delta^2 W}{\delta J(x)\delta J(y)}\frac{\delta}{\delta J(u)}\left[\frac{\delta^2\Gamma}{\delta\phi(y)\delta\phi(z)}\right] = \int \mathrm{d}^4y\; \frac{\delta^2 W}{\delta J(x)\delta J(y)}\int \mathrm{d}^4y'\; \frac{\delta\phi(y')}{\delta J(u)}\frac{\delta}{\delta\phi(y')}\left[\frac{\delta^2\Gamma}{\delta\phi(y)\delta\phi(z)}\right] = -\int \mathrm{d}^4y\; \frac{\delta^2 W}{\delta J(x)\delta J(y)}\int \mathrm{d}^4y'\; G(u,y')\frac{\delta^3\Gamma}{\delta\phi(y)\delta\phi(z)\delta\phi(y')} , \tag{73}$$

because $\delta\phi(y')/\delta J(u) = -G(u,y')$. By combining eqs. (72) and (73) one obtains

$$\int \mathrm{d}^4y\; \frac{\delta^3 W}{\delta J(x)\delta J(y)\delta J(u)}\frac{\delta^2\Gamma}{\delta\phi(y)\delta\phi(z)} = \int \mathrm{d}^4y\; \frac{\delta^2 W}{\delta J(x)\delta J(y)}\int \mathrm{d}^4y'\; G(u,y')\frac{\delta^3\Gamma}{\delta\phi(y)\delta\phi(z)\delta\phi(y')} . \tag{74}$$

To summarise, every derivative of Γ with respect to J could be swapped with a derivative in ϕ and an integration with the Green function G, that is,

$$\frac{\delta}{\delta J(u)} = \int \mathrm{d}^4 y' \, \frac{\delta\phi(y')}{\delta J(u)} \frac{\delta}{\delta\phi(y')} = -\int \mathrm{d}^4 y' \, G(u, y') \frac{\delta}{\delta\phi(y')} \,. \qquad (75)$$

As with Z and W, it is possible to expand Γ as a power series in ϕ:

$$\Gamma(\phi) = \sum_{n=0}^{+\infty} \frac{1}{n!} \int \ldots \int \mathrm{d}x_1 \ldots \mathrm{d}x_n \, \phi(x_1) \ldots \phi(x_n) \Gamma^{(n)}(x_1, \ldots, x_n) \,. \qquad (76)$$

It is possible to show that $\Gamma^{(n)}(x_1, \ldots, x_n)$ is the sum of all 1PI Feynman graphs with n external lines.

We can expand the effective action $\Gamma(\phi)$ in momentum space, in powers of momentum. If one considers renormalisable theory, then the effective action could be written as:

$$\Gamma(\phi) = \int \mathrm{d}x^4 \, \left[-V(\phi) + \frac{1}{2} (\partial\phi)^2 Z_2(\phi) + \ldots \right] \,, \qquad (77)$$

where $Z_2(\phi)$ is the wave function renormalisation, see eq. (14) of (Fabiano, 2021b). The term without derivatives, $V(\phi)$, is called *effective potential*. To express it in terms of 1PI Green functions, we have to write $\Gamma^{(n)}$ in momentum space:

$$\Gamma^{(n)}(x_1, \ldots, x_n) = \int \ldots \int \frac{\mathrm{d}^4 k_1}{(2\pi)^4} \cdots \frac{\mathrm{d}^4 k_n}{(2\pi)^4} \times$$
$$(2\pi)^4 \delta^{(4)}(k_1 + \ldots + k_n) e^{i(k_1 \cdot x_1 + \ldots + k_n \cdot x_n)} \Gamma^{(n)}(k_1, \ldots, k_n) \,. \qquad (78)$$

Putting this expression in eq. (76) and expanding in the powers of momenta k_i gives

$$\Gamma(\phi) = \sum_{n=0}^{+\infty} \frac{1}{n!} \int \ldots \int \mathrm{d}x_1 \ldots \mathrm{d}x_n \, \int \ldots \int \frac{\mathrm{d}^4 k_1}{(2\pi)^4} \cdots \frac{\mathrm{d}^4 k_n}{(2\pi)^4} \times$$
$$\int \mathrm{d}^4 x \, e^{i(k_1 + \ldots + k_n) \cdot x} e^{i(k_1 \cdot x_1 + \ldots + k_n \cdot x_n)} \times$$
$$\left[\Gamma^{(n)}(0, \ldots, 0) \phi(x_1) \ldots \phi(x_n) + \ldots \right] =$$
$$\int \mathrm{d}^4 x \, \sum_{n=0}^{+\infty} \frac{1}{n!} \left\{ \Gamma^{(n)}(0, \ldots, 0) \, [\phi(x)]^n + \ldots \right\} \,, \qquad (79)$$

where we have used the fact that $(2\pi)^4\delta^{(4)}(k_1 + \ldots + k_n) = \int d^4x\ e^{i(k_1+\ldots+k_n)\cdot x}$. Comparing eqs. (79) and (77) we see that the nth derivative of the effective potential $V(\phi)$ is the sum of all 1PI diagrams with n external lines carrying zero momenta, that is, with $k_i = 0$ for all i:

$$V(\phi) = -\sum_{n=0}^{+\infty} \frac{1}{n!}\Gamma^{(n)}(0,\ldots,0)\,[\phi(x)]^n\ . \tag{80}$$

Effective potential: an example

Suppose we have a generic Lagrangian written as

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi)\ . \tag{81}$$

In this case, a closed form for $W(J)$ is impossible to obtain. However, it can be evaluated using the saddle point approximation described in eqs. (31)-(32). The saddle point field $\phi_s(x)$ is determined by the equation

$$\left.\frac{\delta W(J)}{\delta\phi}\right|_{\phi_s} = \left.\frac{\delta[S(\phi) + \int d^4y\ J(y)\phi(y)]}{\delta\phi(x)}\right|_{\phi_s} = 0\ . \tag{82}$$

Writing the explicit form of the Lagrangian of (eq. 81) and integrating by parts the kinetic term, that is, $\int d^4x\ \partial_\mu\phi\partial^\mu\phi = -\int d^4x\ \phi\partial^2\phi$ yields

$$\partial^2\phi_s(x) + V'[\phi_s(x)] = J(x)\ . \tag{83}$$

To estimate Z, we define the integration variable as $\phi = \phi_s + \tilde{\phi}$, restore $\hbar$ and write

$$Z(J) = e^{(i/\hbar)W(J)} = \int \mathcal{D}\phi\ e^{(i/\hbar)[S(\phi)+J\phi]} \simeq$$
$$e^{(i/\hbar)[S(\phi_s)+J\phi_s]} \int \mathcal{D}\tilde{\phi}\ e^{(i/\hbar)\int d^4x\ \frac{1}{2}\left[(\partial\tilde{\phi})^2 - V''(\phi_s)\tilde{\phi}^2\right]}\ , \tag{84}$$

having expanded in Taylor series of $\phi - \phi_s$ as in eq. (31):

$$\left.\frac{\delta^2 S}{\delta\phi^2}\right|_{\phi_s} = \partial^2 + V''(\phi_s)\ . \tag{85}$$

We observe that for any operator A, $\det A = \prod_i a_i$, where a_i are its eigenvalues. So $\prod_i a_i = e^{\sum_i \log a_i}$ and this implies $\det A = e^{\mathrm{Tr}\log A}$. The last part of eq. (84) reads

$$e^{(i/\hbar)[S(\phi_s)+J\phi_s]} \int \mathcal{D}\tilde{\phi}\ e^{(i/\hbar)\int d^4x\ \frac{1}{2}\left[(\partial\tilde{\phi})^2 - V''(\phi_s)\tilde{\phi}^2\right]} =$$

VOJNOTEHNIČKI GLASNIK / MILITARY TECHNICAL COURIER, 2022, Vol. 70, Issue 4

$$e^{(i/\hbar)[S(\phi_s)+J\phi_s]}\left[\frac{2\pi i\hbar}{\det S''(\phi_s)}\right]^{1/2} =$$
$$e^{(i/\hbar)[S(\phi_s)+J\phi_s]+\frac{1}{2}\log(2\pi i\hbar)-\frac{1}{2}\mathrm{Tr}\log S''(\phi_s)} \,. \tag{86}$$

By dropping irrelevant constant terms, we have determined an explicit expression for W:

$$W(J) = [S(\phi_s) + J\phi_s] + \frac{i\hbar}{2}\mathrm{Tr}\log\left[\partial^2 + V''(\phi_s)\right] + \mathcal{O}(\hbar^2) \,. \tag{87}$$

The first term gives the classical contribution to the Green's function. The next term in $\hbar$ gives the first quantum corrections to the Green's functions. Next is the Legendre transformation, for which

$$\phi = \frac{\delta W}{\delta J} = \frac{\delta[S(\phi_s) + J\phi_s]}{\delta\phi_s}\frac{\delta\phi_s}{\delta J} + \phi_s + \mathcal{O}(\hbar) = \phi_s + \mathcal{O}(\hbar) \,, \tag{88}$$

and so for the effective action

$$\Gamma(\phi) = S(\phi) + \frac{i\hbar}{2}\mathrm{Tr}\log\left[\partial^2 + V''(\phi_s)\right] + \mathcal{O}(\hbar^2) \,, \tag{89}$$

and the effective potential

$$V_{\text{eff}}(\phi) = V(\phi) + \frac{i\hbar}{2}\mathrm{Tr}\log\left[\partial^2 + V''(\phi_s)\right] + \mathcal{O}(\hbar^2) \,. \tag{90}$$

It is clear that in general it is not possible to obtain a closed form for the eigenvalues of the operator in eq. (85). We need to introduce some simplifications: the configurations we will study will be the ones for which ϕ is independent of x. In this case, $V''(\phi)$ becomes a constant related to a mass, $\mu(\phi)^2$. The operator $\partial^2 + V''(\phi)$ becomes translationally invariant and is easily evaluable going to momentum space. After obtaining the eigenvalues of the operator, we have to calculate the logarithm and sum over for trace. Therefore

$$\mathrm{Tr}\log\left[\partial^2 + V''(\phi)\right] = \int \mathrm{d}^4x \,\langle x\left|\log\left[\partial^2 + V''(\phi)\right]\right|x\rangle =$$
$$\int \mathrm{d}^4x \int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\langle x|k\rangle\langle k\left|\log\left[\partial^2 + V''(\phi)\right]\right|k\rangle\langle k|x\rangle =$$
$$\int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\log\left[-k^2 + V''(\phi)\right] \,, \tag{91}$$

after having inserted a complete set of states. Going to Euclidean space (Fradkin, 1959) and writing the mass term, we have to deal with the expression

$$\mathcal{I}(\mu^2) = -\frac{1}{(2\pi)^4}\int \mathrm{d}^4k\ \log\left(k^2+\mu^2\right)\ , \tag{92}$$

which as it stands is terribly divergent at infinity, faster than a fourth power. However, if we derive three times with respect to μ^2 we obtain a finite function,

$$\frac{d^3\mathcal{I}(\mu^2)}{(d\mu^2)^3} = \mathcal{I}^{(3)}(\mu^2) = -\frac{2}{(2\pi)^4}\int \mathrm{d}^4k\ \frac{1}{(k^2+\mu^2)^3} = \\ \frac{2\pi^2}{(2\pi)^4}\int_0^{+\infty}\mathrm{d}k\ \frac{k^3}{(k^2+\mu^2)^3} = \frac{1}{32\pi^2\mu^2}\ . \tag{93}$$

By integrating three times in μ^2 we have

$$\mathcal{I}(\mu^2) = \frac{\mu^4\log\mu^2}{64\pi^2} + A + B\mu^2 + C\mu^4\ , \tag{94}$$

as well as three integration constants that can be reabsorbed in the original Lagrangian by renormalisation.

For example, suppose that $V(\phi) = \frac{m}{2}\phi^2 + \frac{g}{4!}\phi^4$, then for the effective potential we would have obtained

$$V_{\text{eff}}(\phi) = \frac{m}{2}\phi^2 + \frac{g}{4!}\phi^4 + \frac{\mu(\phi)^4\log\mu(\phi)^2}{64\pi^2}\ . \tag{95}$$

Loop expansion

We have done perturbative calculations where the expansion parameter is given by the coupling constant of the theory. Now we will organise the perturbation theory in a different form, of *loop expansion*, that is an expansion in increasing number of independent loops of Feynman diagrams. At first order we find the Born diagrams or tree level diagrams, that is, diagrams without loops. The next order consists of diagrams with one loop, with integration on internal momenta. Then diagrams with two loops, and so on. The loop expansion described has a small expansion parameter given by Planck's constant $\hbar$.

Let I be the number of internal lines and V the number of vertices in a Feynman diagram. Then the number of independent loops L will be the number of independent internal momenta after taking into account the momentum conservation in each vertex. One combination of momenta conservation will correspond to the overall conservation of external momenta, so the number of contributing vertices will have to be diminished by 1. The number of independent loops in a given Feynman diagram will therefore be given by the expression

$$L = I - (V - 1) = I - V + 1 \; . \tag{96}$$

In order to relate this loop formula to the powers of $\hbar$, we have to restore first its value. From the equal time commutator of canonical variables, we recall that

$$[\phi(\mathbf{x}, t), \pi(\mathbf{y}, t)] = i\hbar\delta^{(3)}(\mathbf{x} - \mathbf{y}) \; , \tag{97}$$

therefore the propagator in momentum space will furnish us with a factor $\hbar$:

$$G(x) = \langle 0 \,| T\phi(x)\phi(0) |\, 0\rangle = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \, e^{ik\cdot x} \frac{i\hbar}{k^2 - m^2 + i\epsilon} \; . \tag{98}$$

The other place where $\hbar$ appears is in the action of the path integral, $\int \mathcal{D}\phi \; e^{(i/\hbar) S(\phi)}$. As this corresponds to the interaction Lagrangian in the interaction picture

$$e^{\left[\frac{i}{\hbar}\int \mathrm{d}^4 x \, \mathcal{L}_{int}(\phi)\right]} \; , \tag{99}$$

this means that each vertex carries a $1/\hbar$ factor. So for any given Feynman diagram, the power P of $\hbar$ that appears, $\hbar^P$, is given by

$$P = I - V = L - 1 \; . \tag{100}$$

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ИНТЕГРАЛ ПО ТРАЕКТОРИИ В КВАНТОВОЙ ТЕОРИИ ПОЛЯ

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РУБРИКА ГРНТИ: 29.05.03 Математические методы теоретической физики,
29.05.23 Релятивистская квантовая теория. Квантовая теория поля
29.05.33 Электромагнитное взаимодействие
ВИД СТАТЬИ: обзорная статья

Резюме:

Введение / цель: Исходя из гамильтониана в настоящей статье дано альтернативное описание квантовой меха-

ники, основанное на сумме всех возможных траекторий между начальной и конечной точками.

Методы: Теоретические методы математической физики. Интегральный метод на основе интеграла по траекториям.

Результаты: Метод и концепции интеграла по траекториям могут применятся в других областях физики, не ограничиваясь квантовой механикой.

Выводы: Подход интеграла по траекториям дает всестороннее описание полей в отличие от обычного лагранжевого подхода, который представляет локальное описание полей.

Ключевые слова: интеграл по траекториям, квантовая механика, квантовая теория поля.

ИНТЕГРАЛ ПУТА У КВАНТНОЈ ТЕОРИЈАМА ПОЉА

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ОБЛАСТ: математика
ВРСТА ЧЛАНКА: прегледни рад

Сажетак:

Увод / циљ: Полазећи од Хамилтонијана, дат је алтернативни опис квантне механике, заснован на збиру свих могућих путева између почетне и финалне тачке.

Методе: Теоријске методе математичке фиизике. Интегрални метод заснован на интегралу пута.

Резултати: Методе и концепти интеграла пута могу бити примењени и на друге гране физике, нису ограничени на квантну механику.

Закључак: Приступ заснован на интегралу пута даје глобални опис поља, за разлику од уобичајеног приступа заснованог на Лагранжијану који представља локални опис поља.

Кључне речи: интеграл пута, квантна механика, квантна теорија поља.

Fabiano, N., Path integral in quantum field theories, pp.993–1016

EDITORIAL NOTE: The author of this article, Nicola Fabiano, is a current member of the Editorial Board of the *Military Technical Courier*. Therefore, the Editorial Team has ensured that the double blind reviewing process was even more transparent and more rigorous. The Team made additional effort to maintain the integrity of the review and to minimize any bias by having another associate editor handle the review procedure independently of the editor – author in a completely transparent process. The Editorial Team has taken special care that the referee did not recognize the author's identity, thus avoiding the conflict of interest.

КОММЕНТАРИЙ РЕДКОЛЛЕГИИ: Автор данной статьи Никола Фабиано является действующим членом редколлегии журнала *«Военно-технический вестник»*. Поэтому редколлегия провела более открытое и более строгое двойное слепое рецензирование. Редколлегия приложила дополнительные усилия для того чтобы сохранить целостность рецензирования и свести к минимуму предвзятость, вследствие чего второй редактор-сотрудник управлял процессом рецензирования независимо от редактора-автора, таким образом процесс рецензирования был абсолютно прозрачным. Редколлегия во избежание конфликта интересов позаботилась о том, чтобы рецензент не узнал кто является автором статьи.

РЕДАКЦИЈСКИ КОМЕНТАР: Аутор овог чланка Никола Фабиано је актуелни члан Уређивачког одбора *Војнотехничког гласника*. Због тога је уредништво спровело транспарентнији и ригорознији двоструко слепи процес рецензије. Уложило је додатни напор да одржи интегритет рецензије и необјективност сведе на најмању могућу меру тако што је други уредник сарадник водио процедуру рецензије независно од уредника аутора, при чему је тај процес био апсолутно транспарентан. Уредништво је посебно водило рачуна да рецензент не препозна ко је написао рад и да не дође до конфликта интереса.

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TECHNICAL ASPECTS OF FLIGHT SAFETY OF MILITARY AIRCRAFT

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FIELD: Aeronautical technologies and maintenance
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Abstract:

Introduction/purpose: The use of modern military aircraft requires an extraordinary effort of human and material potential to ensure the conditions for the execution of specific tasks. Complex technology, different spatial and temporal conditions of aircraft use impose the need to create organizational and technical conditions to assist the pilot during flight with the aim of safely and completely accomplishing the flight task. The aim of this paper is to identify possible problems in the organization of the flight safety system through the description of the influence of technical factors on flight safety and to find the best solutions for overcoming problems during the life cycle of the aircraft.

Methods: In the research of the subject area, first of all, the analysis of the regulations regulating the field of flight safety was carried out, and then a description of the technical means and their impact on flight safety was carried out.

Results: On the basis of the performed analysis, the activities to ensure the reliability of the aircraft during development and production were defined, and directions for improving flight safety were proposed through the amendments of regulations, organizational and technical as well as technological measures.

Conclusion: The results of the analysis confirm the assumptions about the possible directions of development and improvement of flight safety of military aircraft through improvement and installation of technical systems (devices and equipment), both aviation and ground.

Key words: flight safety, military aircraft, aircraft reliability.

Introduction

Flight safety is an actual and complex problem and not only the lives of passengers and crews depend on its successful solution, but also the efficiency and readiness of the aircraft for use.

The actuality of the issue of flight safety is conditioned by an increase in the speed of flight, take-off and landing, an increase in the intensity of air traffic, considerable complexity of aircraft construction and high requirements of the use of military aviation (Miličević & Bojković, 2021; Duarte et al, 2016). Aviation equipment on modern aircraft is very complex, and crew workload and flight safety largely depend on its quality and operational reliability.

The reliability of aviation technology is an important factor that affects the readiness of the aircraft for flight, the efficiency of its use and flight safety. However, flight safety is primarily affected by in-flight failures of aircraft equipment. Failures also reduce the efficiency of the use of aircraft equipment. Malfunctions (failures and breakdowns) that occur on the ground and in flight require time to be repaired. Malfunctions mainly occur due to structural and manufacturing defects, non-compliance with technical conditions for production, poor assembly, foreign objects entering aggregates and into the flight control system, insufficient reliability of aggregates, equipment and systems of aircraft and their power units, insufficient ability to control the condition of the aircraft itself, before and during the flight (Živaljevic & Siladić, 1997). Flight safety is also affected by operational factors: errors by flight personnel, deficiencies in planning, organizing and managing flights, malfunctions and disruption of the system for managing and securing flights from the ground.

The aim of this paper is to identify possible problems in the organization of the flight safety system through the description of the influence of technical factors on flight safety and to find the best solutions for overcoming problems during the aircraft life cycle.

The paper describes the basic technical factors that affect flight safety, as well as the ways of collecting, systematizing and processing data necessary for flight safety assessment (Honcharenko et al, 2020). The activities in the process of design, production and exploitation of aircraft to ensure high reliability of aviation technology are listed. An insight into the current technical means for ensuring flight safety is given, which include avionics, additional equipment, ground equipment, as well as equipment of objective flight control. After the analysis, a model was proposed for the improvement of the flight safety system with an emphasis on technical factors in accordance with the needs that arise with the introduction into use of new types of aircraft, equipment, tools and control and verification apparatus, as well as changes in terms of procedures and aircraft maintenance technology.

Fundamentals of military aircraft flight safety

The basic act that regulates the flight of military aircraft in the Republic of Serbia is the Rulebook on the Flight of Military Aircraft which more closely prescribes the organization of the flight of military aircraft and risks to flight safety (Službeni vojni list, 2018). The Rulebook provides risk assessment guidelines as well as measures to prevent flight safety risks. These measures are implemented in the instructions for aircraft maintenance, where the duties of aviation technical staff: managers, controllers and technicians, as bearers of aircraft technical support, are defined in terms of performing analysis of malfunctions of aviation technology, determining the cause of failure, developing and implementing measures to prevent and eliminate failure causes within the framework of their duties.

The flight safety system in the narrower sense consists of: expert bodies for flight safety in the Air Force and Air Defense Command, aviation brigade commands, flight safety assistants in flight units, Councils for Methodology and Flight Safety as well as all other bodies involved in the planning process, preparation, organization, performance and analysis of flying, in accordance with their competences.

In the flight safety system, no technical bodies were formed to deal with the organization of flight safety, analyses of the impact of certain factors and taking measures to prevent mishaps, but this function was assigned, as an additional one, to the bodies leading the preparation and execution of flights. In this way, aviation technical managers deal with ensuring the conditions for the safe execution of the planned flight, but without a deeper analysis and prediction of the future impact of technical factors on endangering flight safety.

The periods of the development of flight safety systems are shown in Figure 1, (International civil aviation organization- ICAO, 2018). Until 1970, the main cause of plane accidents was a technical factor. With the improvement of the production technology and the reliability of the aircraft, the period of influence of the human factor in the servicing and maintenance of the aircraft as a primary factor affecting flight safety begins. In the mid-1990s, the causes of flight organization as a cause of aviation accidents were investigated in order to introduce the term total flight safety at the beginning of the 21st century through the management of a flight safety system that combines technical, human, organizational and safety factors into a single system.

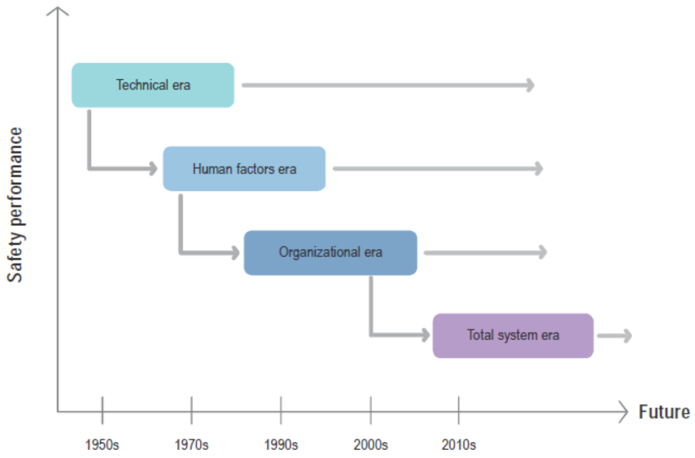

Figure 1 – Flight management system development
Рис. 1 – Разработка системы управления полетом
Слика 1 – Развој система безбедности летења

Technical factors of flight safety

The reliability of aviation technology is an important factor that affects the readiness of the aircraft for flight, the efficiency of its use and flight safety. The requirements that determine the parameters of the given flight technical characteristics and efficiency of aircraft use are set in the phases of aircraft design and production (Bolshakov et al, 2018). However, they are also significantly affected by the process of aircraft maintenance and use: organization, scope and periodicity of maintenance works, methods and equipment used during maintenance, qualification structure and psychophysical condition of technical personnel, quality and reliability of ground equipment for securing flight, etc.

Aviation technology characteristics that affect flight safety arise from the requirements to expand the range of flight speeds and altitudes and from the complexity of tasks performed by flight crews. The complexity of the tasks to be solved causes an increase in the number of instruments, pointers and signalers, which imposed the need to introduce an automatic flight control system.

Flight safety is also affected by operation factors: mistakes by flight crews in piloting techniques, deficiencies in planning, organization and flight management, failures and disruption of the systems for control and security of flights from the ground (Solomonov, 1975).

Mistakes of flight and technical staff as well as mistakes of flight managers can be caused by insufficient qualification of the staff, insufficient knowledge of the technique and the regulations for its exploitation, the absence of solid habits and training, depreciation of the aviation technical staff, deviations from the established regulations for the maintenance and servicing of the aircraft and their ground equipment (Marinković & Drenovac, 2015). Critical situations in flight can also be facilitated by external factors which include: storms, rain and hail clouds, hail, gusts of wind, fog, snow flurries and dense fog, intense icing of the aircraft in flight, strong turbulence, the presence of balloons or birds in the air.

Initial information for identifying critical situations in flight includes pilot reports, objective control data, aircraft operational documentation, personal observations of air traffic controllers, observations of superiors and eyewitness accounts.

Malfunctions are a special factor of flight safety. An important source of data are records of failures of parts, assemblies and aggregates, which are an integral part of maintenance reporting, as part of analyses of work at individual maintenance levels. The analysis of these data can provide a basis for predicting the failure of certain assemblies in the future and serve as a basis for changing procedures in preventive maintenance and exploitation of aircraft.

Ensuring flight safety of military aircraft in the process of their acquisition and exploitation

The constant improvement of the quality and reliability of aircraft requires collective efforts of researchers, test centers, manufacturers and organizations that operate and maintain aircraft. In order to avoid high maintenance costs during aircraft exploitation and large losses due to malfunctions, due to non-use of aircraft during exploitation, research and development and design organizations strive to ensure high reliability of aircraft.

The aircraft reliability depends on the scope and depth of aircraft research and testing in the design and development phase. During the design process, the basics of the reliability of each aircraft, their construction strength and material fatigue, service life, and convenience for maintenance are defined. In the same phase, problems of functional efficiency are solved as well as problems of technological, repair and exploitation indicators.

In order to ensure aircraft reliability, and thus flight safety, high-quality constructive materials are used in the construction of aircraft. The use of composite materials provides strength, stiffness, and stability at elevated temperatures, as well as other positive properties that contribute to the increase of aircraft flight safety.

Constructive methods for increasing reliability include: development and production of reliable aggregates, improvement of structural strength calculations, provision of favorable operating conditions for aggregates, as well as proper selection of aggregate operating parameters. The construction of aircraft, engines, equipment and systems must have a high coefficient of maintenance convenience, and if possible, built-in sensors and measuring elements to signal probable occurrence of malfunctions. In addition to the basic exploitation characteristics, aircraft must also have features that facilitate their exploitation, such as ease of servicing, minimum possible time required to prepare the aircraft for use, mechanization and automation of fueling, special liquids and gases, battery charging, etc.

During aircraft development, tests of aggregates and vital systems are performed in laboratory conditions, and tests are carried out regarding the determination of resources in simulated environment conditions, with the aim of assessing reliability in the design phase. As shown in Figure 2 (Čestić, 2022), an effective way to increase the reliability of aviation technology is the reservation (redundancy) of especially vital aircraft systems, the failure of which directly threatens flight safety.

Based on the analysis of reliability test results in the stages of development and production, an assessment is made on the operational reliability of aviation technology, measures are developed for its maintenance or increase, the life cycles of operation of the aircraft structure, aggregates of its equipment and systems is determined, and operational documentation is defined (Solomonov, 1975).

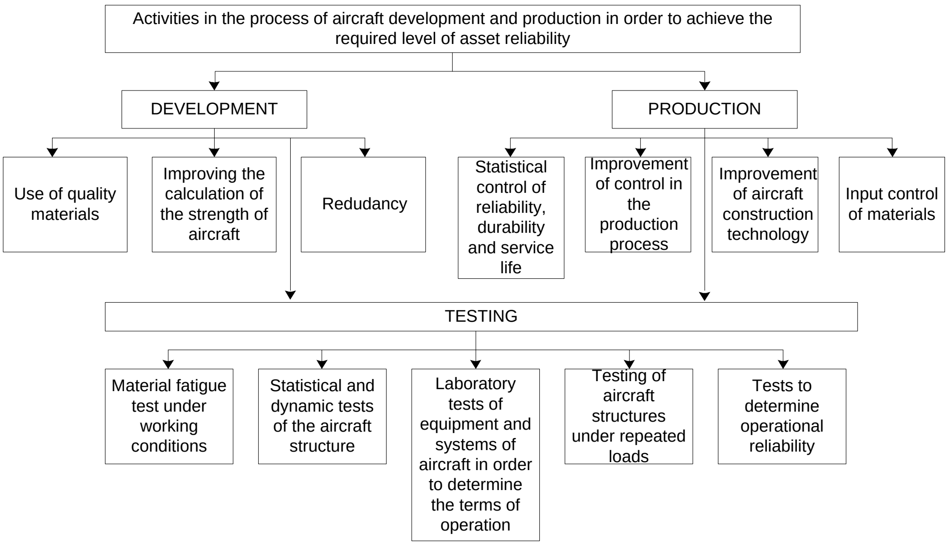

Figure 2 – Activities in the development and production process to ensure aircraft reliability
Рис. 2 – Деятельность в процессе разработки и производства в области обеспечения надежности воздушного судна
Слика 2 –Активности у процесу развоја и производње на обезбеђењу поузданости ваздухоплова (in Serbian)

During operation, ensuring the required level of reliability of aviation technology is done primarily through the aviation technical maintenance system, where modern diagnostic, special testing and measuring equipment and modern data processing methods are used (Pokorni, 2021).

Quantitative evaluations of various factors, according to the importance of their influence on emergency events in flight depend on the permitted methods of testing aircraft in terms of flight safety (Kublanov, 2021).

To determine the initial stage of the development of cracks, corrosion, internal (hidden) deformities of materials and malfunctions of parts, assemblies and aggregates, physical methods of defectoscopy are used (magnetic, electroinduction, gamma defectoscopy, ultrasonic, penetration - using a penetrating liquid and challenger, etc.) .

To determine changes in technical characteristics (parameters) and evaluate the working capabilities of aggregates and hydraulic devices, devices for air, fuel, cabin pressurization and air conditioning systems and other aircraft systems, instrumental checks using automated devices, counters, as well as various devices and instruments are used. To assess the technical condition of the aircraft electronic equipment, instrumental checks are applied using built-in external control systems, control-measuring devices for general and special purposes, special workbenches and various instruments.

Control and assessment of the technical condition of aviation equipment in flight units is carried out using standard and special measuring devices, based on appropriate inspection lists for specific aviation equipment. A particularly important role in ensuring the reliability of aviation technology, preventing failures and aviation accidents is played by periodic inspections and preventive works that are carried out based on the hourly or time limit of aircraft use. Periodic inspections include activities to inspect the condition of all elements of the aircraft according to the prescribed periodic inspection lists for a specific type of aircraft.

To increase the efficiency of the use of aircraft and their operational safety, it is important to quickly and correctly determine the causes of technical failures, as well as to quickly and efficiently eliminate them, prevent their recurrence and forecast their eventual occurrence.

Technical diagnostic methods for early failure detection which are increasingly being developed represent one of the most important modern directions for increasing reliability and reducing aircraft exploitation costs. The improvement of diagnostic methods enables the automated collection and processing of information on parameters for failure forecasting and signaling (warnings to the crew) about malfunctions on the aircraft during flight.

Control in operation is primarily aimed at detecting gradual failure of elements and devices, whose failure indirectly affects flight safety. Particularly important data for the diagnosis of the state of certain aircraft systems are provided by recorders for the analysis and processing of dynamic parameters of aviation technology, such as vibrations, pressure changes in hydraulic and air devices, and the like.

Technical equipment for ensuring flight safety

To ensure flight safety, aircraft are equipped with special technical equipment that make up the safety scheme. They are designed to eliminate the consequences of the failure of vital systems or to prevent the possibility of these systems entering a dangerous mode of operation (Solomonov, 1975). Constructively, safety schemes are applied in the form of vital systems and devices or in the form of the installation of special blocks in aggregates (control, safety), automatic safety, control, restrictions, exclusions and the like. New generations of aircraft are equipped with electronic blocks that have self-checking options, where correctness checks are performed automatically without disassembly and special tests during periodic inspections. It is particularly important that during the self-diagnosis of a certain set (e.g. radio station or navigation devices), in the event of malfunction, accurate information is given on which block or which communication line from the set has failed, which significantly reduces the time of defection and repair and ultimately raises flight safety to a higher level .

On many aircraft, in order to prevent exceeding the permissible value of the workload coefficient, limiters of the speed (tempo) of the movement of the pilot stick are installed, either mechanically or by software in systems with "fly by wire" technology.

Some multi-engine airplanes are equipped with automatic course maintenance (correction) devices when one of the engines fails.

An effective way to increase flight safety is the diagnostic control of the basic structural elements of the aircraft, especially the resistance (firmness) of its engines, based on the analysis and control of vibrations. The vibration control system consists of vibration transmitters, electronic blocks and indicators or vibration recorders (Nacional'nyj centr vertoljotostroenija imeni M.L. Milja i N.I. Kamova, 2019). In addition, aircraft can be equipped with means which control aircraft in emergency situations (Solomonov, 1977). All modern airplanes are equipped with autopilots in which control blocks are installed, which the autopilot automatically shuts down in the event of failure, as well as rudder end deflection transmitters. Many aircraft are equipped with dangerous altitude blocks that signal the autopilot and the pilot to enter climb (Kurdel et al, 2019).

Modern aviation equipment has imposed the need to install a large number of devices, instruments and signals necessary for the pilot to be informed about the state or flight mode of the aircraft. Special attention is paid to the display of parameters and signaling in the form of multifunctional indicators, which significantly relieve the instrument panel from classic analog instruments, and where it is possible to select parameters or groups of parameters that are important for monitoring, as seen in Figure 3 (Airbus Helicopters, 2016).

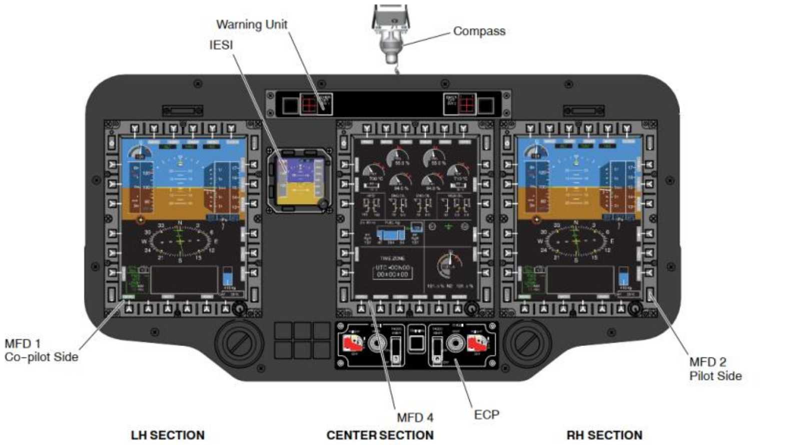

Figure 3 – Layout of multifunctional indicators of the H-145M helicopter
Рис. 3 – Компоновка многофункциональных индикаторов вертолета H145M
Слика 3 – Распоред мултифункционалних индикатора хеликоптера Х-145М

The desired data displayed on multifunctional indicators are processed in the flight management computer system Flight Management System (FMS) in which error signals are processed in parallel, as well as signals about a dangerous situation in flight, on the basis of which warnings to the crew are generated. Signal lights, light boards (inscriptions), pointers and tone (sound) signaling are often used as signals to inform the pilot about the operation of aircraft systems, their failures, the aircraft's entry into critical flight modes and to indicate targets.

An effective tool that facilitates piloting in certain phases of flight and increases flight safety is the display of aiming and navigation data on the windshield through which the pilot observes the area in front of the aircraft, as well as the way of displaying the level of warnings and errors on the systems or the values of certain parameters through colors, where the pilot is aware of the existence of a certain error but does not need to pay attention if that error does not jeopardize the performance of the task (Peysakhovich et al, 2018).

The additional means that aircraft are equipped with to ensure flight safety include collision prevention devices, anti-icing devices and prevention of electrical (atmospheric) discharges to the aircraft, pilot notification devices, rescue equipment, fire fighting equipment, etc.

Special Airborne Collision Avoidance System (ACAS)/Traffic Collision Avoidance System (TCAS) systems have been developed and are being perfected to prevent collisions in flight. The basic criterion of the system for assessing the possibility of an aircraft collision is the flight height. A signal transmitted from one aircraft at a precise moment in time is received by the devices of another aircraft, and their distance is determined according to the time difference between the transmission and reception of the signal (Vorobev et al, 1989).

In flight, rare cases of electrical discharges to the aircraft may occur. To ensure BL, reliable electrical connection of all elements of the aircraft structure is performed with appropriate metallization; and at least 2 mm thick plating is used on all parts of the structure near the fuel system elements to prevent burn-through. The structure of the aircraft is designed in such a way as to prevent the creation of an electric arc in the space of the fuel tanks. Flame extinguishers are also installed at the fuel system outlets (Solomonov, 1977).

For flying at extremely low altitudes, the limitations of the aircraft's maneuverability and the pilot's physiological capabilities are taken into account. Flight controls for flying at extremely low altitudes must provide manual, semi-automatic and automatic aircraft control modes.

To monitor the terrain (relief) situation, a radar is used that provides data on the distance from the nearest elevation and the angle of its elevation in relation to the reference plane (Zuluaga et al, 2017). When overcoming obstacles, there are possible cases of the aircraft arriving at a flight mode close to the landing mode, as well as descending to below the permitted height. For this reason, equipment is installed on the aircraft that warns of obstacles about the approach of the aircraft to the landing mode and signals to the pilot that the flight height above the obstacles is lower than the set one, the so-called Terrain Awareness and Warning System (TAWS) devices.

The basic method of rescuing the crew during emergency situations in flight is to leave the aircraft using the ejection seat. The improvement of ejectable pilot seats represents today the main direction in the fight to increase the efficiency of the means for forced abandonment of the aircraft (Zubkov et al, 2007). Ejectable seats installed in helicopters are also significant, but they are not widely distributed due to the complexity of the construction and the possibility of installation only in helicopters with a coaxial carrier rotor.

Most modern aircraft are equipped with devices for broadcasting the GPS position of the aircraft Emergency Locator Transmitter (ELT), in order to quickly find an aircraft that has suffered an accident or forced landing. Figure 4 (Airbus Helicopters, 2016) shows the configuration of one set of ELT devices, consisting of special modules with the antenna part and the power supply, a pointer and a control module.

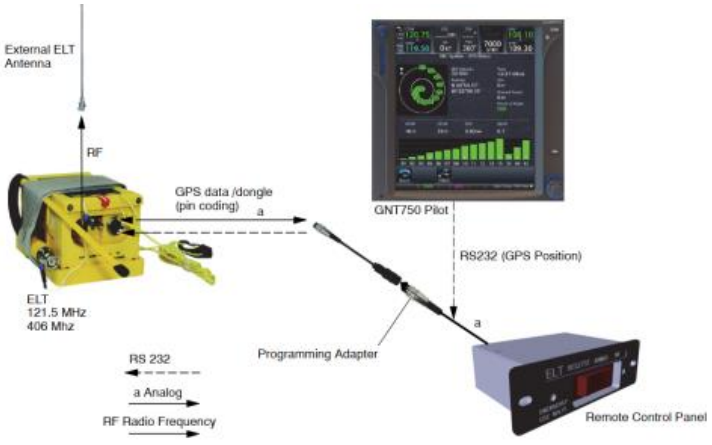

Figure 4 – Components of the ELT device
Рис. 4 – Компоненты устройства ELT
Слика 4 – Саставни елементи ELT уређаја (in Serbian)

Ground means for ensuring flight safety

Ground-based technical means for ensuring flight safety include aircraft guidance and flight control equipment, airport lighting equipment, technical means for dispersing birds, means for preparing and training personnel, and storage means. All major airports also have ILS (Instrument Landing System) systems that ensure precise guidance of the aircraft during landing. Figure 5 shows the navigation device PNP-72 which, with the combined operation of several devices and equipment in aircraft, enables landing in conditions of reduced visibility.

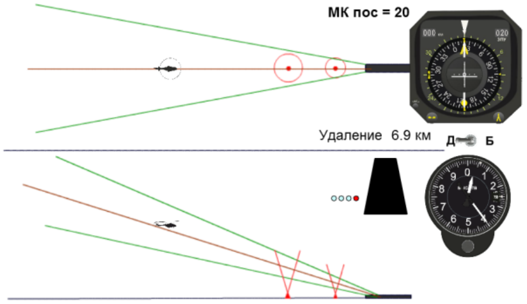

Figure 5 – Display of the operation of the ILS system in conjunction with the helicopter navigation device
Рис. 5 – Отображение работы системы ILS совместно с навигационным устройством вертолета
Слика 5 – Приказ рада ILS система у спрези са хеликоптерским навигационим уређајем (in Serbian)

For the implementation of light, navigation and signal security, starting radio stations, light signaling devices with servers for stationary night start and mobile night start, mobile flight control, as well as other ground radio-navigation equipment are provided.

Special runway curtains and braking devices, which both aircraft and airports are equipped with, are used in the cases of aircraft disembarking from the runway. Systems with arresting nets and arresting ropes are most often used to stop the aircraft on the runway. Along with them, the method of covering the runways with foam for the forced stop of the aircraft is also used.

Experience shows that some critical situations in flight are conditioned by the so-called "human" factor, i.e. mistakes (Poussin et al, 2017). Therefore, the introduction of trainers and simulators into use is of great importance for ensuring flight safety (Vlačić et al, 2022). The possibility of effective training of crew emergency procedures when modeling failure and imitation of emergency events on simulators (engine failure, autopilot and auxiliary systems failure, fire simulation, etc.) creates conditions for systematic training of flight personnel with the aim of acquiring the necessary habits and maintaining them at a certain level (Rudnjanin & Debijađi, 1984).

Also, very often interactive and 3D classrooms are used in addition to flight simulators, as a good technical basis for training pilots and training technical staff.

Technical means of the objective flight control

Determining the real causes of critical situations in most cases is possible only if there is reliable information about the flight parameters of the aircraft, about the correct operation of its systems and the work of the crew during the development of an emergency event in flight. Certain types of aircraft in the world are equipped with means of objective control that have a direct data link with the home base, another aircraft or a flying command post, where it is possible to monitor the parameters of the aircraft in real time (Zubkov, 2007). Also, these types of communication have the possibility of direct transmission of radar and sensor images and are installed on 4++ and 5th generation multipurpose aircraft. Flight parameter recorders, Figure 6 (Nacional'nyj centr vertoljotostroenija imeni M.L. Milja i N.I. Kamova, 2019), are used in the investigation of aviation accidents and critical situations in flight.

Figure 6 – Flight parameters recorder BUR-1-2 of the Mi-17V-5 helicopter
Рис. 6 – Регистратор параметров полета БУР-1-2 вертолета Ми-17В-5
Слика 6 – Регистратор параметара лета БУР-1-2 хеликоптера Ми-17В-5 (in Serbian)

In practice, data on engine operating times, collected from the flight parameters recorder, can also be used during homologation tests of certain engine parts (Banjac et al, 2017). Also, based on recorded data using the recorder, it is possible to observe changes in the parameters of individual parts, assemblies and aggregates that provide a basis for predicting future failures and a basis for preventive inspection or replacement of a specific part or assembly.

Improvement of flight safety of military aircraft by applying organizational and technical-technological measures and procedures

Ensuring flight safety is a constant task and a continuous process to which special attention is paid regardless of the current circumstances in which flight tasks are performed. However, there is room for improving the current situation through the application of organizational, normative and technical-technological measures and procedures (Bogdane et al, 2019).

Organizational measures for flight safety improvement

An important area in flight security is organizational measures, such as: flight management, preparation, training and control of flight and aviation technical personnel and the organization of the use of ground assets for flight security in flight units (Bilbija, 2017). The basis for raising the level of flight safety in terms of technical factors is the adaptation of the administrative authorities of the commands to the requirements of the profession, whereby the work of the technical authorities would be organized and controlled primarily by the technical service and only then by the line of command. A system in which "everyone can do everything", in a situation where it is necessary to provide the maximum of professional capacities, cannot fully meet the requirements of maintaining modern aviation technology.

The recruitment of aeronautical personnel must be based on quality and not on quantity. The process of education and training of staff should follow the needs of the units for professional staff of all aviation technical specialties. Individuals who work on aviation technology must be confident in their knowledge and actions and constantly improve themselves. It is necessary to introduce permanent monitoring and verification of training in specialties, practically as a form of license renewal, with prescribed rights and obligations of individuals. Also, it is necessary to prescribe the training levels of specialists according to the degree of professional competence and experience in working on a specific aircraft, which should be accompanied by appropriate benefits.

Measures for normative regulation of flight safety improvement

Along with organizational measures, an important role is also played by continuous improvement of the content and form of documentation and records that regulate flight training, flight planning, maintenance, especially for newly introduced aircraft. The departments dealing with technical documentation, their updating, systematization, monitoring and distribution of changes and bulletins are of particular importance. Those departments must be the link between the manufacturer and the user of the aircraft in terms of documentation, which is one of the essential requirements of the concept of integral logistics support (ILP). The efficient work of the aforementioned departments creates the conditions for reducing the costs of the aircraft's lifetime through a more efficient organization of technological measures and procedures within the specific level of maintenance, shortens the time for carrying out maintenance procedures, enables efficient planning of the procurement of spare parts, aviation ordnance, electronic equipment as well as keeping the same. Failure to adhere to the concept of ILP results in the exact opposite.

Another important area of ensuring flight safety is the analysis of the causes of aircraft malfunctions and failures. An objective analysis includes the development of scientific methods and recommendations for investigating the causes of critical situations in flight based on registered flight parameters, performing the necessary calculations, flight modeling, tests and research in flight (Hooper & O'Hare, 2013).

In particular, it is necessary to make efforts to prescribe a detailed scheme of the operation of the flight safety system, with the bearers of individual activities, and the duties of each authority when analyzing critical events and failures, or any other negative impacts on aircraft maintenance.

Application of technical-technological measures and procedures for flight safety improvement

Practically all work on aircraft is carried out with prescribed tools, control and measuring equipment, which reduces the degree of improvisation in the work to a minimum. Combat equipment must be stored in appropriate hangars, where it is possible to carry out maintenance activities in microclimate-controlled conditions in a closed space, with heating in the winter months, which, in addition to the technological requirements of maintenance activities, will also enable favorable psychophysical conditions for the technical staff to devote all their attention to the required work on aircraft. The hangars should provide effective conditions for working with modern stationary equipment (electric generators and rectifiers powered from the city network), thus reducing the engagement of electric starters and, therefore, maintenance costs. Also, it is necessary to invest in the continuous equipping of laboratories for testing and checking aircraft equipment, which would create the conditions for compliance with maintenance procedures according to the highest world standards, which is a prerequisite for the use of the most modern aircraft.

In terms of maintaining the correctness and reliability of measuring, control and verification equipment and tools, it is necessary to pay attention to metrology laboratories for verification, their equipment and devices, and to train personnel for the verification of highly sophisticated equipment.

In order to monitor and manage resources as efficiently as possible, it is necessary to reduce the involvement of aviation technical personnel in administrative tasks, by introducing a specialized information system that would improve the way of monitoring resources, keeping records and reporting (Senol, 2020). Practically, in one place it would be possible to create an overview of all important information, from the availability and training of staff, through the state of resources, filling with spare parts, data on the maintenance process through monitoring and analysis of malfunctions. Also, this system would have to include all analyses of flights performed by all types of aircraft equipped with such means, so that the technical authority would have the possibility of a more detailed investigation of the failure of parts and aggregates. In this way, the flight safety system would be improved, significantly reducing the time from the occurrence of the critical event, through the cause analysis to the appropriate reaction of the maintenance system.

Conclusion

Equipping with airplanes, helicopters and remotely piloted aircraft of new generations as a prerequisite requires an extremely high technical culture of pilots and aviation technical personnel. The challenge for aviation technical personnel is to maintain the reliability of modern aircraft, organize the maintenance process, develop methods and means for maintenance, as well as ensure the qualification structure and adequate psychophysical condition of flight and technical personnel.

The aircraft, as the final product of the aviation industry, is subject to modifications, modernizations, changes in the production process and especially changes in the process of preventive and corrective maintenance, the ultimate goal of which, in addition to improving tactical and technical characteristics, is to increase the level of flight safety.

The task for the aviation maintenance and safety system is to enable high-quality distribution, processing and analysis of data on the state of the organization and technique through the introduction of an information system.

Most of the mentioned proposals and requirements in this paper are solved in the phases of deesign and aircraft production, and sometimes during modernization, through the absolute application of the concept of integral logistic support.

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ТЕХНИЧЕСКИЕ АСПЕКТЫ БЕЗОПАСНОСТИ ПОЛЕТОВ ВОЕННЫХ ЛЕТАТЕЛЬНЫХ АППАРАТОВ

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РУБРИКА ГРНТИ: 78.25.13 Военная авиационная техника и вооружение
ВИД СТАТЬИ: обзорная статья

Резюме:

Введение/цель: При использовании современной военной авиации требуются экстраординарные человеческие усилия и материальные средства, особенно в обеспечении условий для выполнения специальных задач. Сложная технология, различные пространственно-временные условия использования воздушного судна диктуют необходимость создания организационных и технических условий для оказания помощи пилоту во время полета с целью безопасного и полного выполнения полетной задачи. Целью данной статьи является выявление возможных проблем в организации системы безопасности полетов путем описания влияния технических факторов на безопасность полетов и поиска наилучших решений в преодолении проблем в течение жизненного цикла воздушного судна.

Методы: При исследовании предметной области, прежде всего был проведен анализ нормативных правовых актов, регулирующих сферу безопасности полетов, а затем было проведено описание технических средств и их влияния на безопасность полетов.

Результаты: На основании проведенного анализа были определены меры по обеспечению надежности воздушного судна в процессе разработки и производства, а также предложены направления повышения безопасности полетов путем внесения изменений в регламенты по организационным, техническим и технологическим мерам.

Выводы: Результаты анализа подтверждают предпосылки о формировании возможных направлений в развитии и повышении безопасности полетов военных летательных аппаратов за счет совершенствования системы и установки авиационных и наземных технических систем (приборов и оборудования).

Ключевые слова: безопасность полетов, военная авиация, надежность летательных аппаратов.

ТЕХНИЧКИ АСПЕКТИ БЕЗБЕДНОСТИ ЛЕТЕЊА ВОЈНИХ ВАЗДУХОПЛОВА

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ОБЛАСТ: ваздухопловне технологије и одржавање
КАТЕГОРИЈА (ТИП) ЧЛАНКА: прегледни рад

Сажетак:

Увод/циљ: Употреба савремених војних ваздухоплова захтева изузетно напрезање људских и материјалних потенција за обезбеђење услова за извршавање наменских задатака. Сложена технологија, различити просторни и временски услови употребе ваздухоплова намећу потребу за стварањем организацјских и техничких услова за испомоћ пилоту у току летења, а ради сигурног и потпуног остварења летачког задатка. Циљ овог рада јесте да се, кроз опис утицаја техничких фактора на безбедност летења, идентификују могући проблеми у организацији система безбедности летења и пронађу најбоља решења за њихово превазилажење током животног циклуса ваздухоплова.

Методе: У истраживању су прво анализирани прописи којима је регулисана област безбедности летења, а затим је извршена дескрипција техничких средстава и њихов утицај на безбедност летења.

Резултати: На основу извршене анализе, дефинисане су активности на обезбеђењу поузданости ваздухоплова у току развоја и производње и предложени су правци унапређења *безбедности летења, кроз измену и допуну регулативе, као и организацијске и техничко-технолошке мере.*

Закључак: Резултати извршене анализе потврђују претпоставке о могућим правцима развоја и унапређења безбедности летења војних ваздухоплова кроз усавршавање и уградњу техничких система (уређаја и опреме), како ваздухопловних, тако и земаљских.

Кључне речи: безбедност летења, војни ваздухоплов, поузданост ваздухоплова.

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Русија развија противбродску балистичку ракету[1]

Русија ради на развоју нове противбродске балистичке ракете у оквиру своје стратегије против приступа/ускраћивања подручја.

Извештаји из Русије сугеришу да она убрзано развија противбродску балистичку ракету, или ASBM (anti-ship ballistic missile), класу оружја која је популарно названа „убица носача авиона”. Раније непознат пројекат ракете, под руским именом *Zmeyevik* (што значи серпентинит, врста стене), додаје снажну компоненту руској стратегији против приступа/одбијања подручја (anti-access/area-denial *A2/AD*), која већ укључује различите ракете за обалску одбрану и хиперсоничне противбродске ракете у развоју.

Према извештају руске државне новинске агенције ТАСС, заснованом на два неименована извора „блиских министарству одбране и војно-индустријском комплексу”, *Zmeyevik* се налази у развоју, а њиме ће бити опремљене јединице обалске одбране руске морнарице.

Пројектил, за који агенција каже да је првенствено намењен за гађање великих површинских платформи, укључујући носаче авиона, заправо је балистичка ракета са завршном фазом маневарског хиперсоничног планирајућег возила (maneuvering hypersonic glide vehicle – *HGV*).

Један извор навео је да је ракета *Zmeyevik* врло слична кинеским балистичким пројектилима *DF-21D* и *DF-26B ASBM* са дометом од око 4. 000 километара.

Детаљи о тренутном статусу развоја ракете *Zmeyevik* нису познати, иако је агенција ТАСС навела да се обратила компанији *NPO Mashinostroeniya* за коментар, претпостављајући да та пројектантска агенција развија ову ракету. Компанија није одговорила, али треба напоменути да NPO „Mashinostroeniya” производи хиперсоничну противбродску ракету Zircon за површинске ратне бродове и подморнице, маневарско хиперсонично планирајуће возило *Avangard HGV* које носе интерконтиненталне балистичке ракете (*ICBM*) и систем обалске одбране *Bastion* који је наоружан суперсоничним ракетама *Onyx.*

Сарадници сајта *War Zone* разговарали су са Дмитријем Стефановичем, научним сарадником у Центру за међународну безбедност *IMEMO RAS*, о његовим запажањима у вези са развојем ракете *Zmeyevik*.

„Изгледа да је овај пројекат заиста био у развоју неко време, иако није било никаквих тестова који су поменути или примећени, што, наравно, не значи да није могло да се спроведе тестирање прототипа”, рекао је

[1] July 13, 2022 www.thedrive.com/the-war-zone

Vučković, D., SAVREMENO NAORUŽANJE I VOJNA OPREMA / MODERN WEAPONS AND MILITARY EQUIPMENT, pp.1039-1048

Стефанович. Пројектовање брзих противбродских ракета је карактеристично за Совјете и Русе.

Међутим, Стефанович је указао и на неке недостатке концепта, бар што се тиче руске морнарице:

„С обзиром на домет и брзину ракете, потребна је озбиљна (intelligence, surveillance, and reconnaissance ISR) активност (обавештајна, надзорна и извиђачка) и могућности циљања, као и навођење у крајњем стадијуму лета. Маневарска возила са повратним уласком и хиперсонична планирајућа возила су примарни тип корисног терета у овом случају али постоје и очигледне празнине у руским *ISR* способностима.”

На основу тога, поставило се питање о томе на који начин Кина добија податке о циљу како би успешно наводила своје *ASBM*, иако се чини да је Пекинг у том погледу више напредовао. Једна опција за Русију била би да користи своје системе за осматрање преко хоризонта (over-the-horizon – *OTH*), за упозорење и циљање, космичке сензоре, као и своју флоту поморских патролних авиона великог домета. За разлику од Кине, Русија тренутно нема велику флоту беспилотних летелица великог домета на великим висинама које би могле да помогну у проналажењу непријатељских флота и пренесу такве информације ракетним јединицама, иако је то подложно промени у будућности.

Руски морнарички патролни авион Il-38N

„Фокус на обалски ракетни систем као примарну платформу (за разлику од пројектила Zircon, на пример, који ће у почетку опремати површинске платформе) вредан је пажње” наставио је Стефанович.

„Овакви системи имају своје генеричке радарске могућности и вероватно их је лакше повезати са целокупном информационом мрежом о ситуацији преко мора у близини руских обала, која између осталог укључује радаре за осматрање иза хоризонта, иако је ово само спекулација".

„Осим тога, руска морнарица је већ показала веома реалну способност копненог напада својим обалским ракетним системима (*Bastion* и подзвучним *Bal*), где се та искуства могу искористити у развоју пројектила *Zmeyevik*. Иначе, Русија тврди да је користила свој ракетни систем обалске одбране *Bastion-P* за ударе на копнене циљеве у Украјини у актуелном сукобу.

Систем обалске одбране Bastion-P

Коначно, распоређивање копнених ракета домета од неколико хиљада километара тешко да је у складу са руским самонаметнутим мораторијумом на неразмештање оружја (Intermediate-Range Nuclear Forces – *INF)* средњег домета (нуклеарне снаге средњег домета) након распада овог споразума", приметио је Стефанович. „Наравно, не знамо временски оквир за стварно распоређивање, тако да је могуће да ће америчко оружје попут Long Range Hypersonic Weapon – *LRHW* (хиперсонично оружје дугог домета), *Dark Eagle* или Mid-Range Capability – *MRC* (способност средњег домета) у међувремену бити распоређено у Европу или Азију на шта треба обратити пажњу."

У прошлости се разговарало о могућностима *ASBM* у кинеском контексту, у облику *DF-21D и DF-26.*

Сада се чини да Русија, такође, покушава да понуди оружје у сличној категорији. Иако нема детаља о врсти лансирне платформе коју ће *Zmeyevik* користити, могуће је да ће то бити врло мобилне копнене платформе, какве користе кинеске *ASBM* и друге руске мобилне ракете за обалску одбрану.

Што се тиче поменутог кинеског оружја, *DF-21D* је балистичка ракета средњег домета (medium-range ballistic missile – *MRBM*) – дефинисана као балистичка ракета са максималним дометом између 1.500 и 3.000 километара. У међувремену, *DF-26 је* класификована као балистичка ракета средњег домета (*IRBM*), категорија која се састоји од оружја већег домета способног да погоди циљеве на удаљености између 1.000 и 5.500 километара. На основу званичника које цитира агенција ТАСС, чини се да је *Zmeyevik* упоредивији са *DF-26B*, у погледу домета.

Попут *DF-21D* и *DF-26B*, захтеви ракете *Zmeyevik* били би да његове *HGV* бојеве главе имају довољно маневарске способности да погоде велике бродове који се релативно споро крећу, као што су носачи авиона, које би били међу примарним циљевима пројектила. *HGV* би, вероватно, користио радар и/или инфрацрвени трагач за терминално навођење, иако је то у овом тренутку није потврђено.

Постоји чак и могућност да би Руси прилагодили постојећу балистичку ракету кратког домета *Iskander-M* као основу за *Zmeyevik*, иако је то оружје много мањег домета, способно да погоди циљеве на удаљености од око 500 километара. Међутим, *Iskander-M* је већ коришћен за стварање балистичке ракете са ваздушним лансирањем *Kinzhal* која има домет између 1.500 и 2.600 километара. У међувремену, Кина истражује могућност употребе балистичких ракета кратког домета у противбродским мисијама, при чему ракета *CM-401* такође има одређене сличности са ракетом *Iskander-M*, у погледу величине и изгледа. Пекинг сада распоређује сличне балистичке ракете на својим површинским ратним бродовима који ће врло вероватно имати противбродске способности или ће их стећи у будућности. На крају, и верзија која се лансира са брода такође може бити опција за програм *Zmeyevik*.

Такође, важно је знати да се не очекује да *Zmeyevik* делује сам, већ би ракета чинила још један слој постојећег руског *A2/AD* „кишобрану” који служи за одбрану њених обала и територије око њих. Конкретно, руска стратегија *A2/AD* фокусира се на поморска подручја од посебног значаја, стварајући такозване „зоне за оптимално ангажовање ракета” (super missile engagement zone – *MEZ*), као што су области у Црном мору и око Калињинграда у Балтичком мору. Они користе различите типове противбродских ракета – морског, копненог и ваздушног лансирања, за одбијање ратних бродова НАТО-а.

Поред успостављених MEZ у Црном мору и на Балтику, Русија је такође у процесу развоја сличних капацитета на Арктику, као и на Далеком истоку, где су спорна Курилска острва у посебном фокусу. Значај *ASBM* дугог домета у оба ова стратешка позоришта је јасан. Русија је већ нагласила распоређивање нових противбродских ракета ваздух-море и копнених лансера противбродских ракета, поред осталог војног хардвера, на овим испоставама, као део своје шире стратегије *A2/AD.* У том контексту, *ASBM* би били утолико кориснији када би били у стању да гађају и копнене циљеве. Имајући у виду руску традицију развоја противбродских/нападних ракета двоструке улоге, то изгледа вероватно. Друга могућа локација за ракету *Zmeyevik* је Медитеран, за који Москва показује све веће интересовање. У оквиру тог подручја постоји опција стационирања *ASBM* у Сирији, нешто што је Русија већ урадила са противбродским ракетама које се лансирају из ваздуха.

Далекометни *ASBM* би омогућио руској морнарици да ангажује ратне бродове на много већим удаљеностима, и до неколико хиљада километара даље од релативне безбедности руског копна, где би били заштићенији од превентивних мера или противудара. За разлику од других обалских одбрамбених система, *ASBM* би, такође, био у стању да ангажује мете на веома широком подручју без неопходног претходног премештања.

У исто време, овакво оружје би помогло да се надокнади релативни недостатак савремених противбродских способности дугог домета површинске флоте руске морнарице, с обзиром на то да се амбициозни програми за изградњу нових класа ратних бродова до сада нису остварили.

У међувремену, постојеће класе површинских бродова руске морнарице развијене су првенствено за гађање НАТО носача авиона и других ратних бродова високе вредности. Ради се о бродовима класе *Project 1144* или *Kirov* и крстарица класе *Project 1164* или *Slava,* и њиховим противбродским ракетама, које датирају из Хладног рата. Релативна рањивост ових типова пловила показала се када је пројекат *1164* крстарица *Moskva* потопљена током рата у Украјини, као циљ украјинских подзвучних противбродских пројектила *Neptune*.

Иако би хиперсонична ракета *Zircon* требало да обезбеди руској морнарици знатно побољшане способности противбродске борбе, ово оружје још увек није у употреби, упркос опсежним испитивањима. Знаци да би овај програм могао наићи на потешкоће појавили су се 2020. године, када је водећи званичник руске морнарице навео неодређене „дечије болести" у својим развојним напорима. Имајући то у виду, развој *ASBM* би могао бити чак и једноставнији, посебно имајући у виду руско искуство са ракетом *Avangard* и могућност коришћења постојеће балистичке ракете за ношење противбродског *HGV*. Чак и са пројектилом *Zircon,* који функционише без проблема, ово је оружје много мањег домета, које вероватно достиже циљеве на максималној удаљености од око 1.000 километара.

Треба рећи да одбрана од напада пројектила *Zmeyevik* није једноставна. Чак и уочавање и откривање балистичких пројектила може бити компликовано, док је њихово пресретање много изазовније од откривања и ангажовања крстарећих ракета које ниско лете. За сада се, међутим, не зна да ли је руски *ASBM* уопште тестиран, а камоли да ли ће моћи поуздано да погоди мету коју представља велики брод у покрету, као што је носач авиона.

Међутим, ако ова технологија профункционише, користи од *ASBM* за руске оружане снаге су јасне. Сада, када је руска инвазија на Украјину довела до даљег заоштравања и потенцијалног сукоба са НАТО-ом и Западом, потреба Русије за снажним *A2/AD* капацитетима вероватно ће постати још већа. Било да буде на Арктику, Балтику, у Црном мору или на Далеком истоку, руски *ASBM* би додатно повећао ризик било којој другој морнарици која би одлучила да делује у овим областима. Имајући то у виду, помно ће се пратити даљи развој програма *Zmeyevik*.

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Подморница класе Kilo са „pump-jet” пропулзијом[2]

Јединствена руска дизел-електрична подморница *Alrosa*, наоружана новим крстарећим ракетама, поринута је у море ради испитивања пре него што буде оперативно распоређена у Црноморску флоту. Ова јединствена подморница, најстарија у Црноморској флоти, вероватно ће се придружити борбеним операцијама против Украјине, у којој Русија води појачану кампању ракетног бомбардовања.

Руска државна новинска агенција ТАСС известила је, у јуну 2022. године, да је *Alrosa*, подваријанта дизел-електричних подморница *Project 877* или класе *Kilo,* поринута у море како би се започела испитивања након надоградње. То је објавила 13. фабрика за ремонт бродова Црноморске флоте у Севастопољу, на Криму.

„Подморница је поринута (у море из Севастопољског залива). Први пут током осам година биће тестирана у различитим режимима рада и мора да докаже своју способност да ради у складу са својим наменом”, саопштила је фабрика.

На друштвеним мрежама појавиле су се фотографије и видео-снимци на којима се види како *Alrosa* напушта сервисно постројење. Важно је напоменути да је нејасно колико ће трајати ова испитивања и када ће тачно подморница бити проглашена спремном за оперативну употребу. Штавише, то су фабричка испитивања која ће вероватно бити праћена даљим истраживањима које ће спровести руска морнарица.

Постоји неколико детаља о степену надоградње, али се наводи да је подморница пренаоружана подзвучним крстарећим ракетама 3М14 *Kalibr,*

[2] www.thedrive.com/the-war-zone Jun 28, 2022

познатих у НАТО-у као SS-N-30A *Sagaris*. Ово оружје је у широкој употреби у Русији од почетка рата у Украјини, а лансира се са бродова и подморница. Пре тога, ове ракете користила је руска морнарица током интервенције Москве у грађанском рату у Сирији. Ракета 3М14 *Kalibr* је у великој мери слична крстарећој ракети *RGM-109 Tomahawk,* а претпоставља се да има домет између 1.500 и 2.500 километара, наоружана је високоексплозивном бојевом главом масе 450 килограма. Ракете *Kalibr* могу се заменити противбродским ракетама 3М54, али с обзиром на одсуство украјинске претње у виду површинских бродова, мало је вероватно да ће подморница бити наоружана њима.

Alrosa, на сувом доку, открива детаље свог прилично напредног дизајна пумпно-млазног погона.

Прошлог месеца, у извештајима руских државних медија, описани су елементи надоградње подморнице *Alrosa* које су укључиле „нове борбене и техничке способности”, које су довеле до стандарда упоредивог са шест унапређених подморница Црноморске флоте последњих година. То се односи на најновије подморнице класе *Project 636.3* или *Improved Kilo,* а које од самог почетка имају ракете *Kalibr*. Ове модерније подморнице испоручене су између 2014. и 2017. године.

Оно што је необично код подморнице *Alrosa*, која је унапређена по јединственом стандарду познатом као Project 877В, јесте њен погонски систем. Док стандардне дизел-електричне подморнице класе *Kilo* покреће пропелер са шест или седам лопатица, *Alrosa* има млазни пропулзор.

Млазно-пумпни погон има низ предности у односу на пропелере, пре свега могућност достизања већих брзина без бучне кавитације, што значи

Vučković, D., SAVREMENO NAORUŽANJE I VOJNA OPREMA / MODERN WEAPONS AND MILITARY EQUIPMENT, pp.1039-1048

да се могу прелазити велике удаљености много брже. Према руским извештајима, подморнице са млазним пумпама називају се и „црне рупе” због својих карактеристика за сузбијање буке.

Штавише, млазнице пумпе су, такође, ефикасније у односу на укупне перформансе подморнице и исказују посебне предности у плитким водама, што се нарочито односи на Црно море. Истовремено, млазне пумпе су тешке, скупље и сложеније од пропелера, али су недавно нашле примену и на неким подморницама класе *Trafalga*r Краљевске морнарице, а касније и на америчким нападним нуклеарним подморницама класе *Seawolw* и *Virginia*.

Чини се да је код подморнице *Alrosa* пумпно-млазни погон био намењен само за експерименталну употребу, те се касније није поново појавио у каснијим итерацијама *Kilo* дизајна. Међутим, пумпно-млазни погон покреће руске подморнице са балистичким пројектилима на нуклеарни погон класе *Borei*.

Alrosa је знатно старија подморница, изграђена у бродоградилишту Красноје Сормово у тадашњем Горком (данас Нижњи Новгород), и први пут је поринута у септембру 1989. године, непосредно пре распада Совјетског Савеза.

Са нестанком совјетске морнарице, *Alrosa* је остала на Криму, који је тада још увек био део нове независне Украјине. Године 1992. већи део Црноморске флоте контролисала је Украјина, а покушано је и формирање независне украјинске морнарице. Руски извештаји наводе да је, у марту те године, Украјина покушала да заплени подморницу, иако је недостатак резервних делова и батерија у то време онемогућио подморници дејство у редовним операцијама.

Без обзира на то, ситуација је брзо довела до тензија са Москвом и договора о заједничкој флоти под билатералном командом, иако је у пракси доминирала руска морнарица, која је задржала своје упориште на Криму. Украјинска морнарица је наставила да постоји, такође, као посебна целина.

Коначно, 1997. године потписан је споразум којим су подељена средства бивше совјетске Црноморске флоте између Украјине и Русије, при чему је Москва плаћала Кијеву за коришћење кримских поморских објеката на основу уговора о закупу. У том тренутку, *Alrosa* је постала једина активна подморница руске Црноморске флоте, чији је стратешки значај опао одмах након завршетка Хладног рата.

Након руске анексије полуострва Крим, 2014. године, украјинска морнарица је исељена, а руска Црноморска флота је преузела контролу над бившим бродовима украјинске морнарице.

У годинама после анексије, *Alrosa* је чамила у луци у Севастопољу. Иако је њена надоградња и поновно активирање започело много пре актуелног рата у Украјини, чини се да је одлука да се подморница врати Црноморској флоти вођена захтевима сукоба, а посебно потребом за крстарећим пројектилима платформе.

Подморница Project 877В Alrosa у Севастопољу, у августу 2005. године

До почетка јуна очекивало се да ће *Alrosa* прећи у Балтичку флоту, када се врати у употребу, а да ће се затим користити као подморница за обуку посада намењених за новије подморнице Project 636.3. Овај трансфер првобитно је био планиран за 2020. годину, али је наводно заустављен због кашњења у радовима на поправци.

Затим, 17. јуна 2022. године, објављено је да ће *Alrosa* ипак остати у саставу Црноморске флоте, у 4. самосталној подморничкој бригади у Севастопољу.

На одлуку да се *Alrosa* задржи у саставу Црноморске флоте можда је утицала и доступност осталих шест подморница. Непотврђени извештаји сугеришу да је, средином јуна 2022, флота имала само две подморнице које су биле потпуно активне и оперативне у Црном мору, док су још две у припреми у Средоземном мору. Од осталих подморница, једна је била на сувом доку, друга је последњи пут виђена у мају, а трећа је била усидрена у Севастопољу.

Чак и без појачавања темпа напада крстарећим ракетама, због тренутне ситуације, Црноморској флоти у великој мери недостају подморнице. Такође, упитна је изводљивост враћања две подморнице из Средоземног мора у Црно море због услова Монтрејске конвенције.

Конвенција поставља ограничења за ратне бродове који не припадају Турској, а пролазе између Црног мора и Средоземног мора и проширује се на руске подморнице. Изузетак се може направити ако се подморница креће ка или из матичне луке на Црном мору ради поправке, што је аргумент који је Русија користила у прошлости. То је био случај са борбеним операцијама подморница Црноморске флоте у Сирији.

Међутим, чак и са свих шест подморница класе Improved Kilo у Црном мору, рутинско одржавање, обука посаде и допуна горива, оружја и залиха биће тешко одрживи.

Током разговора са британским адмиралом Дејвидом Радакином, главнокомандујући украјинских оружаних снага Валериј Залужни изнео је податак да је, само током јуна 2022. године, Русија лансирала на Украјину . преко 150 крстарећих ракета „са разних платформи",

Сада различите платформе, укључујући бомбардере Tu-22M3, лансирају своје ракете изнад Белорусије, стратешки бомбардери Tu-95MS и Tu-160 лансирају крстареће ракете изнад Русије, док су досад поморски бродови, укључујући фрегате, корвете и подморнице Црноморске флоте, дуго имали кључну улогу у наношењу ових удара.

Класа Kilo је релативно компактна, а свака подморница Project 636.3 наводно носи само четири ракете *Kalibr*. Иако није потврђено, вероватно је да надограђена *Alrosa* има сличан капацитет.

Побољшане перформансе пумпно-млазног погона нису од значаја у сукобу у којем нема противподморничке претње. Чињеница да је *Alrosa* раније била намењена за обуку сугерише да било која предност коју јој даје јединствени погон вероватно није процењена као довољна да би се одржала на првој линији фронта.

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Значај приватног сектора у производњи тактичке опреме

Производни програми из домена наменске (одбрамбене) индустрије нису ексклузивно резервисани за државне произвођаче, као ни за оне чији је већински власник држава. Несумњиво, свака национална држава и те како има интерес да у што већој мери контролише и диктира развој националне наменске индустрије, дакле домаћих произвођача, са седиштима и производњом на државној територији. То је посебно уочљиво (и оправдано) када је у питању, на пример, производња муниције, односно развој и производња стратегијског конвенционалног наоружања. Државна управа остварује и штити своје интересе у овој сфери кроз позитивноправно законодавство и бројне институте, затим планирањем издатака у националном буџету, субвенцијама, али и обезбеђивањем међународне војнотехничке сарадње и трансфера технологије, у оквиру политичких и политичко-војних споразума са партнерским земљама. Држава има интерес у руковођењу националном наменском индустријом првенствено због безбедности, јер се, на пример, у случају ратног стања национална држава у највећој мери мора ослањати управо на ресурсе, капацитете и производе које је у стању самостално да експлоатише, односно уводи у употребу[1]. У недавној прошлости дешавало се да су набавке различитих система из иностранства за потребе Министарства одбране и Војске Србије биле или веома отежане или чак стопиране, првенствео услед сложених геополитичких околности, без обзира на то што је српска страна, као купац, била потпуно солвентна. Друга врста разлога који изискују добро организовану националну одбрамбену индустрију је економске природе, јер је ова област пословања веома лукративна, а последњих година су приметна и увећана издвајања средстава за оружане снаге, како у свету, тако и на простору тзв. Западног Балкана[2].

Удео приватног сектора у овој области пословања је много више изражен у традиционално капиталистичким земљама, у којима се током деценија предузетништво, заједно са преузимањем ризика и иновацијама, много више подржава, што је приметно, на пример, и у америчкој Стратегији националне безбедности из децембра 2017. године. Ово је и потпуно логично, јер је у традиционално капиталистичким привредама управо приватни сектор носилац огромног капитала, као и најнапреднијих технологија и знања, а самим тим и иновација које каткад представљају и тзв. *game changer*, имајући потенцијал да пореметe постојећи однос снага, дајући једној страни осетну предност, а ривалима задатак да ту предност

[1] То је очигледно када је реч о аутоматским пушкама које се уводе као стандардно примарно наоружање у националне оружане снаге. Наиме, држава се, по правилу, труди да највећи део својих борбених јединица опреми системима које производи национална наменска индустрија, попут српске пушке „Застава М21”, или „Застава М70”, у различитим моделима.

[2] Више о томе у бази података Стокхолмског међународног института за истраживање мира (СИПРИ): https://milex.sipri.org/sipri

што пре достигну, престигну или анулирају. Следствено томе, управо приватни сектор врло често помера границе и диктира развој технологија, као и њихову војну апликацију. Управо због тога, државни и војни руководиоци труде се да остварују што бољу сарадњу са носиоцима приватног сектора у земљи, обезбеђујући им што повољније пословно окружење на различите начине, попут креирања либералнијих правних прописа и сл. На тај начин омогућавају јачу синергију између врхунских капацитета приватног сектора, с једне стране, и потребу да се испрати технолошка динамика из области војног наоружања и опреме, те задовоље потребе националног система безбедности, с друге стране.

Резултат улагања у националну наменску индустрију: лансер ПМНЛ „Сова Б-2”. Фото: Милош Јевтић

Када је у питању простор ранијег Источног блока, присуство државе је знатно израженије у сфери одбрамбене индустрије, првенствено преко јавних предузећа, односно правних лица чији је већински власник државни капитал. Такав је случај и са Републиком Србијом и српском одбрамбеном индустријом у коју се последњих година осетно значајније инвестира, а српски произвођачи годинама бележе увећан извоз у партнерске земље, посебно када је реч о категоријама конвенционалног наоружања. То је последица више фактора, почевши од мењања државне политике према капацитетима националне одбрамбене индустрије, до чињенице да је највећи део развојних и производних капацитета управо државни капитал.

Поред тога, прописи којима се уређује увоз и извоз производних материјала, полупроизвода и готових производа иде у прилог управо државним привредним субјектима.

За приватни сектор ситуација је нешто повољнија када је у питању опрема, условно речено, мањег тактичког значаја, попут тактичке опреме, резонујући овај појам екстензивно и подразумевајући било коју врсту опреме за професионалце и цивиле која има тактичку примену.

Захваљујући израженијој доступности информација и технологија, испреплетености и динамици међународних економских односа, приватни привредни субјекти често имају већу конкурентност од државних. То значи да обогаћују тржиште, истовремено нудећи крајњим корисницима додатну и често квалитетнију опцију за задовољавање њихових потреба, те на тај начин усмеравају даљи развој тржишта.

Иновације су, истина, веома скупе за реализацију, па их најчешће остварују управо произвођачи који највише средстава улажу у развој. С друге стране, напредак и боље позиционирање на домаћем тржишту се врло често остварује и јефтинијом методом – „пресликавањем” решења која су се већ показала као успешна у иностранству, те постала стандард у изради тактичке опреме и опремању крајњих корисника.

Српски бренд „TACTIX” – тактичка одећа и опрема за високообучене професионалце

Српско тржиште тактичке опреме је током 2018. године постало богатије за бренд „TACTIX”, који потписује „Хорус доо”. Он се састоји од тактичке опреме и одеће која се примарно израђује за професионалце – припаднике полицијских и војних јединица. Оно што је заједничко свим члановима тима који се окупио око бренда „TACTIX” јесте жеља да купцима у Србији, али и у иностранству, понуде опцију која ће по квалитету бити у рангу најпознатијих светских брендова, уз што приступачнију цену.

Значајнији контингенти тактичке опреме из овог програма испоручени су специјалистичким тимовима Управе криминалистичке полиције Министарства унутрашњих послова Републике Србије. Посебно су атрактивне дводелне тактичке униформе у маскирној шари *black multicam*, коју произвођач, за сада једини у оригиналној шари, користи у производњи тактичке одеће.

Детаљ са сајма „Партнер 2021”. Фото: Милош Јевтић

Имајући у виду примарну циљну групу, експерти за развој овог тактичког програма велику пажњу посвећују потребама, сугестијама, као и рецензијама професионалаца, који опрему овакве врсте користе свакодневно и у свим условима, те су и најкомпетентнији у давању мишљења. С тим у вези, корисници имају врло добро мишљење о тактичким панталонама „Tactix P-5 Supreme Combat Pants”, „Tactix P-8”, као и „Tactix P-7 Winter Tactical Pants”. Искуства у изради ових модела представљају основ за развој будућих. Овакав однос произвођача према крајњим корисницима је вишеструко користан, јер омогућава константно усавршавање у процесу производње. То подразумева проучавање захтева са терена које износе непосредни корисници, те конструисање најпрактичнијих решења у финалном решењу сваког производа, који морају да задовољавају високе критеријуме функционалности (попут лаког и брзог приступа џеповима и опреми у различитим положајима тела), максималну удобност при ношењу и што мању силуету корисника. Управо је зато мото бренда и тима који стоји иза њега с правом „Made With Experience”, тј. прављено, произведено са искуством. Истовремено, експерти задужени за развој прате и стандарде и иновације у иностранству и, у складу са могућностима, имплементирају их у свој производни програм. Императив у развоју сваког производа је употреба висококвалитетних материјала, попут „TenCate Tecawork” поли-памук 65/35, рајсфершлуса и чичака„YKK”, ојачања „C.F. Weber Cordura”, „Schoeller-dynamic” и др., као и доказано практичних техничких решења попут система за брзо откопчавање „2M ROC/80”, којим је опремљен носач плоча „N-1”.

Припадници УКП позирају у тактичким униформама „Tactix”. Фото: Милош Јевтић

Представљање у иностранству је важан део стратегије „Хорусовог” тима, па се производи из програма „Tactix” редовно могу видети на најзначајнијим сајмовима у земљи, попут сајма „Партнер”, као и у Европи и свету. То су уједно и прилике за размену искустава са партнерима из иностранства, односно њихову евентуалну имплементацију у даљи развој овог тактичког програма.

Карактеристике тактичке лампе

Тактичке лампе данас представљају популаран и веома практичан део тактичке опреме професионалаца у полицијским, војнополицијским и војним јединицама. Услед све веће популарности ових производа, важно је разликовати тактичке и тзв. лампе за свакодневну употребу (енг. *every day carry*). Треба поменути да се данас на тржишту модели лампи за свакодневну употребу врло често представљају као тактички, како би се купци убедили да за одређени модел одвоје више новца. Стога, прегледом основних карактеристика тактичких модела лампи, покушаћемо да направимо дистинкцију у односу на моделе других намена.

Тактичка лампа „Olight M2R Pro Warrior”. Фото: 365 Plus.

Прва карактеристика тактичких лампи јесте да су намењене професионалцима – припадницима полицијских, војних и војнополицијских јединица, што примарно значи да су израђене да задовоље њихове свакодневне теренске потребе, тј. тактичку апликацију. Међу мноштвом теренских захтева издваја се потреба за обезбеђивањем веома интензивног излаза светла, како би се тактичка лампа ефективно тактички примењивала – при осветљавању тамних области, при трагању и спасавању, односно при самоодбрани. Прегледом тактичких модела лампи најпознатијих произвођача може се закључити да већина задовољава критеријум од 1000 лумена, који многи корисници оцењују као оптималан. Ипак, имајући у виду велику различитост ситуација у којима се тактичка лампа примењује, није захвално постављати овакве уопштене стандарде[3]. На пример, урбано окружење у којем је ваздух загађенији у условима смањене видљивости захтева другачије осветљење циља, за разлику од руралног. У вези с тим, потребно је објаснити начин рада савремене батеријске лампе. Код традиционалне лампе светлост излази у свим правцима, док батеријска лампа има рефлектор који окружује сијалицу, фокусирајући тако сноп светлости у облику конуса. Начин на који су конструисани склоп сијалице и рефлектора утиче на начин на који светлост стиже до циља и осветљава га. Савремене тактичке лампе корисницима нуде и различите „режиме” рада, тј. јачине осветљености (најчешће три), као и тзв. строб (треперење) светло, које се у пракси показало као веома

[3] Управо зато је усвојен стандард *ANSI FL1*, који обједињава седам стандарда (излаз светлости, удаљеност снопа, период рада, максимални интензитет снопа, отпорност на ударце, водонепропусност, водоотпорност) на основу којих се могу тестирати и упоређивати батеријске лампе различитих произвођача.

корисно при суочавању са агресивним појединцима, тј. при самоодбрани. Наиме, емитовање строб светла јаког интензитета може привремено дезоријентисати и заслепити нападача, који покретне објекте опажа несинхронизовано, „исецкано” (тзв. треперећа вртоглавица, или Бучов ефекат), дајући тако браниоцу неколико вредних секунди предности. Конструкција тактичке лампе мора да омогући кориснику логичан и једноставан одабир режима рада, како би у стресним ситуацијама могућност грешке у руковању била сведена на минимум. У том смислу, дугме, или тастер за паљење и гашење (каткад истовремено и за одабир јачине светла или строб светла), врло често се поставља на задњем крају лампе. Корисник му лако може приступити палцем шаке или притискањем лампе о тело, део личне опреме и слично. Поред тога, тиме се кориснику олакшава да у врло стресним ситуацијама лако утврди који је крај лампе предњи, а који задњи, посебно када лампа не ради, што може бити корисно при самоодбрани, јер је предња ивица лампе често ожлебљена, исечена, како би лампа могла да се употреби при самоодбрани, или при ломљењу стакла.

Тактичке лампе монтиране на системе „SIG 516 Patrol”. Фото: Милош Јевтић

Робустност тактичке лампе, која се испољава и у стандардима отпорности на механичке ударце, водоотпорности, водонепропусности из стандарда *ANSI FL1*, такође је императив. Наиме, лампа мора радити у окружењу у којем се налази њен корисник, а у случају припадника јединица за специјалне намене то су сви климатско-теренски услови. Робустност је можда и карактеристика по којој се врхунске тактичке лампе највише разликују од осталих батеријских лампи.

Компактан дизајн и добра ергономија такође су важни стандарди за професионалце. Компактност олакшава одлагање лампе у личној опреми (нпр. на тактичком опасачу), као и брзи приступ по потреби. Такође, ова карактеристика олакшава руковање у скученом окружењу, омогућава интеграцију, комбиновање са личним наоружањем, умањује могућност нежељеног качења лампе за делове одеће, опреме или предмета из окружења, а по правилу умањује и укупну масу лампе. Добра ергономија и профилисање основних команди су важни, јер корисници често морају руковати лампама носећи рукавице, влажним шакама и сл.

Капацитет и поузданост батерије су такође дистинктивна карактеристика квалитетних тактичких лампи у односу на лампе намењене за свакодневну употребу. Тактичке лампе на тржишту често су компатибилне са литијумским батеријама *CR123* 3V, прихватајући најчешће по две батерије истовремено.

У вези са компатибилношћу лампи са наоружањем, треба направити разлику између модела лампи који су посебно конструисани за прихватање на лично наоружање, представљајући тако тактичке додатке (нпр. тактичке лампе за пиштољ монтирају се брављењем на шину на доњој страни рама пиштоља) и модела који су намењени за руковање, али се по потреби и уз одговарајући носач могу монтирати на наоружање. С друге стране, комбиновање лампи за свакодневну употребу са личним наоружањем је нелогично и тактички неоправдано, осим у ургентним ситуацијама. Ове лампе, по правилу, нису компатибилне за монтирање на наоружање, осим импровизацијом (нпр. учвршћивањем лепљивом траком на потхват оружја, тј. на облогу цеви).

На основу наведених карактеристика тактичких лампи и стандарда у њиховој изради може се закључити да су то робустне батеријске лампе, конструисане за потребе припадника полицијских, војних и војнополицијских јединица и намењене за тактичку примену.

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ПОЗИВ И УПУТСТВО АУТОРИМА

ПРИГЛАШЕНИЕ И ИНСТРУКЦИЯ ДЛЯ АВТОРОВ РАБОТ

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ПОЗИВ И УПУТСТВО АУТОРИМА О НАЧИНУ ПРИПРЕМЕ ЧЛАНКА

Упутство ауторима о начину припреме чланка за објављивање у Војнотехничком гласнику урађено је на основу Правилника о категоризацији и рангирању научних часописа Министарства просвете, науке и технолошког развоја Републике Србије ("Службени гласник РС", број 159/20). Примена овог Правилника првенствено служи унапређењу квалитета домаћих часописа и њиховог потпунијег укључивања у међународни систем размене научних информација.

Војнотехнички гласник / Vojnotehnički glasnik / Military Technical Courier (втг.мо.упр.срб, www.vtg.mod.gov.rs, ISSN 0042-8469 – штампано издање, e-ISSN 2217-4753 – online, UDC 623+355/359, DOI: 10.5937/VojnotehnickiGlasnik; https://doi.org/10.5937/VojnotehnickiGlasnik), јесте рецензирани међународни научни часопис.

Власници часописа су Министарство одбране Републике Србије и Војска Србије. Издавач и финансијер часописа је Универзитет одбране у Београду (Војна академија).

Програмска оријентација часописа заснива се на годишњој категоризацији часописа, коју врши надлежно државно министарство у одређеним областима, као и на његовом индексирању у међународним индексним базама.

Часопис обухвата научне, односно стручне области у оквиру образовно-научног поља **природно-математичких наука**, као и у оквиру образовно-научног поља **техничко-технолошких наука**, а нарочито области **одбрамбених наука и технологија**. Објављује теоријска и практична достигнућа која доприносе усавршавању свих припадника српске, регионалне и међународне академске заједнице, а посебно припадника војски и министарстава одбране. Публикује радове са уравнотеженим извештавањем о аналитичким, експерименталним и примењеним истраживањима, као и нумеричким симулацијама, обухватајући различите дисциплине. Објављени материјали су високог квалитета и релевантности, написани на начин који их чини доступним широкој читалачкој публици. Сви радови који извештавају о оригиналним теоријским и/или практично оријентисаним истраживањима или проширеним верзијама већ објављених радова са конференција су добродошли. Радови за објављивање одабиру се двоструко слепим поступком рецензије како би се осигурала оригиналност, релевантност и читљивост. Притом циљ није само да се квалитет објављених радова одржи високим већ и да се обезбеди правовремени, темељни и уравнотежени поступак рецензије.

Уређивачка политика Војнотехничког гласника заснива се на препорукама Одбора за етичност у издаваштву (COPE Core Practices), као и на најбољим прихваћеним праксама у научном издаваштву. Војнотехнички гласник је члан COPE (Committee on Publication Ethics) од 2. маја 2018. године.

Министарство просвете, науке и технолошког развоја Републике Србије утврдило је дана 23. 12. 2021. године категоризацију Војнотехничког гласника, за 2021. годину:

– на листи часописа за рачунарске науке:
категорија истакнути национални часопис **(М52)**,

– **на листи часописа за електронику, телекомуникације и информационе технологије:**
категорија истакнути национални часопис **(М52)**,
– **на листи часописа за машинство:**
категорија врхунски часопис националног значаја **(М51)**,
– **на листи часописа за материјале и хемијске технологије:**
категорија врхунски часопис националног значаја **(М51)**.

Усвојене листе домаћих часописа за 2021. годину могу се видети на сајту Војнотехничког гласника, страница *Категоризација часописа*.

Детаљније информације могу се пронаћи и на сајту Министарства просвете, науке и технолошког развоја Републике Србије.

Подаци о категоризацији могу се пратити и на сајту КОБСОН-а (Конзорцијум библиотека Србије за обједињену набавку).

Категоризација часописа извршена је према Правилнику о категоризацији и рангирању научних часописа Министарства просвете, науке и технолошког развоја Републике Србије ("Службени гласник РС", број 159/20).

Часопис се прати у контексту Српског цитатног индекса – СЦиндекс (база података домаћих научних часописа), Научно-информационог система Redalyc и Руског индекса научног цитирања (РИНЦ). Подвргнут је сталном вредновању (мониторингу) у зависности од утицајности (импакта) у самим базама. Детаљи о индексирању могу се видети на сајту Војнотехничког гласника, страница *Индексирање часописа*.

Војнотехнички гласник омогућава и примењује Creative Commons (CC BY) одредбе о ауторским правима. Детаљи о ауторским правима могу се видети на сајту часописа, страница *Ауторска права и политика самоархивирања*.

Радови се предају путем онлајн система за електронско уређивање АСИСТЕНТ, који је развио Центар за евалуацију у образовању и науци (ЦЕОН).

Приступ и регистрација за сервис врше се на сајту www.vtg.mod.gov.rs, преко странице *АСИСТЕНТ* или *СЦИНДЕКС*, односно директно на линку aseestant.ceon.rs/index.php/vtg.

Детаљно упутство о регистрацији и пријави за сервис налази се на сајту www.vtg.mod.gov.rs, страница *Упутство за АСИСТЕНТ*.

Потребно је да се сви аутори који подносе рукопис за објављивање у Војнотехничком гласнику региструју у регистар ORCID (Open Researcher and Contributor ID), према упутству на страници сајта *Регистрација за добијање ORCID идентификационе шифре*.

Војнотехнички гласник објављује чланке на енглеском језику (arial, величина слова 11 pt, проред Single).

Поступак припреме, писања и уређивања чланка треба да буде у сагласности са *Изјавом о етичком поступању* (http://www.vtg.mod.gov.rs/izjava-o-etickom-postupanju.html).

Чланак треба да садржи сажетак са кључним речима, увод (мотивацију за рад), разраду (адекватан преглед репрезентативности рада у његовој области, јасну изјаву о новини у представљеном истраживању, одговарајућу теоријску позадину, један или више примера за демонстрирање и дискусију о представљеним идејама), закључак и литературу (без нумерације наслова и поднаслова). Обим чланка треба да буде до једног ауторског табака (16 страница формата А4 са проредом Single), а највише 24 странице.

Чланак треба да буде написан на обрасцу за писање чланка, који се у електронској форми може преузети са сајта на страници *Образац за писање чланка*.

Наслов

Наслов треба да одражава тему чланка. У интересу је часописа и аутора да се користе речи прикладне за индексирање и претраживање. Ако таквих речи нема у наслову, пожељно је да се придода и поднаслов.

Текући наслов

Текући наслов се исписује са стране сваке странице чланка ради лакше идентификације, посебно копија чланака у електронском облику. Садржи презиме и иницијал имена аутора (ако аутора има више, преостали се означавају са „et al." или „и др."), наслове рада и часописа и колацију (година, волумен, свеска, почетна и завршна страница). Наслови часописа и чланка могу се дати у скраћеном облику.

Име аутора

Наводи се пуно име и презиме (свих) аутора. Веома је пожељно да се наведу и средња слова аутора. Имена и презимена домаћих аутора увек се исписују у оригиналном облику (са српским дијакритичким знаковима), независно од језика на којем је написан рад.

Назив установе аутора (афилијација)

Наводи се пун (званични) назив и седиште установе у којој је аутор запослен, а евентуално и назив установе у којој је аутор обавио истраживање. У сложеним организацијама наводи се укупна хијерархија (нпр. Универзитет одбране у Београду, Војна академија, Катедра природно-математичких наука). Бар једна организација у хијерархији мора бити правно лице. Ако аутора има више, а неки потичу из исте установе, мора се, посебним ознакама или на други начин, назначити из које од наведених установа потиче сваки од наведених аутора. Афилијација се исписује непосредно након имена аутора. Функција и звање аутора се не наводе.

Контакт подаци

Адреса или е-адреса свих аутора даје се поред имена и презимена аутора.

Категорија (тип) чланка

Категоризација чланака обавеза је уредништва и од посебне је важности. Категорију чланка могу предлагати рецензенти и чланови уредништва, односно уредници рубрика, али одговорност за категоризацију сноси искључиво главни уредник.

Чланци у *Војнотехничком гласнику* класификују се на научне и стручне чланке.

Научни чланак је:

– оригиналан научни рад (рад у којем се износе претходно необјављени резултати сопствених истраживања научним методом);

– прегледни рад (рад који садржи оригиналан, детаљан и критички приказ истраживачког проблема или подручја у којем је аутор остварио одређени допринос, видљив на основу аутоцитата);

– кратко или претходно саопштење (оригинални научни рад пуног формата, али мањег обима или прелиминарног карактера);

– научна критика, односно полемика (расправа на одређену научну тему, заснована искључиво на научној аргументацији) и осврти.

Изузетно, у неким областима, научни рад у часопису може имати облик монографске студије, као и критичког издања научне грађе (историјско-архивске, лексикографске, библиографске, прегледа података и сл.), дотад непознате или недовољно приступачне за научна истраживања.

Радови класификовани као научни морају имати бар две позитивне рецензије.

Ако се у часопису објављују и прилози ваннаучног карактера, научни чланци треба да буду груписани и јасно издвојени у првом делу свеске.

Стручни чланак је:

– стручни рад (прилог у којем се нуде искуства корисна за унапређење професионалне праксе, али која нису нужно заснована на научном методу);

– информативни прилог (уводник, коментар и сл.);

– приказ (књиге, рачунарског програма, случаја, научног догађаја, и сл).

Пожељно је да обим кратких саопштења буде 4 до 7 страница, научних чланака и студија случаја 10 до 14 страница, док прегледни радови могу бити и дужи. Број страница није строго ограничен и, уз одговарајуће образложење, пријављени чланци такође могу бити дужи или краћи.

Ако су радови који су претходно објављени на конференцији проширени, уредници ће проверити да ли је додато довољно новог материјала који испуњава стандарде часописа и квалификује поднесак за поступак рецензије. Додати материјал не сме бити претходно објављен. Нови резултати нису нужно потребни, али су пожељни. Међутим, поднесак треба да садржи проширене кључне идеје, примере, разраде, итд., који су претходно били садржани у поднеску са конференције.

Језик рада

Језик рада треба да буде енглески.

Текст мора бити језички и стилски дотеран, систематизован, без скраћеница (осим стандардних). Све физичке величине морају бити изражене у Међународном систему мерних јединица – SI. Редослед образаца (формула) означава се редним бројевима, са десне стране у округлим заградама.

Сажетак

Сажетак јесте кратак информативан приказ садржаја чланка који читаоцу омогућава да брзо и тачно оцени његову релевантност. У интересу је уредништава и аутора да сажетак садржи термине који се често користе за индексирање и претрагу чланака. Саставни делови сажетка су увод/циљ истраживања, методи, резултати и закључак. Сажетак треба да има од 100 до 250 речи и треба да се налази између заглавља (наслов, имена аутора и др.) и кључних речи, након којих следи текст чланка.

Кључне речи

Кључне речи су термини или фразе које адекватно представљају садржај чланка за потребе индексирања и претраживања. Треба их додељивати ослањајући се на неки међународни извор (попис, речник или тезаурус) који је најшире прихваћен или унутар дате научне области. За нпр. науку уопште, то је листа кључних речи Web of Science. Број кључних речи не може бити већи од 10, а у интересу је уредништва и аутора да учесталост њихове употребе буде што већа. У чланку се пишу непосредно након сажетка.

Систем АСИСТЕНТ у ту сврху користи специјалну алатку KWASS: аутоматско екстраховање кључних речи из дисциплинарних тезауруса/речника по избору и рутине за њихов одабир, тј. прихватање односно одбацивање од стране аутора и/или уредника.

Датум прихватања чланка

Датум када је уредништво примило чланак, датум када је уредништво коначно прихватило чланак за објављивање, као и датуми када су у међувремену достављене евентуалне исправке рукописа наводе се хронолошким редоследом, на сталном месту, по правилу на крају чланка.

Захвалница

Назив и број пројекта, односно назив програма у оквиру којег је чланак настао, као и назив институције која је финансирала пројекат или програм, наводи се у посебној напомени на сталном месту, по правилу при дну прве стране чланка.

Претходне верзије рада

Ако је чланак у претходној верзији био изложен на скупу у виду усменог саопштења (под истим или сличним насловом), податак о томе треба да буде наведен у посебној напомени, по правилу при дну прве стране чланка. Рад који је већ објављен у неком часопису не може се објавити у Војнотехничком гласнику (прештампати), ни под сличним насловом и измењеном облику.

Табеларни и графички прикази

Пожељно је да наслови свих приказа, а по могућству и текстуални садржај, буду дати двојезично, на језику рада и на енглеском језику.

Табеле се пишу на исти начин као и текст, а означавају се редним бројевима са горње стране. Фотографије и цртежи треба да буду јасни, прегледни и погодни за репродукцију. Цртеже треба радити у програму word или corel. Фотографије и цртеже треба поставити на жељено место у тексту.

За слике и графиконе не сме се користити снимак са екрана рачунара програма за прикупљање података. У самом тексту чланка препоручује се употреба слика и графикона непосредно из програма за анализу података (као што су Excel, Matlab, Origin, SigmaPlot и други).

Навођење (цитирање) у тексту

Начин позивања на изворе у оквиру чланка мора бити једнообразан.

Војнотехнички гласник за референцирање (цитирање и навођење литературе) примењује Харвардски систем референци, односно Харвардски приручник за стил (Harvard Referencing System, Harvard Style Manual). У самом тексту, у обичним заградама, на месту на којем се врши позивање, односно цитирање литературе набројане на крају чланка, обавезно у обичној загради написати презиме цитираног аутора, годину издања публикације из које цитирате и, евентуално, број страница. Нпр. (Petrović, 2012, pp.10–12).

Детаљно упутство о начину цитирања, са примерима, дато је на страници сајта *Упутство за Харвардски приручник за стил*. Потребно је да се позивање на литературу у тексту уради у складу са поменутим упутством.

Систем АСИСТЕНТ у сврху контроле навођења (цитирања) у тексту користи специјалну алатку CiteMatcher: откривање изостављених цитата у тексту рада и у попису референци.

Напомене (фусноте)

Напомене се дају при дну стране на којој се налази текст на који се односе. Могу садржати мање важне детаље, допунска објашњења, назнаке о коришћеним изворима (на пример, научној грађи, приручницима), али не могу бити замена за цитирану литературу.

Листа референци (литература)

Цитирана литература обухвата, по правилу, библиографске изворе (чланке, монографије и сл.) и даје се искључиво у засебном одељку чланка, у виду листе референци. Референце се не преводе на језик рада и набрајају се у посебном одељку на крају чланка.

Војнотехнички гласник, као начин исписа литературе, примењује Харвардски систем референци, односно Харвардски приручник за стил (Harvard Referencing System, Harvard Style Manual).

Литература се обавезно пише на латиничном писму и набраја по абецедном редоследу, наводећи најпре презимена аутора, без нумерације.

Детаљно упутство о начину пописа референци, са примерима, дато је на страници сајта *Упутство за Харвардски приручник за стил*. Потребно је да се попис литературе на крају чланка уради у складу са поменутим упутством.

Нестандардно, непотпуно или недоследно навођење литературе у системима вредновања часописа сматра се довољним разлогом за оспоравање научног статуса часописа.

Систем АСИСТЕНТ у сврху контроле правилног исписа листе референци користи специјалну алатку RefFormatter: контрола обликовања референци у складу са Харвардским приручником за стил.

Изјава о ауторству

Поред чланка доставља се *Изјава о ауторству* у којој аутори наводе свој појединачни допринос у изради чланка. Такође, у тој изјави потврђују да су чланак урадили у складу са *Позивом и упутством ауторима* и *Изјавом о етичком поступању часописа*.

Сви радови подлежу стручној рецензији.

Списак рецензената Војнотехничког гласника може се видети на страници сајта *Списак рецензената*. Процес рецензирања објашњен је на страници сајта *Рецензентски поступак*.

Уредништво

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ПРИГЛАШЕНИЕ И ИНСТРУКЦИЯ ДЛЯ АВТОРОВ О ПОРЯДКЕ ПОДГОТОВКИ СТАТЬИ

Инструкция для авторов о порядке подготовки статьи к опубликованию в журнале «Военно-технический вестник» разработана согласно Регламенту о категоризации и ранжировании научных журналов Министерства образования, науки и технологического развития Республики Сербия («Службени гласник РС», № 159/20). Применение этого Регламента способствует повышению качества отечественных журналов и их более полному вовлечению в международную систему обмена научной информацией.

Военно-технический вестник (**Vojnotehnički glasnik / Military Technical Courier**), втг.мо.упр.срб, www.vtg.mod.gov.rs/index-ru.html, ISSN 0042-8469 – печатное издание, e-ISSN 2217-4753 – online, UDK 623+355/359, DOI: 10.5937/VojnotehnickiGlasnik; https://doi.org/10.5937/VojnotehnickiGlasnik, является рецензируемым международным научным журналом.

Собственники журнала: Министерство обороны и Вооруженые силы Республики Србия.

Издатель журнала: Университет обороны в г. Белград (Военная академия).

Программная ориентация журнала основана на ежегодной категоризации журнала, которая производится соответствующим отраслевым министерством, в зависимости от области исследований, а также на его индексировании в международных наукометрических базах данных.

Журнал охватывает научные и профессиональные сферы в рамках учебно-научной области **естественно-математических наук**, а также в рамках учебно-научной области **технико-технологических наук**, особенно в области **оборонных наук и технологии**. В журнале публикуются теоретические и практические достижения, которые способствуют повышению квалификации представителей сербского, регионального и международного академического сообщества, особенно служащих Министерств Обороны и Вооружённых сил. В журнале публикуются статьи со соответствующими обзорами об аналитических, экспериментальных и прикладных исследованиях, а также о численном моделировании, охватывая различные дисциплины. Публикуемые материалы отличаются высоким качеством и актуальностью. Они написаны научным, но понятным и доступным для широкого круга читателей языком. Приветствуются все статьи, сообщающие об оригинальных теоретических и/или практических исследованиях и/или расширенные версии ранее опубликованных статей, представленных на конференциях. Статьи для публикации отбираются путем двойного слепого рецензирования, которое гарантирует оригинальность, актуальность и удобочитаемость. Цель состоит не только в поддержании высокого качества публикуемых статей, но и в обеспечении своевременного, тщательного и соответствующего процесса рецензирования.

Редакционная политика журнала «Военно-технический вестник» основана на рекомендациях Комитета по этике научных публикаций (COPE Core Practices), а также на лучшей практике в научно-издательской деятельности. «Военно-технический вестник» является членом COPE со 2 мая 2018 года.

Министерством образования, науки и технологического развития Республики Сербия утверждена 23 декабря 2021 г. категоризация журнала «Военно-технический вестник» за 2021 год:

– Область компьютерные науки:
высококачественный национальный журнал **(M52)**,

– **Область электроники, телекоммуникаций и информационных технологий:** высококачественный национальный журнал **(M52)**,
– **Область машиностроения:** ведущий журнал государственного значения **(M51)**,
– **Область материалов и химической технологии:** ведущий журнал государственного значения **(M51)**.

С информацией относительно категоризации за 2021 год можно ознакомиться на странице сайта «Военно-технического вестника» *Категоризация Вестника*.

Более подробную информацию можно найти на сайте Министерства образования, науки и технологического развития Республики Сербия.

С информацией о категоризации можно ознакомиться и на сайте КОБСОН (Консорциум библиотек Республики Сербия по вопросам объединения закупок).

Категоризация Вестника проведена согласно Регламенту о категоризации и ранжировании научных журналов Министерства образования, науки и технологического развития Республики Сербия («Службени гласник РС», № 159/20)

Журнал соответствует стандартам Сербского индекса научного цитирования (СЦИндекс/SCIndeks) - наукометрической базы данных научных журналов Республики Сербия, Научно-информационного система Redalyc, а также Российского индекса научного цитирования (РИНЦ). Журнал постоянно подвергается мониторингу и оценивается количественными наукометрическими показателями отражающими его научную ценность.

С информацией об индексировании можно ознакомиться на странице сайта журнала *Индексирование Вестника*.

«Военно-технический вестник» обеспечивает читателям возможность открытого доступа, в соответствии с положениями об авторских правах, утверждёнными Creative Commons (CC BY). С инструкцией об авторских правах можно ознакомиться на странице *Авторские права и политика самоархивирования*, перейдя по ссылке http://www.vtg.mod.gov.rs/index-ru.html.

Рукописи статей направляются в редакцию журнала с использованием online системы ASSISTANT, запущенной Центром поддержи развития образования и науки (ЦПРОН).

Регистрация в системе и оформление прав доступа выполняется по адресу http://www.vtg.mod.gov.rs/index-ru.html, через страницу *ASSISTANT* или *СЦИНДЕКС* (aseestant.ceon.rs/index.php/vtg).

С инструкцией по регистрации и правам доступа можно ознакомиться по адресу http://www.vtg.mod.gov.rs/index-ru.html, на странице *Инструкция по ASSISTANT*.

Все авторы, предоставляющие свои рукописи для публикации в редакцию журнала «Военно-технический вестник» должны пройти предварительную регистрацию в реестре ORCID (Open Researcher and Contributor ID). Эта процедура осуществляется в соответствии с инструкцией, размещенной на странице сайта *Регистрация в реестре ORCID для присвоения идентификационного кода*.

«Военно-технический вестник» публикует статьи на английском языке (Arial, шрифт 11 pt, пробел Single).

Процесс подготовки, написания и редактирования статьи должен осуществляться в соответствии с принципами *Этического кодекса* (http://www.vtg.mod.gov.rs/etichyeskiy-kodyeks.html).

Статья должна содержать резюме с ключевыми словами, введение (цель исследования), основную часть (соответствующий обзор представительного исследования в данной области, четкое изложение научной новизны в представленном исследовании, соответствующую теоретическую основу, один или несколько примеров для демонстрирования и обсуждения представленных тезисов), заключение и список литературы (без нумерации заголовков и подзаголовков). Объем статьи не должен превышать один авторский лист (16 страниц формата А4 с одинарным интервалом, максимум до 24 страниц, включая ссылки и приложения).

Статья должна быть набрана на компьютере с использованием специально подготовленного редакцией макета, который можно скачать на странице сайта *Правила и образец составления статьи*.

Заголовок

Заголовок должен отражать тему статьи. В интересах журнала и автора необходимо использовать слова и словосочетания, удобные для индексации и поиска. Если такие слова не содержатся в заголовке, то желательно их добавить в подзаголовок.

Текущий заголовок

Текущий заголовок пишется в титуле каждой страницы статьи с целью упрощения процесса идентификации, в первую очередь копий статьей в электронном виде. Заголовок содержит в себе фамилию и инициал имени автора (в случае если авторов несколько, остальные обозначаются с «et al.» или «и др.»), название работы и журнала (год, том, выпуск, начальная и заключительная страница). Заголовок статьи и название журнала могут быть приведены в сокращенном виде.

ФИО автора

Приводятся полная фамилия и полное имя (всех) авторов. Желательно, чтобы были указаны инициалы отчеств авторов. Фамилия и имя авторов из Республики Сербия всегда пишутся в оригинальном виде (с сербскими диакритическими знаками), независимо от языка, на котором написана работа.

Наименование учреждения автора (аффилиация)

Приводится полное (официальное) наименование и местонахождение учреждения, в котором работает автор, а также наименование учреждения, в котором автор провёл исследование. В случае организаций со сложной структурой приводится их иерархическая соподчинённость (напр. Военная академия, кафедра военных электронных систем, г. Белград). По крайней мере, одна из организаций в иерархии должна иметь статус юридического лица. В случае если указано несколько авторов, и если некоторые из них работают в одном учреждении, нужно отдельными обозначениями или каким-либо другим способом указать в каком из приведённых учреждений работает каждый из авторов. Аффилиация пишется непосредственно после ФИО автора. Должность и специальность по диплому не указываются.

Контактные данные

Электронный адрес автора указываются рядом с его именем на первой страницы статьи.

Категория (тип) статьи

Категоризация статьей является обязанностью редакции и имеет особое значение. Категорию статьи могут предлагать рецензенты и члены редакции, т.е. редакторы рубрик, но ответственность за категоризацию несет исключительно главный редактор. Статьи в журнале распределяются по следующим категориям:

Научные статьи:

– оригинальная научная статья (работа, в которой приводятся ранее неопубликованные результаты собственных исследований, полученных научным методом);

– обзорная статья (работа, содержащая оригинальный, детальный и критический обзор исследуемой проблемы или области, в который автор внёс определённый вклад, видимый на основе автоцитат);

– краткое сообщение (оригинальная научная работа полного формата, но меньшего объёма или имеющая предварительный характер);

– научная критическая статья (дискуссия-полемика на определённую научную тему, основанная исключительно на научной аргументации) и научный комментарий.

Однако, в некоторых областях знаний научная работа в журнале может иметь форму монографического исследования, а также критического обсуждения научного материала (историко-архивного, лексикографического, библиографического, обзора данных и т.п.) – до сих пор неизвестного или недостаточно доступного для научных исследований. Работы, классифицированные в качестве научных, должны иметь, по меньшей мере, две положительные рецензии.

В случае если в журнале объявляются и приложения, не имеющие научный характер, научные статьи должны быть сгруппированы и четко выделены в первой части номера.

Профессиональные статьи:

– профессиональная работа (приложения, в которых предлагаются опыты, полезные для совершенствования профессиональной практики, но которые не должны в обязательном порядке быть обоснованы на научном методе);

– информативное приложение (передовая статья, комментарий и т.п.);

– обзор (книги, компьютерной программы, случая, научного события и т.п.).

Объем кратких сообщений составляет 4-7 страниц, исследовательские статьи и тематические исследования с проблемно-ситуационным анализом − 10-14 страниц, однако объем обзорных статей может быть больше. Ограничения по количеству страниц не являются строгими, следовательно при соответствующем обосновании предоставленные работы могут быть длиннее или короче. В случае подачи расширенных версий ранее опубликованных докладов, представленных на конференции, редакция проверит было ли добавлено достаточно новых материалов для того, чтобы статья соответствовала стандартам журнала и условиям рецензирования. Добавленный материал должен быть новым, неопубликованным ранее. Новые результаты приветствуются, но не являются обязательным условием; однако ключевые тезисы, примеры, разработки и пр. должны быть более подробно представлены в статье по сравнению с первичным докладом на конфереции.

Язык работы

Статья должна быть написана на английском языке.

Текст должен быть в лингвистическом и стилистическом смысле упорядочен, систематизирован, без сокращений (за исключением стандартных). Все физические величины должны соответствовать Международной системе единиц измерения – СИ. Очередность формул обозначается порядковыми номерами, проставляемыми с правой стороны в круглых скобках.

Резюме

Резюме является кратким информативным обзором содержания статьи, обеспечивающим читателю быстроту и точность оценки её релевантности. В интересах редакции и авторов, чтобы резюме содержало термины, часто используемые для индексирования и поиска статьей. Составными частями резюме являются введение/цель исследования, методы, результаты и выводы. В резюме должно быть от 100 до 250 слов, и оно должно находится между титулами (заголовок, ФИО авторов и др.) и ключевыми словами, за которыми следует текст статьи.

Ключевые слова

Ключевыми словами являются термины или фразы, адекватно представляющие содержание статьи, необходимые для индексирования и поиска. Ключевые слова необходимо выбирать, опираясь при этом на какой-либо международный источник (регистр, словарь, тезаурус), наиболее используемый внутри данной научной области. Число ключевых слов не может превышать 10. В интересах редакции и авторов, чтобы частота их встречи в статье была как можно большей. В статье они пишутся непосредственно после резюме.

Программа ASSISTANT предоставляет возможность использования сервиса KWASS, автоматически фиксирующего ключевые слова из источников/словарей по выбору автора/редактора.

Дата получения статьи

Дата, когда редакция получила статью; дата, когда редакция окончательно приняла статью к публикации; а также дата, когда были предоставлены необходимые исправления рукописи, приводятся в хронологическом порядке, как правило, в конце статьи.

Выражение благодарности

Наименование и номер проекта, т.е. название программы благодаря которой статья возникла, совместно с наименованием учреждения, которое финансировало проект или программу, приводятся в отдельном примечании, как правило, внизу первой страницы статьи.

Предыдущие версии работы

В случае если статья в предыдущей версии была изложена устно (под одинаковым или похожим названием, например, в виде доклада на научной конференции), сведения об этом должны быть указаны в отдельном примечании, как правило, внизу первой страницы статьи. Работа, которая уже была опубликована в каком-либо из журналов, не может быть напечатана в «Военно-техническом вестнике» ни под похожим названием, ни в изменённом виде.

Нумерация и название таблиц и графиков

Желательно, чтобы нумерация и название таблиц и графиков были исполнены на двух языках (на языке оригинала и на английском). Таблицы подписываются таким же способом как и текст и обозначаются порядковым номером с верхней стороны. Фотографии и рисунки должны быть понятны, наглядны и удобны для репродукции. Рисунки необходимо делать в программах Word или Corel. Фотографии и рисунки надо поставить на желаемое место в тексте. Для создания изображений и графиков использование функции снимка с экрана (скриншота) не допускается. В самом тексте статьи рекомендуется применение изображений и графиков, обработанных такими компьютерными программами, как: Excel, Matlab, Origin, SigmaPlot и др.

Ссылки (цитирование) в тексте

Оформление ссылок на источники в рамках статьи должно быть однообразным. «Военно-технический вестник» для оформления ссылок, цитат и списка использованной литературы применяет Гарвардскую систему (Harvard Referencing System, Harvard Style Manual). В тексте в скобках приводится фамилия цитируемого автора (или фамилия первого автора, если авторов несколько), год издания и по необходимости номер страницы. Например: (Petrović, 2010, pp.10-20). Рекомендации о способе цитирования размещены на странице сайта *Инструкция по использованию Гарвардского стиля*. При оформлении ссылок, цитат и списка использованной литературы необходимо придерживаться установленных норм. Программа ASSISTANT предоставляет при цитировании возможность использования сервиса CiteMatcher, фиксирующего пропущенные цитаты в работе и в списке литературы.

Примечания (сноски)

Примечания (сноски) к тексту указываются внизу страницы, к которой они относятся. Примечания могут содержать менее важные детали, дополнительные объяснения, указания об использованных источниках (напр. научном материале, справочниках), но не могут быть заменой процедуры цитирования литературы.

Литература (референции)

Цитированной литературой охватываются, как правило, такие библиографические источники как статьи, монографии и т.п. Вся используемая литература в виде референций размещается в отдельном разделе статьи.

Названия литературных источников не переводятся на язык работы.

«Военно-технический вестник» для оформления списка использованной литературы применяет Гарвардскую систему (Harvard Style Manual). В списке литературы источники указываются в алфавитном порядке фамилий авторов или редакторов. Рекомендации о способе цитирования размещены на странице сайта *Инструкция по использованию Гарвардского стиля*. При оформлении списка использованной литературы необходимо придерживаться установленных норм.

При оформлении списка литературы программа ASSISTANT предоставляет возможность использования сервиса RefFormatter, осуществляющего контроль оформления списка литературы в соответствии со стандартами Гарвардского стиля.

Нестандартное, неполное и непоследовательное приведение литературы в системах оценки журнала считается достаточной причиной для оспаривания научного статуса журнала.

Авторское заявление

Авторское заявление предоставляется вместе со статьей, в нем авторы заявляют о своем личном вкладе в написание статьи. В заявлении авторы подтверждают, что статья написана в соответствии с *Приглашением и инструкциями для авторов*, а также с *Кодексом профессиональной этики журнала*.

Все рукописи статей подлежат профессиональному рецензированию.

Список рецензентов журнала «Военно-технический вестник» размещён на странице сайта *Список рецензентов*. Процесс рецензирования описан в разделе *Правила рецензирования*.

Редакция

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CALL FOR PAPERS AND ARTICLE FORMATTING INSTRUCTIONS

The instructions to authors about the article preparation for publication in the Military Technical Courier are based on the Regulations on categorization and ranking of scientific journals of the Ministry of Education, Science and Technological Development of the Republic of Serbia (Official Gazette of the Republic of Serbia, No 159/20). This Regulations aims at improving the quality of national journals and raising the level of their compliance with the international system of scientific information exchange.

The Military Technical Courier / Vojnotehnički glasnik (www.vtg.mod.gov.rs/index-e.html, втг.мо.упр.срб, ISSN 0042-8469 – print issue, e-ISSN 2217-4753 – online, UDC 623+355/359, DOI: 10.5937/VojnotehnickiGlasnik; https://doi.org/10.5937/VojnotehnickiGlasnik), is an international peer-reviewed scientific journal.

The owners of the journal are the Ministry of Defence of the Republic of Serbia and the Serbian Armed Forces. The publisher and financier of the Military Technical Courier is the University of Defence in Belgrade (Military Academy).

The program of the journal is based on the annual classification of journals performed by a relevant Ministry as well as on its indexing in international indexing databases.

The journal covers scientific and professional fields within the educational-scientific field of **Natural-Mathematical Sciences**, as well as within the educational-scientific field of **Technical-Technological Sciences**, and especially the field of **defense sciences and technologies**. It publishes theoretical and practical achievements leading to professional development of all members of Serbian, regional and international academic communities as well as members of the military and ministries of defence in particular. It publishes papers with balanced coverage of analytical, experimental, and applied research as well as numerical simulations from various disciplines. The material published is of high quality and relevance, written in a manner that makes it accessible to a wider readership. The journal welcomes papers reporting original theoretical and/or practice-oriented research as well as extended versions of already published conference papers. Manuscripts for publication are selected through a double-blind peer-review process to validate their originality, relevance, and readability. This being so, the objective is not only to keep the quality of published papers high but also to provide a timely, thorough, and balanced review process.

The editorial policy of the Military Technical Courier is based on the COPE Core Practices and the journal articles are consistent with accepted best practices in their subject areas. As of 2 May 2018, the Military Technical Courier is a member of COPE (Committee on Publication Ethics).

The Ministry of Education, Science and Technological Development of the Republic of Serbia classified the Military Technical Courier for the year 2021, on December 23, 2021

– on the list of periodicals for computer sciences,
category: quality national journal **(M52)**,

– on the list of periodicals for electronics, telecommunications and IT,
category: quality national journal **(M52)**,

– on the list of periodicals for mechanical engineering,
category: reputed national journal **(M51)**,

– on the list of periodicals for materials and chemical technology,
category: reputed national journal **(M51)**.

The approved lists of national periodicals for the year 2021 can be viewed on the website of the Military Technical Courier, page *Journal categorization*.

More detailed information can be found on the website of the Ministry of Education, Science and Technological Development of the Republic of Serbia.

The information on the categorization can be also found on the website of KOBSON (Consortium of Libraries of Serbia for Unified Acquisition).

The periodical is categorized in compliance with the Regulations on categorization and ranking of scientific journals of the Ministry of Education, Science and Technological Development of the Republic of Serbia (Official Gazette of the Republic of Serbia, No 159/20). More detailed information can be found on the website of the Ministry of Education, Science and Technological Development.

The journal is in the Serbian Citation Index – SCIndex (data base of national scientific journals), in the Scientific Information System Redalyc, and in the Russian Index of Science Citation/Российский индекс научного цитирования (RINC/РИНЦ) and is constantly monitored depending on the impact within the bases themselves. More detailed information can be viewed on the website of the Military Technical Courier, page *Journal indexing*.

Military Technical Courier enables open access and applies the Creative Commons Attribution (CC BY) licence provisions on copyright. The copyright details can be found on the *Copyright notice and Self-archiving policy* page of the journal's website.

Manuscripts are submitted online, through the electronic editing system ASSISTANT, developed by the Center for Evaluation in Education and Science – CEON.

The access and the registration are through the Military Technical Courier site http://www.vtg.mod.gov.rs/index-e.html, on the page *ASSISTANT* or the page *SCINDEKS* or directly through the link (aseestant.ceon.rs/index.php/vtg).

The detailed instructions about the registration for the service are on the website http://www.vtg.mod.gov.rs/index-e.html, on the page *Instructions for ASSISTANT*.

All authors submitting a manuscript for publishing in the Military Technical Courier should register for an ORCID ID following the instructions on the web page *Registration for an ORCID identifier*.

The Military Technical Courier publishes articles in English, using Arial and a font size of 11pt with Single Spacing.

The procedures of article preparation, writing and editing should be in accordance with the *Publication ethics statement* (http://www.vtg.mod.gov.rs/publication-ethics-statement.html).

The article should contain an abstract with keywords, introduction (motivation for the work), body (adequate overview of the representative work in the field, a clear statement of the novelty in the presented research, suitable theoretical background, one or more examples to demonstrate and discuss the presented ideas), conclusion, and references (without heading and subheading enumeration). The article length should not normally exceed 16 pages of the A4 paper format with single spacing, up to a maximum of 24 pages with references and supplementary material included.

The article should be formatted following the instructions in the Article Form which can be downloaded from website page *Article form*.

Title

The title should be informative. It is in both Journal’s and author’s best interest to use terms suitable for indexing and word search. If there are no such terms in the title, the author is strongly advised to add a subtitle.

Letterhead title

The letterhead title is given at a top of each page for easier identification of article copies in an electronic form in particular. It contains the author’s surname and first name initial (for multiple authors add “et al”), article title, journal title and collation (year, volume, issue, first and last page). The journal and article titles can be given in a shortened form.

Author’s name

Full name(s) of author(s) should be used. It is advisable to give the middle initial. Names are given in their original form (with diacritic signs if in Serbian).

Author’s affiliation

The full official name and seat of the author’s affiliation is given, possibly with the name of the institution where the research was carried out. For organizations with complex structures, give the whole hierarchy (for example, University of Defence in Belgrade, Military Academy, Department for Military Electronic Systems). At least one organization in the hierarchy must be a legal entity. When some of multiple authors have the same affiliation, it must be clearly stated, by special signs or in other way, which department exactly they are affiliated with. The affiliation follows the author’s name. The function and title are not given.

Contact details

The postal addresses or the e-mail addresses of the authors are given in the first page.

Type of articles

Classification of articles is a duty of the editorial staff and is of special importance. Referees and the members of the editorial staff, or section editors, can propose a category, but the editor-in-chief has the sole responsibility for their classification.

Journal articles are classified as follows:

Scientific articles:

– Original scientific papers (giving the previously unpublished results of the author’s own research based on scientific methods);

– Review papers (giving an original, detailed and critical view of a research problem or an area to which the author has made a contribution demonstrated by self-citation);

– Short communications or Preliminary communications (original scientific full papers but shorter or of a preliminary character);

– Scientific commentaries or discussions (discussions on a particular scientific topic, based exclusively on scientific argumentation) and opinion pieces.

Exceptionally, in particular areas, a scientific paper in the Journal can be in a form of a monograph or a critical edition of scientific data (historical, archival, lexicographic, bibliographic, data survey, etc.) which were unknown or hardly accessible for scientific research.

Papers classified as scientific must have at least two positive reviews.

If the journal contains non-scientific contributions as well, the section with scientific papers should be clearly denoted in the first part of the Journal.

Professional articles:

– Professional papers (contributions offering experience useful for improvement of professional practice but not necessarily based on scientific methods);

– Informative contributions (editorial, commentary, etc.);

– Reviews (of a book, software, case study, scientific event, etc.)

Short communications are usually 4-7 pages long, research articles and case studies 10-14 pages, while reviews can be longer. Page number limits are not strict and, with appropriate reasoning, submitted manuscripts can also be longer or shorter. If extended versions of previously published conference papers are submitted, Editors will check if sufficient new material has been added to meet the journal standards and to qualify such manuscripts for the review process. The added material must not have been previously published. New results are desired but not necessarily required; however, submissions should contain expansions of key ideas, examples, elaborations, etc. of conference papers.

Language

The language of the article should be in English.

The grammar and style of the article should be of good quality. The systematized text should be without abbreviations (except standard ones). All measurements must be in SI units. The sequence of formulae is denoted in Arabic numerals in parentheses on the right-hand side.

Abstract and summary

An abstract is a concise informative presentation of the article content for fast and accurate evaluation of its relevance. It contains the terms often used for indexing and article search. A 100- to 250-word abstract has the following parts: introduction/purpose of the research, methods, results and conclusion.

Keywords

Keywords are terms or phrases showing adequately the article content for indexing and search purposes. They should be allocated heaving in mind widely accepted international sources (index, dictionary or thesaurus), such as the Web of Science keyword list for science in general. The higher their usage frequency is, the better. Up to 10 keywords immediately follow the abstract and the summary, in respective languages.

For this purpose, the ASSISTANT system uses a special tool KWASS for the automatic extraction of key words from disciplinary thesauruses/dictionaries by choice and the routine for their selection, i.e. acceptance or rejection by author and/or editor.

Article acceptance date

The date of the reception of the article, the dates of submitted corrections in the manuscript (optional) and the date when the Editorial Board accepted the article for publication are all given in a chronological order at the end of the article.

Acknowledgements

The name and the number of the project or programme within which the article was realised is given in a separate note at the bottom of the first page together with the name of the institution which financially supported the project or programme.

Article preliminary version

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